PREFERENCE UNCERTAINTY:
ELICITATION AND IMPLICATIONS

Liu Shi

Institute for
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Preference Uncertainty: Elicitation and Implications

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Author: Liu Shi
Title: Preference Uncertainty: Elicitation and Implications

Radboud Dissertations Series
ISSN: 2950-2772 (Online); 2950-2780 (Print)

Published by RADBOUD UNIVERSITY PRESS
Postbus 9100, 6500 HA Nijmegen, The Netherlands
www.radbouduniversitypress.nl

Design: Proefschrift AIO/[Guus Gijben]
Cover image: Guus Gijben
Printing: DPN Rikken/Pumbo

ISBN: 9789465150635
DOI: 10.54195/9789465150635
Free download at: https://doi.org/10.54195/9789465150635

This work was funded by China Scholarship Council (CSC, Grant No.201906280055).

Preference Uncertainty: Elicitation and Implications

Proefschrift ter verkrijging van de graad van doctor
aan de Radboud Universiteit Nijmegen
op gezag van de rector magnificus prof. dr. J.M. Sanders,
volgens besluit van het college voor promoties
in het openbaar te verdedigen op

dinsdag 20 mei 2025
om 10.30 uur precies

door

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geboren op 16 maart 1995
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Preference Uncertainty: Elicitation and Implications

Dissertation to obtain the degree of doctor
from Radboud University Nijmegen
on the authority of the Rector Magnificus prof. dr. J.M. Sanders,
according to the decision of the Doctorate Board
to be defended in public on

Tuesday, May 20, 2025
at 10.30 am

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Chapter 1

Introduction

1.1 Uncertainty in preferences

A fundamental postulate of standard economics is that preferences are complete, deterministic, and stable across different elicitation contexts. However, in most decisions, individuals need to trade off conflicting objectives, e.g., risk vs. return, time vs. reward, price vs. quality, and efficiency vs. equality. These trade-offs are difficult, and thus individuals may not always be certain about what their most preferred option is. Consistent with this idea, mounting evidence suggests that individuals often exhibit uncertainty in their preferences, consciously or subconsciously. They may change their decisions frequently, leading to stochastic choices and potential reversals in repeated choices (Loomes and Pogrebna, 2017; Agranov and Ortoleva, 2017). Or, when evaluating an asset, they may report a range of possible values instead of a precise point value (Butler and Loomes, 2007, 2011; Cubitt et al., 2015; Agranov and Ortoleva, ress). They may also postpone their decisions at a cost even if the delay does not give access to more information (Danan and Ziegelmeyer, 2006; Gerasimou, 2017; Costa-Gomes et al., 2022). Such uncertainty in preference poses serious challenges not only to economic theories but also to the design of economic policies.

Due to the significance of uncertainty in preferences in economic analyses, there have been substantial efforts to account for it theoretically, including random utility (Luce and Suppes, 1965; Eliashberg and Hauser, 1985; Gul and Pesendorfer, 2006; Blavatskyy, 2009), preference incompleteness (Levi, 1990; Ok et al., 2002; Dubra et al., 2004; Ok et al., 2012), preference imprecision (MacCrimmon and Smith, 1986; Butler and Loomes, 2007, 2011; Cubitt et al., 2015) and the hedging of preference uncertainty (Fudenberg et al., 2015). Theoretical models based on uncertainty in preferences have made significant progress in explaining important economic anomalies, e.g., the WTA-WTP gap (Dubourg et al., 1994), preference reversals (Butler and Loomes, 2007; Blavatskyy, 2009), insensitivity to variation in probabilities (Enke and Graeber, 2023), common ratio effect (McGranaghan et al., 2024) and many other violations of standard decision theory (Butler and Loomes, 2011; Cubitt et al., 2015). However, theoretical progress has not been matched with empirical development. One of the main challenges in investigating uncertainty in preferences is to reveal and measure the preference uncertainty with an incentivized measure.

This dissertation aims to contribute to the empirical investigation of preference uncertainty by proposing novel methods to reveal and measure preference uncertainty with incentives. We then apply those methods in economic experiments and examine whether preference uncertainty has the potential to explain some important economic anomalies, such as preference reversal and present bias.

1.2 Experimental method

This dissertation primarily relies on experimental methods. Since the last century, experiments have become a vital tool in economic research. By creating controlled environments where variables can be systematically manipulated, economic experiments enable researchers to observe how individuals make decisions under specific conditions, and the causal relationship between a manipulation and behavior.

The existing experimental literature on eliciting preference uncertainty often relies on non-incentivized methods. For example, Butler and Loomes (2007) asked subjects to report their confidence level in a binary choice between options A and B using one of the following statements: "I definitely prefer A," "I think I prefer A but I'm not sure,", "I definitely prefer B," or "I think I prefer B but I'm not sure." Similarly, Cubitt et al. (2015) adapted the traditional multiple price list by introducing a third option, "I'm not sure about my preference," for subjects to indicate their preference uncertainty. To demonstrate the economic relevance of cognitive uncertainty, Enke and Graeber (2023) elicited cognitive uncertainty by requesting subjects to choose a percentage between 0% and 100% in 5% increments to report their confidence in their choices.

However, incentivized methods have been largely missing. While non-incentivized methods can provide useful information, incentivized methods have the advantages of encouraging individuals to think more clearly about tasks and reducing noise. The lack of incentivized methods for revealing preference uncertainty is due to several reasons. First, the revealed preference typically assumes subjects have a complete preference once they have made a choice, ignoring whether the decision is made with confidence or not. Second, even among studies of preference uncertainty, there is no consensus on what preference uncertainty is and the mechanism underlying it. Thus, we have little theoretical guidance in connecting a behavioral measure to preference uncertainty. Probably because of these reasons, Butler and Loomes (2011, p. 516) claim that they doubt that "such a mechanism can be devised – at least, not in a form simple and transparent enough to work without creating additional uncertainty."

In this dissertation, we propose novel incentivized methods to elicit preference uncertainty. In one method, we offer subjects, besides the standard binary choices, the randomization option, in which the computer randomly chooses one of the two options. Individuals can either pay a small cost (€0.1 in the experiment) to choose one of the two options, or choose the randomization option for free. In another method, we adapt the classical multiple price list by replacing the binary choice in each row with a randomization slider. In each row, instead of choosing one of the two options, subjects can use the slider to determine the probability (from 0% to 100%, in increments of 10%) of receiving either option. This allows individuals to express their uncertainty by delegating their choices to the computer. We apply each method in an experiment and use the measured preference uncertainty to explain preference reversal in one study and present bias in another study.

1.3 Contribution and overview

This dissertation consists of three papers, each exploring revealed preference uncertainty from a different perspective. In the first two papers, we investigate whether stochastic choices caused by preference uncertainty can fully explain the preference reversal phenomenon. Specifically, in the first paper, we implemented our novel method to measure the uncertainty in preferences in a preference reversal experiment and studied whether preference reversal could be explained after allowing for stochastic choices under conscious uncertainty models. The findings revealed that the majority of subjects exhibited stochastic choice, and further that their choices were significantly affected by the elicitation procedures. Building on the findings in the first paper, in the second paper, we study whether stochastic choice raised from noise in preference as in random utility models can explain the preference reversal phenomenon. Using two independent data sets (Loomes and Pogrebna, 2017; Shi et al., 2024), we performed our analysis based on a range of random utility functions: the Logit stochastic function, the stochastic function in Blavatskyy (2009) with homoscedastic random errors, and the stochastic function in Blavatskyy (2009) with heteroscedastic random errors. The results indicate that stochastic choice arising from noise in preference alone is insufficient to explain the preference reversal. We find robust evidence that choices are not only stochastic but also sensitive to the elicitation approach. In the third paper, we studied whether uncertainty in valuing timed rewards, combined with caution toward this uncertainty, can lead to present bias. In two pre-registered experiments, we use our novel method to elicit the uncertainty in valuing the timed rewards, finding that valuation uncertainty plays an important role in explaining present bias and its canonical signature, front-end delay, in intertemporal valuations.

1.3.1 Chapter 2

In Chapter 2, we proposed a novel method to examine the prevalence and implications of stochastic choice and implemented it in a preference reversal experiment. In particular, we focus on conscious stochastic choices. When individuals are aware of their inability to make a definitive choice, they may intentionally exhibit stochastic choices. In the experiment, we adapt valuation choices between one of the bets in preference reversal experiments (P-bet or $-bet) and a varying reference option by adding a randomization option.[1] Subjects could either pay a small cost to select a specific option or opt for a free randomization choice where a computer randomly selects an option. We demonstrate that this method, together with repeating the direct choice between the P-bet and the $-bet nine times (Loomes and Pogrebna, 2017), allows us to consider the three major models of conscious stochastic choice (incomplete preference, preference imprecision, and hedging of preference uncertainty).

In the analysis, we separately estimate stochastic choice functions from repeated direct choices and the randomization range of valuations elicited through our novel method. With the estimated stochastic functions, we examine the prevalence of conscious stochastic choice among subjects and check whether stochastic choice is sensitive to the different elicitation procedures (direct choices vs indirect valuations). We further evaluate quantitatively the explanatory power of stochastic choice for preference reversals by noting that, when subjects' direct choices and indirect valuations are potentially stochastic, an experimental outcome would be a random realization of the stochastic process. We use the estimated stochastic functions to perform one million simulations, as if having these subjects participating in the experiment one million times, and compare the simulated preference reversal patterns with the actual preference reversal patterns in the experiment. If preference reversals arise from conscious stochastic choice alone, the actual choice patterns in the experiment should not differ significantly from the simulated patterns.

Our experimental results suggest prevalent stochastic choice among subjects and substantial preference reversals comparable to previous studies. However, the anal-

[1] In a preference reversal experiment, the P-bet is a safer lottery with a lower payoff, while the $-bet is a riskier lottery with a higher payoff. In our experiment, the P-bet offers a 70% chance of winning ¥24 and nothing otherwise, and the $-bet offers a 25% chance of winning ¥80 and nothing otherwise.

ysis of our experimental results also clearly shows that the estimated stochastic functions depend on elicitation procedures (the direct choices or the valuation choices), the type of bets (the P-bet or the $-bet), and the type of valuation (certainty equivalents or probability equivalents). Moreover, the simulated choice patterns based on one consistent stochastic function for both the direct choice and the indirect valuations fail to capture the magnitude of preference reversals and the asymmetry between the standard preference reversals and the non-standard preference reversals in the experiment. In contrast, when using R-ranges to estimate the probability of valuing the P-bet higher than the $-bet and using the nine-repeated direct choices to estimate the probability of choosing the P-bet separately, the simulated proportions capture the magnitude and the asymmetry of preference reversals well. We conclude that subjects' choices were not only stochastic but also procedure-dependent, and that both are important in explaining preference reversals.

1.3.2 Chapter 3

In Chapter 3, we took a different perspective to examine the role of valuation uncertainty in economic anomalies, in particular preference reversals. Specifically, we mainly focus on the stochastic choice arising from noises in preferences and base the analysis on a wide range of random utility models. Using two independent data sets (Loomes and Pogrebna, 2017; Shi et al., 2024), we estimate a stochastic choice function from the direct choices and valuation choices for each subject and use it to estimate the probabilities of observing different preference reversal patterns.

In the analysis, we consider the following random utility functions: the Logit stochastic function, the stochastic function in Blavatskyy (2009) with homoscedastic random errors, and the stochastic function in Blavatskyy (2009) with heteroscedastic random errors. For each stochastic function form, we estimate a consistent individual-level stochastic function from the choices made in the direct choice and the valuation choice tasks. With the estimated stochastic functions, we compute the aggregate-level probability distribution of preference reversal patterns by simulations.

By comparing the estimated preference reversal pattern with the actual preference reversal pattern, our results suggest that stochastic choices alone are insufficient to explain the preference reversal phenomenon. Although a considerable proportion of subjects exhibited stochastic behaviors in repeated choices, one consistent stochastic function cannot simultaneously estimate the decisions in the direct choice and the valuation choice tasks. The simulated proportions of preference reversal patterns are significantly different from the actual preference reversal proportions. More specifically, the stochastic choices under a consistent stochastic function cannot capture the preference reversal phenomenon in terms of magnitude and the asymmetry between standard preference reversals and nonstandard preference reversals.

1.3.3 Chapter 4

In Chapter 4, we investigate whether uncertainty in valuing delayed rewards, coupled with caution toward this uncertainty, could result in present bias. While the valuation of immediate payment is certain, determining the present equivalent of a delayed reward is complex, and individuals are rarely fully certain about their valuation of the delayed reward. When individuals are uncertain about the present equivalent of a delayed reward, they may select a value at the lower end of the range of possible values out of caution as the single present equivalent of the delayed reward. This leads to "extra" discounting of the delayed reward, which causes the present bias effect.

In this chapter, we formalize a behavioral model of intertemporal valuation, in which valuation uncertainty and caution lead to an extra discount of the delayed reward, however short the delay is. We show that present bias can arise from the discontinuous jump in valuation uncertainty as soon as a delay is introduced. Following our theoretical analysis, we conducted two pre-registered experiments. The experiments mainly consisted of two tasks. The first task elicited the monetary equivalent of timed rewards in terms of the monetary payment received at an earlier date via multiple price lists. The second task elicited the randomization range using our novel method. We presented subjects with the same price lists as the first task, instead of choosing the preferred option, subjects were asked to choose a randomization probability according to which they would receive the fixed delayed reward or the earlier monetary payment in each row. We had three treatments: the baseline treatment (eliciting present monetary equivalents and R-ranges for €10 received in 4 and 24 weeks), the coupon treatment (assessing present monetary equivalents and R-ranges for €10 Amazon coupons received today, in 4 weeks, and in 24 weeks), and the front-end delay treatment (determining the present value for €10 received in 4 weeks and the 4-week value for €10 received in 8 weeks, along with corresponding R-ranges).

Our findings suggest the crucial role of valuation uncertainty and caution in present bias: 1) When the delayed rewards were monetary, subjects exhibited significantly stronger present bias with greater valuation uncertainty. 2) For the non-monetary reward of €10 Amazon coupon, subjects exhibited much weaker present bias because valuation uncertainty existed even for the present Amazon coupon. 3) subjects without valuation uncertainty had a significantly weaker front-end delay effect than those who exhibited valuation uncertainty. Additionally, we found that when using the upper bound of an R-range, which excludes valuation uncertainty under our model, subjects exhibited significantly weaker present bias.

Chapter 2

Consciously stochastic in preference reversals

Stochastic choice, the act of choosing differently in repeated decisions, can be a conscious decision made by individuals who are aware of their inability to make a definitive choice. To examine the prevalence and implications of conscious stochastic choice, we developed a novel method and implemented it in a preference reversal experiment: in each valuation choice between the bet and a varying reference option, subjects could either pay a small cost to select a specific option or opt for a free randomization choice where a computer randomly selects an option. Our findings revealed that the majority of subjects exhibited conscious stochastic choice, and further that their choices were significantly affected by the elicitation procedures.

This chapter has been published as: Shi, L., Qiu, J., Li, J., and Bohn, F. (2024). Consciously stochastic in preference reversals. Journal of Risk and Uncertainty, 68:255–297.

2.1 Introduction

Individuals often make stochastic choices, selecting different options in repeated decisions (Mosteller and Nogee, 1951; Camerer, 1989; Starmer and Sugden, 1989), even when they are informed of the repetitions in advance (Agranov and Ortoleva, 2017). Stochastic choice can be either unconscious or conscious. Unconscious stochastic choice occurs when random preference shocks influence decision-making, as in random (expected) utility models (Luce and Suppes, 1965; Eliashberg and Hauser, 1985; Gul and Pesendorfer, 2006; Apesteguia and Ballester, 2018), where individuals have a clear preference in each decision and are not aware that they are choosing differently. In contrast, conscious stochastic choice can arise from preference incompleteness (Aumann, 1962), preference imprecision (MacCrimmon and Smith, 1986; Dubourg et al., 1994; Butler and Loomes, 2007), or hedging of preference uncertainty (Fudenberg et al., 2015), where individuals are aware that they do not have a clear preference and are intentionally randomizing their choices.[1]

Models of stochastic choice have been extensively studied and applied in various fields. However, previous research has primarily focused on models of unconscious stochastic choices (Harless and Camerer, 1994; Hey and Orme, 1994; Rieskamp et al., 2006), while the study of conscious stochastic choice has been limited. One of the main challenges in investigating conscious stochastic choice is the difficulty of distinguishing between unconscious stochastic choices that are made based on a clear preference (as in random utility models) and conscious stochastic choices that individuals find difficult to make and randomize intentionally.

This paper presents a novel method to address the challenge of distinguishing between conscious and unconscious stochastic choice, and demonstrates its usefulness in one of the most studied anomalies in economics and management: preference reversals (PR). A typical PR experiment involves a pair of lotteries with similar expected payoffs: a P-bet with a more attractive winning probability and a $-bet with a more attractive payoff. PR occurs when individuals choose one bet over the other in the direct choice between the two bets, but place a higher value on the other in the indirect valuation of each bet separately. PR poses a fundamental challenge to the theory of revealed preferences and has important policy implications because it

[1] Stochastic choice can also be explained by deterministic non-expected utility theory (Machina, 1985), such as Rank-dependent expected utility theory (Quiggin, 1982), in which individuals choose randomly not because they are unsure about their preferences, but because they violate the independence axiom (more precisely, the betweenness axiom, Chew, 1989) and have a strict preference for randomization. It is important to recognize, however, that the violation of betweenness may be due to imprecise preferences (Butler and Loomes, 2011).

questions whether preferences can be reliably revealed from choices. While alternative explanations for PR exist, such as regret aversion (Loomes and Sugden, 1983; Loomes et al., 1989, 1991), flaws in some particular experimental designs such as the Becker-DeGroot-Marschak (Becker et al., 1964) mechanism (Holt, 1986; Karni and Safra, 1987; Segal, 1988), or scale compatibility (Tversky et al., 1990a), there has been a persistent interest in explaining PR by stochastic choice. The stochastic choice explanation is attractive because it can account for PR while retaining procedurally invariant preferences, a fundamental postulate of many theories.

Our novel method involves incentivizing individuals to provide an interval of values that they consider comparable to each bet. In each valuation choice between the bet and a varying reference option, subjects could either pay a small cost to select a specific option or opt for a free randomization choice, where the selection is made randomly by the computer. We demonstrate that this method, together with repeating the direct choice between the P-bet and the $-bet nine times (Loomes and Pogrebna, 2017), allows us to consider the three major models of conscious stochastic choice (incomplete preference, preference imprecision, and hedging of preference uncertainty). Our experimental results reveal substantial conscious stochasticity in choices, and further suggest that subjects' stochastic choice differs across the elicitation procedures (direct choice vs indirect valuation).

Specifically, our experiment used the same P-bet and $-bet as in Butler and Loomes (2007), except that the currency was Chinese Yuan (¥) instead of Australian dollar. The experiment consisted of mainly two parts. Part I was a standard PR experiment. Subjects chose directly between the P-bet and the $-bet, which was later repeated eight more times (separated by other choices). They also evaluated the two bets in terms of certainty equivalent (CE) and probability equivalent (PE) via a sequence of binary valuation choices between a fixed bet (either the P-bet or the $-bet) and a varying reference option (a sure payment when eliciting CE and a reference lottery when eliciting PE). In Part II, we elicited subjects' intervals of possible CE and PE for each bet through a series of valuation choices. In each of them subjects faced a fixed bet and a varying reference option. They could 1) pay a small cost (¥0.10) and choose the fixed bet, 2) pay a small cost (¥0.10) and choose the reference option, or 3) choose the randomization option for free, according to which a computer would randomly select one of the two options.

We demonstrate that the intervals of possible values (CE or PE) are captured by the randomization ranges (referred to as R-ranges hereafter): the ranges of sure payments (or the ranges of winning probabilities in the reference lottery) in which subjects choose the randomization option. We further show how our experiment allows us to estimate one stochastic function from the nine-repeated direct choices between the two bets and a separate one from the randomization ranges. This allows us to examine the prevalence of conscious stochastic choice among subjects, and check whether stochastic choice is sensitive to the different elicitation procedures (direct choices vs indirect valuations). Finally, we evaluate quantitatively the explanatory power of stochastic choice for PR by noting that, when subjects' direct choices and indirect valuations are potentially stochastic, an experimental outcome would be a random realization of the stochastic process. We use the estimated stochastic functions to perform one million simulations, as if having these subjects participating in the experiment one million times, and compare the simulated patterns with the actual patterns in the experiment. If PR arises from conscious stochastic choice alone, the actual choice patterns in the experiment should not differ significantly from the simulated patterns.

Our experimental results suggest prevalent stochastic choice among subjects and substantial PR comparable to previous studies. However, the analysis of our experimental results also clearly shows that the estimated stochastic functions depend on elicitation procedures (the direct choices or the valuation choices), the type of bets (the P-bet or the $-bet), and the type of valuation (certainty equivalents or probability equivalents), inconsistent with theoretical predictions. Moreover, the simulated choice patterns based on one consistent stochastic function for both the direct choice and the indirect valuations fail to capture the magnitude of PR and the asymmetry between the standard PR and the non-standard PR in the experiment. In contrast, when using R-ranges to estimate the probability of valuing the P-bet higher than the $-bet and using the nine-repeated direct choices to estimate separately the probability of choosing the P-bet, the simulated proportions capture the magnitude and the asymmetry of PR well. We conclude that subjects' choices were not only stochastic but also procedure dependent, and that both are important in explaining PR.

Our study contributes to the literature in two ways. First, our novel method allows us to expand the experimental investigation of stochastic choice by also considering models of conscious stochastic choice, which have received limited attention compared to studies on unconscious stochastic choice, such as random utility models. In a pioneering study by Agranov and Ortoleva (2017), choices were repeated three times in a row and subjects were explicitly informed of the repetitions in advance.

They found that 71% of their subjects selected different options, and attributed the observed pattern to conscious stochastic choice instead of random expected utility (Gul and Pesendorfer, 2006) or drift diffusion (Ratcliff, 1978). Similar to our design, Cettolin and Riedl (2019) offered subjects a randomization option in addition to standard binary choices to identify incomplete preferences. Their method, while suitable for their research purpose, may have underestimated the extent of conscious stochastic choice. This is because, without the choosing cost like ours, subjects could flip a virtual coin in their head to randomize instead of using the free external randomization option. In contrast, our design incentivizes subjects to use the external randomization option to fully reveal the extent of conscious stochastic choice. Close to our design, Bouacida (2021) elicited the set of "choosable" alternatives by offering subjects reward for keeping additional options and then randomly selecting one for payment. His focus is to identify choice correspondences. Our study is also related to the measurement of preference imprecision: a range of values for which subjects report unsureness about their preferences (Dubourg et al., 1994, 1997; Butler and Loomes, 2007, 2011; Cubitt et al., 2015) or the strength of preferences: "the relative degree of difference between the two options as perceived by the decision maker" (Butler et al., 2014b, p.538).[2] Those studies relied on *unincentivized* self-reports (see Bayrak and Hey, 2020b, for a survey of the recent literature); in contrast, our method is incentivized. Furthermore, we contribute to the understanding of the deliberate stochastic model by Fudenberg et al. (2015). The investigation of this model is not straightforward because the cost function, a crucial element in their model, is not directly observable. We show that eliciting CE or PE randomization ranges offers a convenient way to identify the cost function. Our result shows that the cost parameter of the P-bet is significantly higher than the one of the $-bet, consistent with the interpretation that subjects perceive significantly higher preference uncertainty for the $-bet. This result supports the hypothesis that the cost parameter depends on the elicitation environment, as in the two extensions (item-invariant and menu-invariant) of their working paper version (Fudenberg et al., 2014).

Second, we link preference reversals with a broader class of theories of stochastic choice. Building on the decision field theory (Busemeyer and Townsend, 1993), Johnson and Busemeyer (2005) proposed that valuations and choices are based on the same stochastic process, and the higher variance about the value of the $-bet than that of the P-bet drives PR. Eliaz and Ok (2006) proposed that PR may arise

[2] Alós-Ferrer et al. (2016) and Alós-Ferrer et al. (2021) related stochastic choice to the strength of preferences and ingeniously infer stochasticity in choices from decision time.

from preference incompleteness. Blavatskyy (2009) presented a particular stochastic specification of random utility models, later axiomatized in Blavatskyy (2012), and showed that this model can account for classical PR. These studies are theoretical. Butler and Loomes (2007) assumed that subjects may have imprecise understanding of their preferences, and showed experimentally that the systematic difference in the imprecision of preferences between the two bets and stochastic choice could explain PR. Collins and James (2015) elicited the probability equivalents for nine endogenously constructed pairs of lotteries using the dual-to-selling version of BDM (Becker et al., 1964; James, 2011), in addition to the original design of Lichtenstein and Slovic (1971). They found that subjects responded differently to probability equivalents than to certainty equivalents, and that reversals based on probability equivalents were less frequent and could be explained by Blavatskyy (2009). Our research is also related to two recent studies of Loomes and Pogrebna (2017) and Alós-Ferrer et al. (2020). Loomes and Pogrebna (2017) provided evidence that preference reversal (PR) arose from distinct cognitive processes in choices and valuations, inconsistent with Johnson and Busemeyer (2005). Their study deviated from standard PR studies in two significant ways: mixing valuation choices for different bets and repeating them four times each; and defining the valuation of bets as the stochastic indifference (SI) point where the bet and the sure amount are chosen equally likely, which is the natural stochastic analogue of certainty equivalence in deterministic theories. Alós-Ferrer et al. (2020) demonstrated theoretically and experimentally that PR could arise from not just behavioral biases but also risk attitudes and experimental design factors like the parameterization of two bets. Our study complements these studies by assessing and differentiating also models of conscious stochastic choice for PR. Working with the estimated stochastic functions and the simulations based on them allows us to extend previous analyses to evaluate qualitatively and quantitatively the potential of the stochastic choice *alone* for PR.

The paper proceeds as follows. Section 2.2 presents the experimental design. Section 2.3 provides a theoretical analysis based on the design and each theory of conscious stochastic choice. Results are reported in Section 2.4. Section 2.5 concludes.

2.2 Experimental design

In the experiment, we use the P-bet and the $-bet as in Butler and Loomes (2007), except that the currency is Chinese Yuan (¥) instead of Australian dollar. Specifically, the P-bet gives ¥24 with a 70% probability and ¥0 otherwise; the $-bet offers

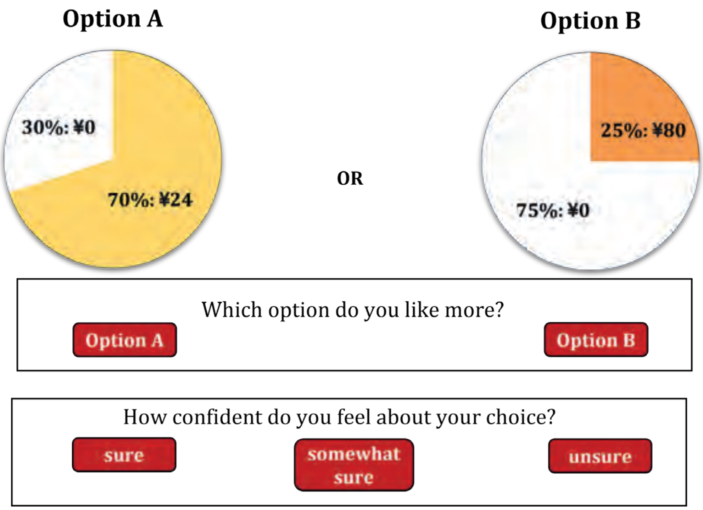


Figure 2.1: The translated decision screen for the direct choice between the P-bet and the $-bet. Subjects first indicated their preferred bet, and then reported their confidence about their choice.

a 25% chance of earning ¥80 and ¥0 otherwise. The experiment mainly consists of two parts, plus some auxiliary tasks.

2.2.1 Part I: standard PR

Similar to most PR experiments, subjects are confronted with a direct choice between the P-bet and the $-bet. The P-bet and the $-bet are presented in pie charts. Following Butler and Loomes (2007), we also ask subjects for their decision confidence after each choice. Subjects can state their confidence on three levels: "*Sure*", "*Somewhat sure*", and "*Unsure*". Figure 2.1 shows an example of the computer screen that subjects face in the direct choice. We repeat the direct choice nine times, with each repetition separated by other choices, and counter-balance the screen position (left or right) of the P-bet and the $-bet for each subject. This repetition allows us to observe whether subjects' choices between the P-bet and the $-bet are stochastic, and if so, to estimate the stochastic function from the nine repeated direct choices.

To make choice and valuation tasks as similar as possible, we elicit subjects' CE and PE of each bet with binary choices. We present subjects with a sequence of binary choices between a fixed bet and a varying reference option (a sure payment in CE valuation, and a reference lottery of paying ¥100 with probability p and ¥0

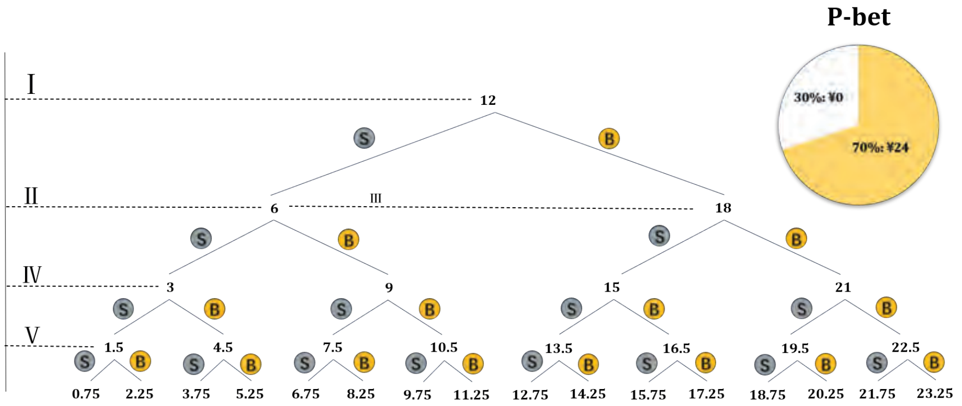


Figure 2.2: The bisection process for eliciting CE of the P-bet. S stands for choosing the sure payment, B stands for choosing the bet. The numbers at the top four rows (I, II, III, IV) are the possible sure payments subjects faced in four binary choices. The numbers at the bottom are the elicited CE for the corresponding choices.

otherwise in PE valuation). Similar to the direct choice, subjects indicate their preferred option and the confidence about their decision. Across choices the bet is fixed while the reference option varies according to a bisection process: each time, the reference option is the mid-point of a certain interval. The original interval lies between 0 and the winning amount of money (or the winning probability) in the fixed bet, and in each succession the computer bisects the interval by using its mid-point as the new lower (upper) limit when subjects choose the fixed bet (or the reference option, respectively). Figure 2.2 illustrates the sequence of sure payment values used when eliciting CE for the P-bet. For example, suppose a subject chooses the sure payment in the first three choices and the bet in the last choice, her CE lies between 1.5 and 3, and we use the mid-point $2.25 = (1.5 + 3)/2$ as her CE. The bisection process allows us to find a relatively accurate CE or PE with a limited number of choices. For the elicitation of the CE, we implement the bisection process six times for the \$-bet and four times for the P-bet, due to the larger payoff range of the \$-bet than the P-bet (80 versus 24). For the elicitation of the PE, we implement the bisection process four times for the \$-bet and five times for the P-bet, due to the larger winning probability in the P-bet than in the \$-bet (0.70 versus 0.25).

While the bisection method has the advantage of avoiding biases in top-down or bottom-up methods such as choice lists (Békésy, 1947), it is well known that the bisection method is not fully incentive compatible. If subjects knew the complete procedure, they would have an incentive to misreport their preferences in the initial

Figure 2.3: The translated decision screen of eliciting the CE R-range of the P-bet.

choices so that they face pairs of options with higher payoffs in later choices. However, as pointed in Bleichrodt et al. (2019, pp. 253): manipulation is "a theoretical possibility but it is practically impossible" (see also Bostic et al., 1990; Abdellaoui, 2000; Abdellaoui et al., 2008; van de Kuilen and Wakker, 2011; Qiu and Steiger, 2011, for more discussion and evidence). Nevertheless, to mitigate this concern, we included some extra choices at the beginning of each bisection process to make it less obvious. See Appendix A.5.3 for more details. We will also specifically examine this potential in the data analysis.

2.2.2 Part II: monetary and probability randomization ranges

In Part II of the experiment, we elicit monetary and probability randomization ranges (CE R-range and PE R-range) for the P-bet and the $-bet. The procedure of eliciting these ranges is similar to the aforementioned CE or PE valuation. The difference is that in each choice subjects have three instead of two options: they can 1) pay a small cost (¥0.10) and choose the fixed bet, 2) pay a small cost (¥0.10) and choose the reference option, or 3) choose a randomization option for free, according to which a computer randomly selects one of the two options. Figure 2.3 illustrates a decision screen (translated to English) to elicit the CE R-range for the P-bet. We use two bisection processes, one for the lower bound and another one for the upper bound, to determine the R-ranges. For more experimental details, please see Appendix A.5.4. In Section 2.3 we demonstrate the theoretical implications of the choice cost.

2.2.3 Additions: risk and ambiguity attitudes

We include two additional tasks to separate the two main tasks in the experiment from each other. One task elicits risk attitudes, and we will use this to estimate subjects' utility function independent of the PR experiment in one of the analyses. Attitudes toward utility uncertainty play a key role in Fudenberg et al. (2015), but there is no established method to elicit them. Therefore, we use the attitude toward belief uncertainty as an exploratory substitute. Details about these two tasks can be found in Appendix A.5.2.

2.2.4 Experimental procedure

Before the experiment began, the experimenter first read the experimental instructions aloud and asked the subjects if they had any questions about the tasks. To ensure that the subjects understood the experiment, we presented them with three incentivized control questions (¥2 for each correctly answered control question) before the real experimental tasks. As mentioned earlier, the experiment consisted of two main parts. To control for potential order effects, we counterbalanced the order of the different parts. Table 2.1 summarizes the experimental order. Finally, the subjects completed a questionnaire at the end of the experiment. Apart from standard demographic information such as gender, age, and field of study, the questionnaire also asked the subjects whether they ever chose the randomization option and, if so, why. Additionally, we administered a measure of control desirability, which elicits how subjects perceive the importance or benefits of maintaining control (Burger and Cooper, 1979; Gebhardt and Brosschot, 2002). See Appendix A.5.5 for more details. Subjects received a show-up fee of ¥10. Additionally, after the experiment, the computer randomly selected one choice from all the choices that the subjects made during the experiment and played it for real.

Order	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
1	PR	Risk	CE R-range	Ambiguity	PE R-range
2	CE R-range	Risk	PE R-range	Ambiguity	PR
3	PR	Ambiguity	CE R-range	Risk	PE R-range
4	CE R-range	Ambiguity	PE R-range	Risk	PR

Table 2.1: The four orders in the experiment. Subjects were randomly assigned into one of the four orders.

Despite COVID restrictions, we were able to conduct a laboratory experiment at the Lab of Experimental Economics and School of Finance, Dongbei University of Finance and Economics (DUFE) from May to July in 2021. We recruited 175 undergraduate and graduate students at DUFE through the laboratory's WeChat recruiting system. Of the subjects, 33.71% were male and 66.29% were female. There were eight experimental sessions, and the average duration of each session was about one hour. The average payment was approximately ¥37.82, which was about 6 dollar. This payment is higher than the average students' hourly wage in China.

2.3 Theoretical analysis

In this section, we provide the theoretical analysis of our experiment. We mainly consider models of incomplete preferences (Eliaz and Ok, 2006) or imprecise preferences (Butler and Loomes, 2007, 2011; Cubitt et al., 2015), and deliberately stochastic preferences (Fudenberg et al., 2015).

2.3.1 Models of incomplete/imprecise preferences

The behavioral predictions of imprecise preferences models and incomplete preferences models are qualitatively the same in our experiment. Below we present the analysis based only on the incomplete preference model of Eliaz and Ok (2006), which explicitly considers incomplete preferences as a potential to explain PR. In this model, there exist pairs of options that individuals are unable to compare (neither strict preference nor indifference). To incorporate incomplete preferences into revealed choices, Eliaz and Ok (2006) relax the standard Weak Axiom of Revealed Preferences (WARP) and consider instead the Weak Axiom of Revealed Non-Inferiority (WARNI). In WARP individuals choose a weakly preferred option, while in WARNI individuals may choose an option that is not clearly inferior to other options (e.g. according to the first-order stochastic dominance criterion), including options that individuals cannot compare. When there are multiple such options, subjects would state "I don't know/care, I'd flip a coin, I guess" (Eliaz and Ok, 2006, p.65). Preference incompleteness can arise from uncertainty in beliefs or preferences (Ok et al., 2012). Our study considers only lotteries with objective probabilities, and thus we focus on uncertainty in preferences: individuals are uncertain about the true utility of a lottery and consider a range of valuations possible. Dubra et al. (2004) model this as individuals having a set of utility functions and ranking options only when all utility functions produce the same ordering.

We now demonstrate how R-ranges reveal preference incompleteness or imprecision. Intuitively, subjects are willing to pay a cost to select a particular option only when they strictly prefer that option over the other, and they should choose the free randomization option when they are unsure about their preference. Concretely, when subjects cannot compare the bet and the reference option or happen to be indifferent between them, in the spirit of Eliaz and Ok (2006), both options are choosable and subjects randomize. There are two ways for subjects to randomize: they can randomize subjectively in their head (e.g., by flipping a virtual coin), or they can choose the explicit randomization option. However, due to the cost of choosing, the explicit option is preferred over the subjective one since the payoff of the explicit randomization (receiving either the bet or the sure payment/the reference lottery) dominates that of the subjective randomization (receiving either the payoff for the bet - ¥0.10 or the sure payment/the reference lottery - ¥0.10). Thus, unlike previous studies such as Cettolin and Riedl (2019), our novel method incentivizes subjects to use the external randomization option to fully reveal preference incompleteness/imprecision. We further note that it is highly unlikely that subjects happen to be indifferent between the bet and one of the reference options in the experiment. Even if we allow for indifference, we can distinguish unsure preference from indifference by recognizing that, since the cost of choosing is small, indifference can occur for at most one value of the reference options (Cettolin and Riedl, 2019). Additionally, we expect that the elicited CE (PE) of a bet lies within the CE R-range (PE R-range). The idea is straightforward: if a subject is willing to pay a cost to pick the bet or the sure payment in the CE R-range task (the reference lottery in the PE R-range task), they should continue doing so when they can freely pick that option in the valuation tasks.

Using the R-ranges of the two bets, we can determine the probability of subjects valuing the P-bet more than the $-bet by assuming that the value of a bet is a random realization within the R-range. According to Eliaz and Ok (2006), when one bet's lower bound is higher than the upper bound of the other bet, subjects have a clear preference and will choose the bet with the higher valuation. When the R-ranges of the P-bet and the $-bet overlap, subjects should choose randomly because their preference is incomplete. Unfortunately, theory does not offer clear guidance on how subjects should randomize when they cannot compare. Therefore, in the analysis of the experimental results, we will consider a few alternative rules of randomization. By using the probabilities of valuing the P-bet higher than the $-bet and the probability of choosing the P-bet in the direct choice from the nine-repeated choices, we can calculate the probability of exhibiting each of the four PR patterns for each subject.

We then run one million simulations. The basic idea is that, since subjects' choices and valuations are stochastic, each experimental outcome can be treated as a random realization of a stochastic process. In each simulation we generate a random number between 0 and 1 for each subject. The randomly generated number and the calculated probabilities together determine the PR choice pattern that this subject would exhibit. For example, suppose a subject exhibits the consistent P-bet choice pattern with probability 20%, PR with 50%, non-standard PR with 10%, and the consistent \$-bet choice pattern with 20%. Then the subject is said to exhibit the consistent P-bet pattern if the randomly generated number is below 0.2; PR if it is between 0.2 and 0.7; non-standard PR if it is between 0.7 and 0.8; and the consistent \$-bet pattern if it is between 0.8 and 1.0. Aggregating across subjects, we obtain one possible realization of the experiment, e.g., k% of subjects exhibit the standard PR, j% the non-standard PR and so on. By running one million simulations, it is as if these subjects participated in the experiment one million times, and we obtain a distribution of proportions for each PR pattern, e.g., the proportions of exhibiting the standard PR across simulations range from a% to b%, the proportions of exhibiting the non-standard PR range from x% to y%, and so on. If PR arises from preference incompleteness/imprecision alone, we should expect the simulated proportions to match the actual proportions. We summarize the predictions of models of incomplete/imprecise preferences below.

Prediction 2.1. *In models of incomplete/imprecise preferences: I) subjects may choose the randomization option frequently; II) the elicited CE or PE must lie within the R-range. Subjects choose differently in the nine-repeated direct choices only when the R-ranges of the two bets overlap; III) the simulated proportions of the PR patterns are comparable to the actual proportions.*

2.3.2 Hedging of preference uncertainty

In models of incomplete/imprecise preferences, individuals choose their preferred option when they have a clear preference and randomize by choosing equally likely options that are not dominated when they have incomplete/imprecise preferences. This is a strong assumption and may be too restrictive. It is possible that individuals are not fully confident about their preferences between two options but may be more likely to choose one option over another. Below we perform the analysis based on the deliberately stochastic choice model of Fudenberg et al. (2015) to capture this idea. In this model individuals have uncertainty about their true preferences, and they randomize deliberately to hedge this preference uncertainty.[3] When facing

[3]Cerreia-Vioglio et al. (2019) is also an important model in this direction, where they consider a class of cautious expected utility preferences (Cerreia-Vioglio et al., 2015). While it could generate preferences for randomization over non-degenerate lotteries, by the axiom of Weak Stochastic Certainty Effect, individuals do not randomize when one of the two options is a sure payment.

two options A and B, individuals behaving according to Fudenberg et al.'s (2015) functional form choose the optimal randomization probability as

$$p^* = \text{argmax}_p \quad pU(A) + (1-p)U(B) - c(p) - c(1-p),$$

where $U(\cdot)$ is the Von Neumann–Morgenstern expected utility, and $c(p)$ is a convex and continuously differentiable cost function. The decision utility of choosing the lottery X for sure is $U(X) - c(1)$, and the decision utility of choosing the reference option y for sure is $U(y) - c(1)$. The first order condition is simply $c'(p) - c'(1-p) = U(A) - U(B)$, which predicts that the randomization probability p is close to 0.5 when $U(A) - U(B)$ is small. Fudenberg et al. (2015) show that their representation corresponds to a form of an uncertainty averse individual who is unsure about her true utility and uses randomization to hedge her preference uncertainty. We assume a cost function of $c(p) = plog(p)/\eta$, which leads to the popular logit choice function: $p = \frac{e^{\eta U(A)}}{e^{\eta U(A)} + e^{\eta U(B)}}$. The sensitivity parameter η can be related to preference uncertainty, with a larger η corresponding to lower preference uncertainty.

We now show that we can infer η, the critical parameter in Fudenberg et al. (2015), from the R-ranges. Let X denote the bet and y the sure payment. When the cost is sufficiently small, as in our experiment, we can ignore it and define the smallest ($\underline{y}$) and largest ($\overline{y}$) values of y such that an individual prefers the randomization option as

$$\begin{aligned} u(\underline{y}) + 0.5[U(X) - u(\underline{y})] - 2c(0.5) &\geq U(X) - c(1.0) - c(0) \\ \Rightarrow u(\underline{y}) &= U(X) - 2[c(1.0) + c(0) - 2c(0.5)], \\ u(\overline{y}) + 0.5[U(X) - u(\overline{y})] - 2c(0.5) &\geq u(\overline{y}) - c(1.0) - c(0), \\ \Rightarrow u(\overline{y}) &= U(X) + 2[c(1.0) + c(0) - 2c(0.5)]. \end{aligned}$$

It follows that $U(X) = [u(\overline{y}) + u(\underline{y})]/2$. Since $c(0)$ or $c(1)$ approaches to infinity when the cost function is $c(p) = plog(p)/\eta$, we approximate these values by $c(0.01)$ and $c(0.99)$, and accordingly $\eta = \frac{2.55}{u(\overline{y}) - u(\underline{y})}$. Similarly, we can also infer η from the PE R-range and obtain $\eta = \frac{2.55}{(\overline{p} - \underline{p})u(100)}$. We assume a parametric utility function $u(x) = \frac{x^\gamma}{\gamma}$ ($u(x) = log(x)$ when $\gamma = 0$), and estimate the parameter γ from the separate risk attitude elicitation tasks.[4] If subjects behave according to the de-

[4]This is possible despite stochastic choice in Fudenberg et al. (2015) because, in binary choices without the randomization option, $u(\underline{y}) < U(X) < u(\overline{y})$, where $\underline{y}$ and $\overline{y}$ are the switching values

liberately stochastic model of Fudenberg et al. (2015), we expect the estimated η from the direct choices to be the same as those from valuations. With the estimated stochastic function for each subject, we can easily calculate the probability of choosing the P-bet and the probability of valuing the P-bet higher than the \$-bet. We can then perform the same analysis as in subsection 2.3.1, and we should expect the simulation proportions of PR patterns to match the actual proportions.

The above analysis allows for the possibility that the elicited CE or PE is outside the R-range because the optimal randomization probability p^* can be positive even when $y < \underline{y}$ and p^* can be less than 1 even when $y > \overline{y}$. We can derive a further hypothesis. In light of Fudenberg et al. (2015), the cost function may relate to a measure of control desirability because, as Fudenberg et al. (2015) point out, a preference for randomization arises because the individual trades off the probability of errors against the cost of making the desired choice. An individual who prefers to maintain control may perceive the implementation cost to be small and is less willing to randomize. In addition, we explore the relationship between the cost function and the attitude toward ambiguity, bearing in mind that this attitude may differ from attitude toward preference uncertainty. We summarize the predictions below.

Prediction 2.2. *In light of Fudenberg et al. (2015): I) subjects' R-ranges relate positively to uncertainty aversion and negatively to the desirability of control; II) the estimated η does not differ between the stochastic function estimated from direct choices or from valuations; III) the simulated proportions should be comparable to the actual proportions in the experiment.*

2.4 Experimental results and analysis

In Appendix A.1 we demonstrate that our experiment replicates the results in Butler and Loomes (2007), and the bisection method did not cause any significant distortion in subjects' behavior. Moving on to our main findings, we first present the prevalence of conscious stochastic choice among subjects. The following result summarizes the finding.

Result 1. *The majority of subjects exhibited conscious stochastic choice: they chose the randomization option more than twice in at least one task, and their R-ranges*

in the multiple price list. We can then follow the standard practice and use the midpoint of the switching values as the certainty equivalent of the lottery.

	0 times	1 time	≥ 2 times
Aggregate	39 (22.3%)	23 (13.1%)	113 (64.6%)
CE_P	90 (51.4%)	38 (21.7%)	47 (26.9%)
$CE_\$$	58 (33.1%)	33 (18.9%)	84 (48.0%)
CE aggregate	50 (28.6%)	29 (16.6%)	96 (54.9%)
PE_P	75 (42.9%)	31 (17.7%)	69 (39.4%)
$PE_\$$	76 (43.4%)	23 (13.1%)	76 (43.4%)
PE aggregate	61 (34.9%)	23 (13.1%)	91 (52.0%)

Table 2.2: The proportions of subjects who chose the randomization option zero times, once, and twice or more. The aggregate numbers across the four tasks (CE_P, $CE_\$$, PE_P, and $PE_\$$) are the proportions of subjects who never chose the randomization option, chose the randomization at most once in any of the four tasks, and twice or more in at least one of the four tasks. CE aggregate (PE aggregate) considers only the CE R-range task (PE R-range task, respectively).

related to self-reported decision confidence systematically.

Support: Table 2.2 presents the proportions of subjects who chose the randomization option zero times, once, and twice or more. On aggregate, a significant proportion of subjects (77.7%) chose the randomization option at least once during the experiment. As it is unlikely that subjects would happen to be indifferent between the bet and one of the presented reference options, this suggests that many subjects may not always have clear preferences. Even when we narrow our focus to subjects who chose the randomization option twice or more in at least one of the four tasks, this proportion is still substantial (64.6%). The proportions of subjects who chose the randomization option twice or more are similar across the CE R-range and the PE R-range tasks, with 54.9% and 52.0%, respectively.

R-ranges differ significantly between bets and across the type of valuation. The mean size of the CE R-range for the P-bet is 1.35, which is about 8% of the expected value of the P-bet. It is 4.74 for the \$-bet, which is about 24% of the expected value of the \$-bet. Thus, the range of CE values that subjects do not have a clear preference is significantly larger for the \$-bet than for the P-bet, consistent with the finding in Butler and Loomes (2007). Similar results hold for the PE R-range, with the mean size of 0.06 for the P-bet and 0.04 for the \$-bet, corresponding to the larger winning probability of the P-bet (70%) than the \$-bet (25%). These differences between the two bets are statistically significant (Wilcoxon signed rank tests, $p < 0.01$ for both CE and PE R-ranges). The histogram in Figure A.2 in

Appendix A.2 illustrates the distribution of the size of the CE and PE R-range for both bets.

We further find that the R-range relates systematically to the interval of less than full confidence ("somewhat sure" or "unsure" according to self-reported confidence statement). The average width of the CE interval of less than full decision confidence is 1.84 for the P-bet and 6.23 for the \$-bet. A series of Spearman rank correlation tests suggest that the size of the CE R-range is positively correlated with the interval of limited confidence ($\rho = 0.30$ for the P-bet; $\rho = 0.32$ for the \$-bet; and $p < 0.01$ for both tests). In addition, the two bounds of the CE R-range positively and significantly correlate with the two bounds of the interval of limited confidence (the lower bound: $\rho = 0.53$ for the P-bet and $\rho = 0.37$ for the \$-bet; the upper bound: $\rho = 0.63$ for the P-bet; $\rho = 0.43$ for the \$-bet, and $p < 0.01$ for all tests). Similar results hold when the valuation is based on PE. The average size of the interval of limited decision confidence is 0.09 for the P-bet and 0.03 for the \$-bet, which is positively correlated with the size of the PE R-range (Spearman rank correlation $\rho = 0.32$ for the P-bet; $\rho = 0.26$ for the \$-bet; and $p < 0.01$ for both tests). Moreover, the two bounds of the PE R-range positively and significantly correlate with the two bounds of the interval of limited confidence (the lower bound: $\rho = 0.57$ for the P-bet; $\rho = 0.49$ for the \$-bet; and $p < 0.01$ for both tests; the upper bound: $\rho = 0.70$ for the P-bet and $p < 0.01$; $\rho = 0.23$ for the \$-bet and $p > 0.10$). These results are consistent with the interpretation that the CE R-ranges and the PE R-ranges capture the range in which subjects do not have clear preferences, and they chose the randomization option consciously.

We proceed to examine the explanatory power of conscious stochastic choice for PR. Under each conscious stochastic choice theory, we perform two analyses:
Analysis 1: we test whether the stochastic functions from different elicitation procedures differ significantly from each other by estimating the stochastic function separately from the nine-repeated direct choices and the CE (PE) R-ranges.
Analysis 2: we evaluate whether it is possible to use one consistent stochastic function for both the direct choice and the CE (or PE) valuation to generate the PR pattern in the experiment. We also check whether allowing for two stochastic functions - one for the direct choice and a different one for the CE valuation - brings simulated proportions closer to the actual proportions.

2.4.1 Models of incomplete/imprecise preferences

There is some potential for the role of preference incompleteness/imprecision for PR. As we can see from Table 2.3, subjects with overlapping R-ranges were more likely to exhibit the standard PR than those without overlapping R-ranges (50.0% vs 39.5%) when the valuation is based on CE, although the difference is not statistically significant (p=0.252 according to a two-sided proportional test). Similarly, when the valuation is based on PE, the subjects with overlapping R-ranges exhibit higher non-standard PR than the subjects without overlapping R-ranges (23.5% vs 19.1%), but the difference is not statistically significant (p=0.738 in a two-sided proportional test).

The CE PR patterns

	Consis. P	Std. PR	Nonstd. PR	Consis. $
G1 (56)	16.1%	50.0%	0.0%	33.9%
G2 (119)	12.6%	39.5%	6.7%	41.2%

The PE PR patterns

	Consis. P	Std. PR	Nonstd. PR	Consis. $
G1 (34)	35.3%	14.7%	23.5%	26.5%
G2 (141)	50.4%	7.8%	19.1%	22.7%

Table 2.3: The proportions of each PR pattern for two groups of subjects based on the incomplete/imprecise preferences. The numbers in parentheses are the number of subjects in each group. Group 1 includes the subjects who had positive (CE or PE) R-ranges and the R-range for the P-bet overlapped with that of the $-bet; Group 2 includes other subjects: those who had a positive (CE or PE) R-range for at least one of the bets but their R-ranges did not overlap, or those who had no positive (CE or PE) R-ranges.

To evaluate more rigorously whether incomplete/imprecise preferences can account for PR patterns, we follow Eliaz and Ok (2006) and assume that subjects choose randomly when they cannot compare the two bets. This happens when their CE or PE R-range overlaps. They select the bet with the higher valuation otherwise.

In the main text we focus on the PR patterns when the valuation is based on CE (referred to as CE PR patterns hereafter), which are the focus of most previous studies. Results on the PR patterns when the valuation is based on PE (referred to as PE PR patterns) are in general similar to those of the CE PR patterns. For conciseness, we report them in Appendix A.3. Here our Analysis 1 consists of three parts: (i) we compare subjects' CE with their CE R-range. Subjects' CE should fall in the CE R-range if their behavior arises from preference incompleteness/imprecision; (ii) we use subjects' CE R-ranges for the P-bet and the $-bet to predict their direct choices. Subjects who had no overlapping CE R-ranges should not choose differently in the nine-repeated choices according to models of incomplete/imprecise preferences; and (iii) we use subjects' direct choices to predict whether their CE R-ranges for the P-bet and the $-bet overlap. Subjects who chose differently in the nine-repeated choices must have overlapping CE R-ranges for the P-bet and the $-bet. The result below summarizes the findings:

Result 2. *Inconsistent with models of incomplete/imprecise preferences, (i) the elicited CE of a non-negligible proportion of subjects (28.0%) is outside the CE R-range; (ii) among the subjects who had no overlapping CE R-ranges in the P-bet and the $-bet, 50.4% (60 out of 119) chose differently in the nine-repeated direct choices; and (iii) among the subjects who chose differently in the nine-repeated direct choices, 68.2% (60 out of 88) had no overlapping R-ranges in the P-bet and the $-bet.*

Support: as we can see from the sub-table a) of Table 2.4, among the 96 subjects who have a positive CE R-range for at least one bet, 51.0% (49 out of 96) had CE outside the R-range for at least one of the two bets. Examining each bet separately, we see that this occurs more often with the $-bet than the P-bet (52.4% vs 21.3%). As we explained in Subsection 2.3.1, when subjects behave according to preference incompleteness/imprecision, their CE should fall within their CE R-range. Thus, these subjects do not behave according to models of incomplete/imprecise preferences.

Sub-table b) of Table 2.4 reports the number of subjects who had overlapping CE R-ranges or non-overlapping CE R-ranges as well as who chose consistently or differently in the nine-repeated direct choices. As we can see, among subjects who had no overlapping CE R-ranges, 50.4% (60 out of 119) chose differently in the nine-repeated choices. Similarly, among subjects who chose differently in the nine-repeated choices, 68.2% (60 out of 88) had no overlapping CE R-ranges for the P-bet and the $-bet. Relaxing the group "chose consistently" by including also the subjects who chose the same bet eight out of nine times, interpreting the one different choice as a mistake, does not improve these proportions, as we can see in Table A.7 in Appendix A.3. Preference incompleteness/imprecision alone cannot explain the behavior of these subjects.

The above result suggests that a large proportion of subjects' behavior in direct choices and valuations cannot be explained by preference incompleteness or

Sub-table a)

	No R-range	In the R-range	Outside the R-range	
			larger than the upper bound	lower than the lower bound
CE_P	128 (73.1%)	37 (21.1%)	6 (3.4%)	4 (2.3%)
$CE_\$$	91 (52.0%)	40 (22.9%)	36 (20.6%)	8 (4.6%)
CE aggregate	79 (45.1%)	47 (26.9%)	49 (28.0%)	

Sub-table b)

	In the nine-repeated direct choices	
	Chose consistently	Chose differently
Overlapping CE R-ranges	28	28
Non-overlapping CE R-ranges	59	60

Table 2.4: Sub-table a) reports the numbers of subjects who had no R-range, who had positive R-ranges and with CE in the R-range, and who had positive R-range but whose CE was outside the R-range. The group "No R-range" includes subjects who had no positive R-range in any of the tasks. The group "In the R-range" includes subjects who had positive R-range in at least one task, and their elicited CE or PE were in the R-range. The group "Outside the R-range" includes subjects who had positive R-range in at least one task and their elicited CE or PE were outside the R-range in at least one task. Sub-table b) reports the number of subjects who had overlapping CE R-ranges or non-overlapping CE R-ranges as well as who chose consistently or differently in the nine-repeated direct choices.

imprecision. To quantitatively evaluate the potential of preference incompleteness/imprecision in the explanation of CE PR patterns, we calculate the probability of valuing the P-bet higher than the $-bet by assuming subjects randomly drawing a value in the CE R-range. To calculate the choice probability, we assume subjects choose randomly (50%-50%) when their R-ranges for the P-bet and the $-bet overlap. As a robustness check, we additionally calculate the choice probability from CE R-ranges, assuming that subjects' choices are determined by randomly sampling the potential values of the bet in the CE R-ranges. By combining the valuation probability with the choice probability, we calculate the probability of each of the four choice patterns for each subject and perform one million simulations, as described in Subsection 2.3.1. This leads to

Result 3. *In light of preference incompleteness/imprecision, simulated proportions based on one consistent stochastic function overestimate consistent preferences and fail to capture the magnitude as well as the asymmetry of CE PR patterns. In contrast, when we estimate the probability of valuing the P-bet higher than the $-*

bet using CE R-ranges and separately estimate the probability of choosing the P-bet through nine-repeated direct choices, the simulated proportions provide a more accurate representation of the magnitude and asymmetry of PR.

Support: as we can see from M1-M3 in Figure 2.4 (more details in Table A.5 in Appendix A.3), simulated proportions based on one consistent stochastic function tend to overestimate the proportion of consistent preferences (ranging from 74.9% to 90.1%), which is significantly higher than the actual proportion of 52.5% (Wilcoxon signed rank tests, $p < 0.10$ for all comparisons). The estimated mean proportions of standard and non-standard PR ranges from 9.8% to 25.1%, which is significantly and substantially lower than the actual proportion of 47.5% (Wilcoxon signed rank tests, p<0.10 for all comparisons). The simulated proportions underestimate the asymmetry between the standard PR and non-standard PR as well: the asymmetry ratio (standard PR/non-standard PR) ranges from 1 to 1.67 according to the simulated proportions, which is significantly and substantially lower than the actual ratio of 9.32 (Wilcoxon signed rank test, p<0.010).

When allowing for two separate stochastic functions, one estimated from R-ranges and another from the nine-repeated choices, the overall simulated proportions of the CE PR patterns now match some of the important properties of the actual proportions in the experiment. As we can see from M4 and M5 in Figure 2.4, the simulated proportions estimate the proportion of PR (the standard and non-standard) to be 40.0% or 43.1%, which is close in magnitude to the actual proportion of 47.5%. Importantly, while still significantly different, the simulated proportions capture the asymmetry of the standard and non-standard PR: the mean asymmetry ratio of the standard PR divided by the non-standard PR is 3.63 and 4.26, although it is still statistically different from the actual ratio of 9.32. This result suggests further that choices are sensitive to the elicitation procedures, and we need separate stochastic functions for choices and valuations in order to account for PR.

2.4.2 Hedging of preference uncertainty (Fudenberg et al., 2015)

To examine whether hedging of preference uncertainty as in Fudenberg et al. (2015) can explain subjects' behavior, we first investigate the relationship between R-range and ambiguity attitude, as well as the desirability of control. Table A.10 in Appendix A.4 provides some supporting evidence, as R-ranges are positively associated with weaker desirability of control and ambiguity aversion, although the relation-

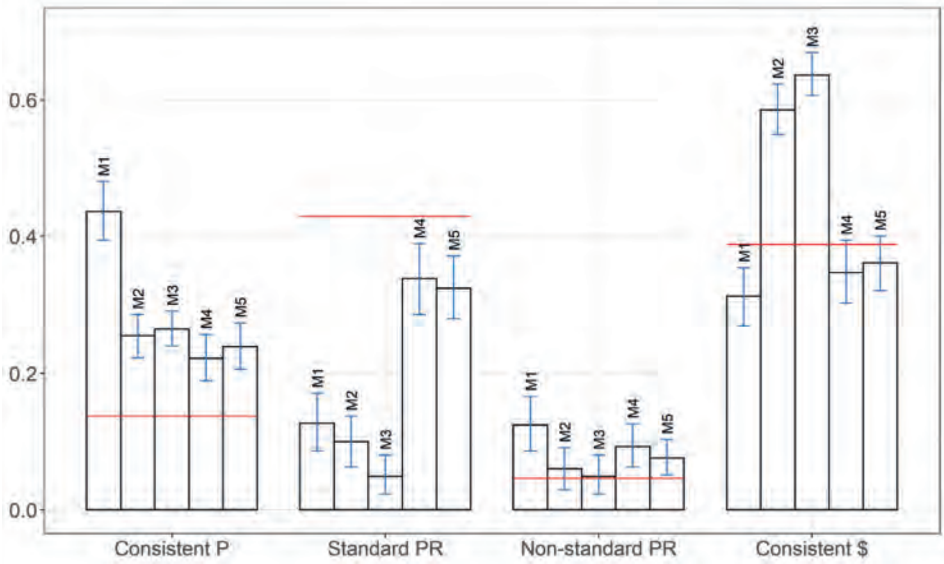


Figure 2.4: The comparison of means and 95% confidence intervals of simulated proportions of CE PR patterns under incomplete/imprecise preferences models across different simulation methods. The error bars are the 95% confidence intervals of the simulated proportions. The horizontal red lines are the actual proportions in the experiment. In M1, the probability of valuing the P-bet higher than the $-bet is 50% if subjects chose different bets in the nine repeated direct choices and it is 1 (or 0) if subjects chose the P-bet (or the $-bet) for nine times. In M2, subjects choose randomly (50%-50%) when their R-ranges for the P-bet and the $-bet overlap. In M3, the choice probability is calculated from CE R-ranges by assuming that subjects' choices are determined by randomly sampling the potential values of the bet in the CE R-range. In M4 and M5, we calculate the probability of choosing the P-bet in the direct choice from the nine-repeated direct choices and use CE R-ranges to calculate the probability of valuing the P-bet higher than the $-bet. In M4, we assume random choice (50%:50%) if subjects chose differently in the nine-repeated direct choices. In M5, we use the actual choice ratio in the nine repeated direct choices as the probability of choosing the P-bet in the direct choice.

ship is statistically significant only for the former (p<0.05 for weaker desirability of control and p>0.10 for ambiguity aversion).

As we show in the deliberately stochastic model of Fudenberg et al. (2015) in Subsection 2.3.2, individuals randomize when they are uncertain about their preferences over the two bets. In particular, subjects could have CE or PE values outside the R-ranges because these ranges do not capture the full extent of their preference uncertainty. To infer subjects' stochastic function for the CE valuations, we follow the theoretical analysis and make use of their R-ranges: $\eta_X = \frac{2.55}{u(\overline{y})-u(\underline{y})}$ when the valuation is based on CE, and $\eta_X = \frac{2.55}{(\overline{p}-\underline{p})u(100)}$ when the valuation is based on PE, where

X is the P-bet or the \$-bet. To infer the stochastic function for the direct choice, we estimate the η by using the actual choice ratio in the nine-repeated direct choices and assuming the Logit stochastic function $P(P - bet \succ \$ - bet) = \frac{e^{\eta U(P)}}{e^{\eta U(P)} + e^{\eta U(\$)}}$. To calculate $u(\overline{y})$, $u(\underline{y})$, $U(\$)$, and $U(P)$ we use the mean estimated risk parameter $\bar{\gamma}$ from the risk measurement task. Below we report the results about the CE PR pattern. The results about PE PR pattern, which are generally similar to those of CE PR pattern, are reported in Appendix A.4. The following result summarizes the finding:

	The estimated η		
	from the direct choices	from the CE R-ranges	
		P-bet	\$-bet
Mean	-0.24	10.85***	10.96***
	(3.80)	(9.51)	(15.65)
Median	0.28	6.45	4.61

Table 2.5: The fitted η in the deliberately stochastic models. Standard deviations are in parentheses. The η of the stochastic function from the direct choices is estimated from the equation $P(P - bet \succ \$ - bet) = \frac{e^{\eta U(P)}}{e^{\eta U(P)} + e^{\eta U(\$)}}$. The η of the stochastic function from the CE R-ranges is estimated from the equation $\eta_X = \frac{2.55}{u(\overline{y}) - u(\underline{y})}$, where X is the P-bet or the \$-bet. Wilcoxon signed rank tests check the difference in the estimated η from the direct choices and the CE R-ranges. * denotes $p < 0.10$, ** denotes $p < 0.05$, *** denotes $p < 0.01$.

Result 4. *The estimated η from CE R-ranges based on Fudenberg et al. (2015) differs significantly from the η estimated from the nine-repeated direct choices.*

Support: Table 2.5 reports the mean and the median estimated η of the stochastic functions from the direct choices and CE R-ranges. As we can see, the mean estimated η from CE R-ranges is 10.85 for the P-bet and 10.96 for the \$-bet (medians are 6.45 and 4.61, respectively). In contrast, the mean estimated η from the nine-repeated direct choices is -0.24 (median of 0.28). Wilcoxon signed rank tests suggest that the difference in the estimated η from CE R-ranges and nine-repeated direct choices is statistically significant ($p < 0.01$ for both the P-bet and the \$-bet comparisons). The η estimated from the choice ratio of the nine-repeated direct choices is significantly lower, even with a negative mean. This suggests that the stochastic function for the direct choice does not respond sensitively (or even wrongly) to the expected utility difference of the two bets. The low values of η estimated from the direct choices thus suggest that subjects may behave differently in the direct choice and in the risk measurement tasks.

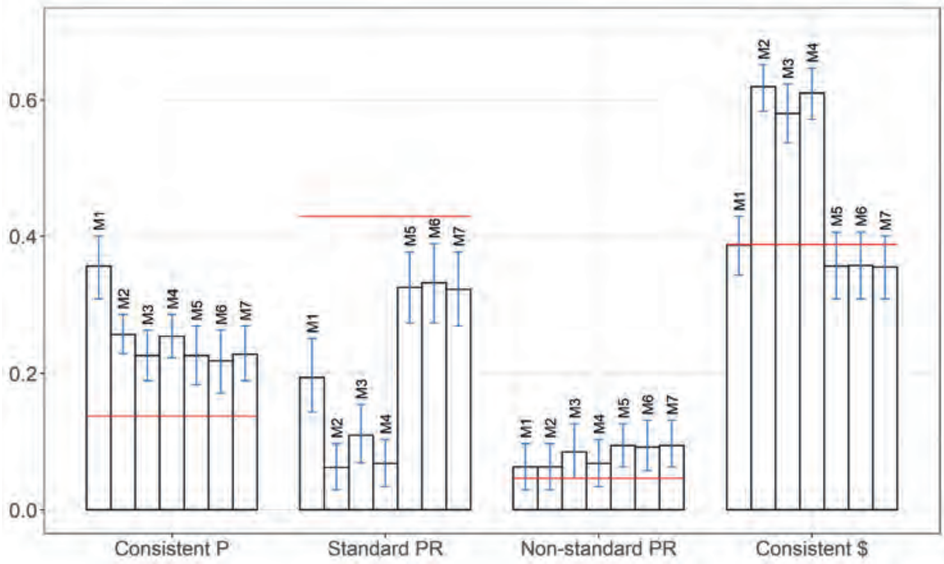


Figure 2.5: The comparison of means and 95% confidence intervals of simulated proportions of CE PR patterns under the deliberately stochastic model of Fudenberg et al. (2015) across different simulation methods. The error bars are the 95% confidence intervals of the simulated proportions. The red horizontal lines are the actual proportions in the experiment. The simulations M1 to M4 use a consistent stochastic function estimated from the nine-repeated direct choices, from the CE R-ranges of the P-bet, the $-bet, and the mean η across the P-bet and the $-bet, respectively. The simulations M5 to M7 use the stochastic function estimated from the nine-repeated direct choices for the direct choice and the stochastic function for valuations estimated from the CE R-ranges of the P-bet, the $-bet, and the mean η across the P-bet and the $-bet.

With the estimated stochastic functions from the CE R-ranges of the P-bet or the $-bet as well as from the nine-repeated direct choices for each subject, following the analysis in Subsection 2.3.2, we use each of these stochastic functions to calculate the probability of choosing the P-bet in the direct choice and the probability of valuing the P-bet higher than the $-bet.

Result 5. *Under Fudenberg et al. (2015), simulated proportions based on one consistent stochastic function overestimate consistent preferences and fail to capture the magnitude as well as the asymmetry of CE PR patterns. In contrast, when using the CE R-ranges to estimate valuation probabilities and using nine-repeated direct choices to estimate the probability of choosing the P-bet, the simulated proportions capture the magnitude and the asymmetry of PR well.*

Support: Figure 2.5 (more details in Table A.9 in Appendix A.4) reports the means and 95% confidence intervals of these simulated proportions of the CE PR patterns for each stochastic function. As we can see, the mean simulated proportions in M1 to M4 overestimate the proportions of consistent preferences (choosing the P-bet or the $-bet consistently): the simulated proportion ranges from 74.3% to 87.6%, which is significantly higher than the actual proportion of 52.5% (Wilcoxon signed rank test, $p < 0.10$). Further, they fail to capture the magnitude and the asymmetry of standard PR and non-standard CE PR. The simulated proportions in M1 to M4 predict 12.5% to 25.7% of standard and non-standard PR, which is significantly lower than the actual proportion of 47.5% (Wilcoxon signed rank test, $p < 0.10$). Most importantly, the simulated proportions do not fully capture the asymmetry of standard PR and non-standard PR (asymmetry ratio of standard PR divided by non-standard PR ranging from 0.98 to 3.08 according to simulated proportions in M1 to M4, which is different from the ratio of 9.33 according to the actual proportions).

We now use the stochastic function from CE R-ranges to calculate the valuation probabilities, and use the actual choice ratio as the choice probability of choosing the P-bet in the direct choice. We find that, while the actual levels are not in the 95% confidence intervals of the simulated proportions, the magnitude of the simulated levels in M5 to M7 are close to the actual proportions. The proportion of PR is 41.6% to 42.4% according to the simulated proportions, which is close to the actual proportion of 47.5%. Importantly, the simulated proportions are able to capture the asymmetry of the standard PR and non-standard PR, although not fully (asymmetry ratio of standard PR vs non-standard PR 3.43 to 3.61 according to simulated proportions and 9.33 according to the actual proportions). Figure 2.5 illustrates directly the comparison of these two approaches.

2.5 Concluding remarks

In this paper we implemented a novel method in a PR experiment to examine the prevalence of conscious stochastic choice and its potential to explain PR. In this method, subjects need to pay a small cost for choosing a particular option or they can choose a free randomization option. We find that the majority of subjects exhibited conscious stochastic choices. While we also find substantial PR, our experimental results do not coincide with the theoretical predictions. Our results suggest that choices are not only stochastic but also depend on the elicitation procedures.

Our study highlights the importance of understanding the stochastic nature of choices and the influence of elicitation procedures in generating values and choices. The findings that choices can be consciously stochastic and violate procedural invariance have several important implications. First, they suggest that, when subjects' choices are stochastic, the standard method of asking subjects to state just one value may not be particularly informative. Instead, eliciting a range of possible values, as demonstrated in earlier works (Butler and Loomes, 2007; Cubitt et al., 2015; Loomes and Pogrebna, 2017) and our study, can offer richer insights. Together with earlier findings in the literature (Tversky et al., 1988, 1990a), the violation of procedural invariance poses a significant challenge to the theory of revealed preferences. For example, when different elicitation tasks reveal systematically different levels of risk tolerance (see e.g., Loomes, 1988; Friedman et al., 2022), it is unclear which task we should rely on to capture risk preferences. Therefore, it is advisable to elicit values in the setting in which they are intended to be used, as this can also greatly influence the obtained value.

Second, the procedure dependency of stochastic choice underscores the need for caution when extrapolating preferences inferred from one elicitation procedure to represent those obtained through another. For example, a recent body of literature makes an insightful observation that noise can bias choices in paired tasks differently (Ballinger and Wilcox, 1997; Loomes, 2005; Blavatskyy, 2007, 2010; Bhatia and Loomes, 2017), but not valuations (Bernheim and Sprenger, 2020; Carrera et al., 2022). Consequently, McGranaghan et al. (2024) suggest to use valuations over choices to uncover preferences. Our finding that choices are stochastic but violate procedural invariance suggests such an interpretation may not always be warranted. While the violation of procedural invariance may not be a concern in McGranaghan et al. (2024) because the payoff ranges did not change across their lottery pairs, in general, individuals may have context-dependent utility functions (Loomes and Sugden, 1983; Tversky et al., 1988, 1990a), and preferences revealed through valuations may not be informative of those underlying choices (Butler et al., 2014a).

Third, our finding that stochastic choice can be conscious opens new ways for revealing and measuring the stochasticity in choices. Instead of repeating similar choices multiple times as in random utility models, we may be able to directly elicit subjects' subjective assessments of randomness and use them to capture stochastic choice. For example, we could allow subjects to assign randomization probabilities to various options in a choice set and pay them according to these randomization probabilities (Feldman and Rehbeck, 2022; Agranov and Ortoleva, ress; Ong and

Qiu, 2023; Halevy et al., 2023). Arts et al. (2024) provided some indicative evidence for the potential of such an approach for capturing stochastic choices: Across subjects and decisions, a higher randomization probability for an option corresponded to choosing that option more frequently (but not always) in binary choices. Another promising direction is to directly ask subjects for their belief about their intended choices out of a set and use that belief as a proxy for stochastic choice probabilities.

Chapter 3

Preference reversal and stochastic choices

This paper provides a comprehensive analysis of the role of stochastic choices in explaining preference reversals (PR). Using two independent data sets (Loomes and Pogrebna, 2017; Shi et al., 2024), we estimate a consistent stochastic choice function for each subject from their direct choices and valuation choices. We then compare the estimated probabilities of PR patterns based on the stochastic choice function with the observed PR patterns. Analyses based on a wide range of random utility stochastic choice models reveal that, while stochastic choices are frequently observed in PR experiments, a consistent stochastic choice function cannot simultaneously estimate the choice probability and the monetary valuations. Further, the computed distribution of PR patterns based on the estimated stochastic function differs significantly from the observed distribution, in particular underestimating the magnitude and asymmetry of PR. These results suggest that choices are not only stochastic but also sensitive to the elicitation approaches.

This chapter is currently under review.

3.1 Introduction

The preference reversal phenomenon (PR), the inconsistency of revealed preferences between choices and valuations, raises a fundamental challenge to the idea that preferences can be reliably inferred from different elicitation procedures. In a typical PR experiment, there is a safer lottery with a lower payoff (P-bet) and a riskier lottery with a higher payoff ($-bet), and an individual exhibits PR when she chooses one bet while valuing the other higher. Previous studies find substantial PR (Lindman, 1971; Grether and Plott, 1979; Tversky et al., 1990a; Seidl, 2002), and, more importantly, a systematic asymmetry in PR: PR occurs much more frequently in a “standard” pattern by choosing the P-bet and valuing the $-bet higher, compared to the “non-standard” pattern of choosing the $-bet and valuing the P-bet higher (Tversky et al., 1990a; Cubitt et al., 2004; Schmidt and Hey, 2004; Butler and Loomes, 2007; Loomes and Pogrebna, 2017). PR has been a prominent research topic in economics, with extensive theoretical attempts to provide explanations, including regret aversion (Loomes and Sugden, 1982, 1983; Loomes et al., 1989, 1991), context-sensitive preference (Tversky et al., 1990a; Cubitt et al., 2004; Alós-Ferrer et al., 2016), preference imprecision (MacCrimmon and Smith, 1986; Butler and Loomes, 2007; Cubitt et al., 2015), and preference incompleteness (Eliaz and Ok, 2006; Ok et al., 2012).

In this paper, we focus on the role of decision noise in the direct choice and the valuation process in explaining PR. The idea that noise can generate systematic biases has received wide attention. A series of works has documented the stochasticity in choices. For example, Butler and Loomes (2007) asked subjects to make a straight choice between the $-bet and the P-bet on three separate occasions and observed that 23 subjects (out of the total 89 subjects) made different choices. Loomes and Pogrebna (2017) made the valuation choice four separate times and the direct choice between the P-bet and the $-bet eight separate times, revealing a large ratio of variation in decisions. Agranov and Ortoleva (2017) made subjects aware of three repetitions of choices and still observed different chosen options in repetitions. Recent studies suggest that random decision errors can generate systematic biases. Khaw et al. (2021) suggest that perceptual biases from noise in internal representations can explain the small-stakes risk aversion. Enke and Graeber (2023) and Enke et al. (2023) propose that cognitive uncertainty, serving as a measurable proxy for the unobserved noisiness, can explain the probability weighting and present bias. McGranaghan et al. (2024) find no aggregate common ratio preference and instead demonstrate that noise can generate a common ratio effect even for individuals without an associated common ratio preference.

We base our analysis on random utility models, one of the most commonly used stochastic choice models. In a random utility model, the decision utility of each alternative has two components: a deterministic component that reflects the intrinsic attributes of the alternative and a random component that captures unobserved random variation in the individual's preferences. Due to the existence of the random component, individuals may exhibit stochastic choices in repeated decision scenarios. Random utility models have been widely studied in the field of decision-making (Becker et al., 1963; Luce and Suppes, 1965; Hey and Orme, 1994; Gul and Pesendorfer, 2006; Von Gaudecker et al., 2011; Apesteguia and Ballester, 2018). In particular, Blavatskyy (2009) developed a random utility model, later axiomatized in Blavatskyy (2012). This model can account for both the systematic asymmetry of PR when the valuations are elicited from certainty equivalents (CE) and the opposite asymmetry of PR when the valuations are elicited from probability equivalents (PE).[1] In our main analysis, we consider the classical Logit choice model and the stochastic choice model proposed by Blavatskyy (2009). For the distribution of the random noise, in addition to the homoscedastic distribution, we also consider the heteroscedastic random errors. Heteroscedastic random errors in stochastic functions allow the variance of the error term to depend on the range of payoffs in the options (Buschena and Zilberman, 2000).

Using two independent data sets (Loomes and Pogrebna, 2017; Shi et al., 2024), we estimate a consistent stochastic choice function at the individual level from the direct choices and the valuation choices. This is possible because in both Shi et al. (2024) and Loomes and Pogrebna (2017) the direct choice between the P-bet and the $-bet was repeated several times (separated by other choices). Additionally, in the latter experiment, each valuation choice was also repeated several times. With the estimated stochastic functions, we compute the aggregate-level probability distribution of PR patterns. The four PR choice patterns are: choosing the $-bet and pricing it higher (consistent $-bet), choosing the P-bet and pricing it higher (consistent P-bet), choosing the P-bet and pricing the $-bet higher (standard PR), choosing the $-bet and pricing the P-bet higher (non-standard PR). To do so, we first compute the choice probability of choosing the P-bet and the valuation probability of valuing the P-bet higher based on a stochastic choice function. By combining the choice probability and the valuation probability together, we obtain

[1] When valuations are elicited using probability equivalents, more subjects exhibit more often the "non-standard" PR than the standard PR (Cubitt et al., 2004; Butler and Loomes, 2007).

the individual-level probability distribution of PR patterns. Second, we run one million simulations as if each subject participated in the experiment one million times. In each simulation, one PR pattern is realized for each subject following their individual-level probability distribution. By aggregating the simulation results, we obtain the aggregate-level simulated probability distribution of PR patterns. We then compare the simulated probability distribution with the observed proportions of PR patterns. Because the above analysis can be sensitive to the parametric forms of the stochastic functions, we employ several random utility models in our main analysis. For the utility curvature, we apply the CRRA utility function in our main analysis and perform a robustness check with the CARA utility function in Appendix B.1.2.

Our analysis results suggest that the stochastic choice alone is insufficient to explain the PR phenomenon. Although a considerable proportion of subjects exhibited stochastic behaviors in repeated choices, when estimating the PR patterns with one consistent stochastic function, the simulated proportions of PR patterns differ significantly from the observed PR proportions in both the magnitude of PR and the asymmetry between standard PR and non-standard PR. Specifically, in Shi et al. (2024), while 50.3% of subjects exhibit stochastic choices in direct choices, the simulated PR proportions range from 26.1% to 32.5%, far below the observed 47.5%. The simulated asymmetry ratios range from 1.48 to 1.58, much lower than the actual ratio of 9.33. Similarly, in Loomes and Pogrebna (2017), 18.5% of subjects exhibited stochastic choices in direct choices and 90.2% in CE valuation choices. The simulated PR proportions ranged from 21.7% to 27.7%, lower than the observed 66.8%. The simulated asymmetry ratios are between 1.07 and 3.12, substantially less than the observed ratio of 122.00. To understand the source of the differences between the simulated and the actual PR patterns, we also examine the stochastic choice in direct choices and valuation choices individually. The results suggest that although the estimated direct choice probability and the monetary valuation of the P-bet are close to the observed data, all stochastic models significantly underestimate the monetary valuation of the $-bet.

Our study contributes to the literature in two ways: First, we contribute to the literature on stochastic choices by applying a typical stochastic choice model, the random utility model, to the analysis of the PR phenomenon. Stochastic choice has been extensively studied in recent literature (Blavatskyy, 2009, 2012; Manzini and Mariotti, 2014; Fudenberg et al., 2015; Loomes and Pogrebna, 2017; Agranov and Ortoleva, 2017; Cerreia-Vioglio et al., 2019). In the analysis, we directly estimate each subject's stochastic functions from their choices in the experiment and conduct analyses of the PR based on the estimated functions. Loomes and Pogrebna (2017) studied whether PR disappears after allowing for stochastic choice. Our paper adopts a different analytical approach by basing our analysis on stochastic choice functions. The estimated stochastic functions enable us to estimate the probabilities of PR patterns. Shi et al. (2024) analyzed PR based on conscious stochastic choice models, while this paper employed random utility models, which attributed the stochastic choice to random noise.

Second, we contribute to the literature on context-sensitive preferences by incorporating stochastic elements in choices. In this paper, we find that a consistent stochastic function cannot simultaneously estimate the direct choices and the valuation choices. The PR that reveals the systematic disparity between choice and valuation elicitation methods cannot be fully explained after allowing for stochastic choice. This provides a piece of evidence that preferences are sensitive to elicitation methods, even if one allows for stochastic choices. Our result is consistent with the idea of scale compatibility (Tversky et al., 1988, 1990a; Slovic et al., 1990), which suggests that, in multi-attribute decision tasks, an attribute of alternatives is weighted more heavily when it matches the response scale than when it does not. The scale compatibility explains the cause of PR in that the payoff is weighted more heavily in valuation tasks and leads to an overvaluation of the $-bet.

The paper is organized as follows: In section 2, we perform the theoretical analysis of investigating PR under random utility models. In section 3, we provide an overview of the experiments involved in our analysis. In section 4, we report the analysis results. In section 5, we conclude our findings and discuss them in connection with the existing literature.

3.2 Theoretical analysis

We consider several stochastic choice function forms, including the Logit stochastic function, the stochastic function in Blavatskyy (2009) with homoscedastic random errors, and the stochastic function in Blavatskyy (2009) with heteroscedastic random errors.

3.2.1 stochastic choice function

In random utility models, individuals' preferences are subject to random shocks, which may affect the preference ordering of options and result in stochastic choices.

In a binary choice, the probability of choosing one option over another is the probability of that option having a higher expected utility than the other option. Let $\{A, B\}$ denote the two available options in a binary choice and $p(A, B)$ denote the probability of choosing A over B. For the stochastic choice function, we first consider the standard Logit choice function in Mcfadden (1974):

$$p(A, B) = \frac{e^{\eta U(A)}}{e^{\eta U(A)} + e^{\eta U(B)}} \tag{3.1}$$

where η is a parameter that captures the sensitivity of choice probability to the utility difference.

Given that varying specifications can result in differing predictions (Blavatskyy and Pogrebna, 2010), we also consider the stochastic model proposed by Blavatskyy (2009). We employ this stochastic model in the analysis as its potential to account for the PR phenomenon has been demonstrated. Under this stochastic choice model, the probability of choosing A over B is

$$p(A, B) = \frac{\phi\left[U(A) - U(A \wedge B)\right]}{\phi\left[U(A) - U(A \wedge B)\right] + \phi\left[U(B) - U(A \wedge B)\right]} \tag{3.2}$$

where $U(.)$ is the von Neumann-Morgenstern expected utility function, $\phi(.)$ is a non-decreasing function with $\phi(0) = 0$, and $A \wedge B$ is the greatest lottery that is dominated both by option A and option B in terms of the first-order stochastic dominance. For example, the P-bet in Shi et al. (2024) is receiving ¥24 with a probability of 0.7 and ¥0 otherwise, and the \$-bet is receiving ¥80 with a probability of 0.25.[2] The greatest lottery dominated both by the P-bet and the \$-bet is the lottery of receiving ¥24 with a probability of 0.25 and ¥0 otherwise. For the $\phi(.)$, we mainly consider $\phi(x) = e^{\eta x} - 1$ as suggested by Blavatskyy (2009).

Moreover, we also consider heteroscedastic random errors in the stochastic function of Blavatskyy (2009). Heteroscedastic random errors allow the variance of the error term to depend on the range of payoffs in the options. One approach is to assume that the standard deviation of the random error is proportional to the utility difference between the highest payoff and the lowest payoff of the available options. Holman and Marley show that the parameter $\frac{1}{\eta}$ can be linked to the variance of the i.i.d. Gumbel preference shocks in a random utility representation (Luce and Suppes, 1965, p.338). To capture heteroscedastic random errors, we assume $\sqrt{\frac{1}{\eta}}$ is proportional to the *payoff* difference between the highest and the lowest payoffs in

[2] ¥ is the symbol for Chinese currency.

each choice. Denoting the payoff range of the P-bet as 1 unit and that of the \$-bet as r ($r > 1$) units, the payoff range of the direct choice (DC) task and the certainty equivalent (CE) tasks involving the \$-bet is r units, and the payoff range of the CE tasks involving the P-bet is 1 unit. Assume $\eta = \eta_0$ in the DC and the $CE_\$$ tasks, then $\eta = \frac{\eta_0}{(1/r)^2} = r^2\eta_0$ in the CE_P tasks.

In the analysis, the stochastic choice functions are estimated using maximum likelihood estimation (MLE). Assuming subjects face a series of n binary choices, with A_i and B_i being the two alternatives in the ith choice, the log-likelihood of the observed choice outcomes across the series of binary choices is

$$LL^n_{A_iB_i} = \sum_{i=1}^{n} \log(I^C_i \times p(A_i, B_i) + (1 - I^C_i) \times p(B_i, A_i)) \tag{3.3}$$

where I^C_i is an indicator that captures the option chosen in the ith choice, equalling 1 when choosing A_i and equalling 0 when choosing B_i. For the utility curvature, we assume a CRRA utility function $u(x) = \frac{x^\gamma}{\gamma}$ in our main analysis ($u(x) = \log(x)$ when $\gamma = 0$) and perform a robustness check with the CARA utility function in Appendix B.1.2. For the dataset, our main analysis is based on Shi et al. (2024), and we conduct additional robustness checks using data from Loomes and Pogrebna (2017). In the analysis, we jointly estimate each subject's utility curvature and stochastic function using all of their choices in DC and CE valuation tasks. Although Shi et al. (2024) includes additional risk tasks in their experiment, we do not perform separate estimations using utility curvature derived from these tasks as it can be unstable under stochastic choice conditions.[3]

3.2.2 From stochastic choice to probabilistic PR

Denoting an arbitrary pair of PR lotteries with the P-bet as P and the \$-bet as \$, the certainty equivalents of the P-bet and the \$-bet are denoted as CE_P and $CE_\$$, respectively. Following equation (3.2), the probability of choosing the P-bet in DC is $p(P, \$)$. The CE valuations in Shi et al. (2024) are elicited following the bisection process and the calculation of the probabilities of each possible CE is illustrated in Figure B.3 in Appendix B.3. The probability density function of CE of the P-bet

[3]For two reasons we do not estimate utility curvature separately from additional tasks in our main analysis: 1) the risk tasks are framed differently from the main tasks, which may result in inconsistent behaviors between risk tasks and PR tasks; 2) choices in risk tasks may also involve stochasticity, a single response without repetition may not be sufficient to capture the utility curvature.

under the bisection process is:

$$CE_P(x) = \prod_{i=1}^{4} p(P, x_i)^{I_i^P} p(x_i, P)^{1-I_i^P} \tag{3.4}$$

where x_i is the sure payment a subject faces in the ith binary choice under the bisection process and x is the CE value elicited after binary choices under the bisection process. I_i^P is an indicator which equals 1 if $x > x_i$ and 0 if $x < x_i$. Shi et al. (2024) elicited CE_P with four binary choices and $CE_\$$ with six choices. The probability density function of $CE_\$$ under the bisection process can be calculated similarly:

$$CE_\$(y) = \prod_{i=1}^{6} p(\$, y_i)^{I_i^\$} p(y_i, \$)^{1-I_i^\$} \tag{3.5}$$

where y_i is the sure payment the subject faces in the ith binary choices under the bisection process and y is the CE value elicited after binary choices under the bisection process. $I_i^\$$ is an indicator which equals to 1 if $y > y_i$ and 0 if $y < y_i$. Based on equations 3.4 and 3.5, the probability that the certainty equivalent of the P-bet is not less than that of the \$-bet is

$$p(CE_p \geq CE_\$) = \sum_{x \geq y, x \in X, y \in Y} CE_P(x) CE_\$(y) \tag{3.6}$$

where X and Y are the sets of all possible CE valuations of the P-bet and the \$-bet obtained through the bisection process.

In the analysis, we estimate a consistent stochastic function for each subject based on all of their choices made in the DC and CE tasks. Then we rely on the estimated stochastic function and compute the direct choice probability $p(P, \$)$ and the CE valuation probability $p(CE_P \geq CE_\$)$. By combining these probabilities, we obtain the individual-level estimated probability of each CE PR pattern. To calculate the aggregate-level distribution of PR patterns, we run simulations. In each simulation, we randomly select one PR pattern for each subject following the predicted probabilities of PR patterns. Then we compute the proportions of PR patterns in the current simulation, e.g., with probability p to have k% of subjects exhibit the standard PR, j% the non-standard PR, and so on. Since it is cumbersome to calculate these probability distributions, we perform the simulation one million times and obtain the distribution of aggregate-level simulated proportions. The basic idea is that, since subjects' choices and valuations are stochastic, each experimental outcome would be a random realization of the stochastic process. By running one million simulations, it is as if all subjects participated in the experiment one million times.

Prediction. *If PR arises from stochastic choice, the simulated proportions based on the estimated stochastic choice functions should be comparable with the actual PR proportions.*

If the PR phenomenon can be fully explained by stochastic choice, a consistent stochastic choice function for each subject should be sufficient to estimate their choices in direct choices as well as valuation choices in a PR experiment. Therefore, this individual-level stochastic function should capture the probability of each PR pattern exhibited by the subject. At the aggregate level, after running one million simulations, the simulated proportions of PR patterns should be comparable with the actual proportions.

3.3 Data sources

The main analysis is based on data from Shi et al. (2024). For robustness, we also perform analysis on data from Loomes and Pogrebna (2017). To help readers understand our analysis, we summarize the data that we included in our analysis in this section. For detailed information regarding the experiments, please refer to Shi et al. (2024) and Loomes and Pogrebna (2017).

3.3.1 Data from Shi et al. (2024)

The experiment of Shi et al. (2024) consists of three main parts: classical PR, randomization range, and additional risk and ambiguity attitudes. Only data from the classical PR part is included in our analysis. We provide an overview of this part here.

The classical PR part consists of direct choice (DC) tasks and valuation tasks. In the choice task, subjects faced a direct choice (DC) between the P-bet and the \$-bet. The P-bet is a relatively safe lottery with a 70% chance of receiving ¥24 and 0 otherwise, while the \$-bet is a relatively risky lottery with a 25% chance of receiving ¥80 and 0 otherwise. These bets are from Butler and Loomes (2007), with the only difference being that the payoffs in Shi et al. (2024) are in Chinese yuan instead of Australian dollars as in Butler and Loomes (2007). The DC is presented to subjects nine times in the experiment, interspersed with other choices. After each DC, they also elicited self-reported confidence of this choice. In the valuation task, Shi et al. (2024) elicited CE and PE valuations through binary choices to reduce the differences in the framing between the choice task and the valuation task. In CE valuation, subjects faced a series of binary choices between the P-bet (or the $-bet) and sure payments. The changes of sure payments follow the bisection process (see Appendix B.3 for details). In PE valuation, subjects faced a series of binary choices between the P-bet (or the $-bet) and reference lotteries. The payoffs of the reference lotteries are fixed while the winning probabilities are changed following the bisection method. In the analysis, we mainly focus on the CE valuation as most literature does, and provide the analysis on PE valuations in the Appendix B.2.

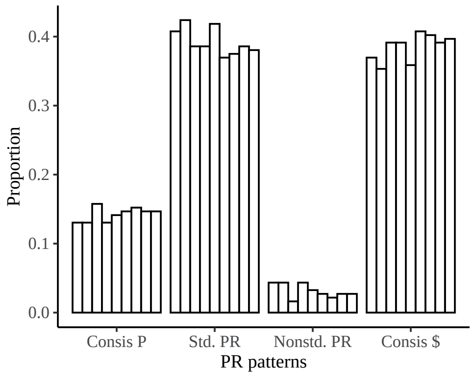


Figure 3.1: Proportions of PR patterns categorized according to nine different DC in Shi et al. (2024).

In the experiment, a substantial proportion of subjects exhibited PR. Figure 3.1 illustrates the proportions of PR patterns categorized according to CE valuations and nine different DC in Shi et al. (2024). On average, 44.6% of subjects exhibited PR. The large proportion of PR indicates an inconsistency between DC and CE. Specifically, 54.3% of subjects preferred the P-bet over the $-bet in DC by opting for the P-bet five or more times out of nine repeated DC, whereas only 18.2% of subjects valued the P-bet more than the $-bet in CE valuations. The means of CE_P and $CE_\$$ are 14.6 and 21.9 respectively. The one-sided Wilcoxon signed-rank test suggests that CE_P is significantly less than $CE_\$$ ($p < 0.01$). In addition to the high prevalence of PR, a strong PR asymmetry was observed. Among those who exhibited PR, the majority exhibited standard PR. The average asymmetry ratio of standard PR to non-standard PR is 13.73. For more details, readers can refer to Appendix A in Shi et al. (2024). Given that Shi et al. (2024) used the same PR lottery pair as in Butler and Loomes (2007), they demonstrated the consistency between the results of the two studies.

Shi et al. (2024) found more than half of the subjects exhibited stochastic choices. Specifically, 50.3% of subjects reversed their choices at least once in the nine repeated DC between the P-bet and the \$-bet. Even after considering potential choice errors by allowing for one different choice among nine repeated direct choices, 24.5% of subjects exhibited inconsistent choices and changed their choices more than once. The frequent occurrence of both PR and stochastic choice in the experiment provides a basis for analyzing whether stochastic choice can explain the PR phenomenon. For the subsequent analysis, we follow Butler and Loomes (2007) and employ the PR pattern categorized by CE valuations and the first DC.

3.3.2 Data from Loomes and Pogrebna (2017)

Loomes and Pogrebna (2017) conducted two PR experiments. In this paper, we perform analysis on their second experiment, as it elicited CE valuations through two approaches: binary choices (BC approach) and direct valuations (DV approach). Our analysis includes data from direct choices (DC), CE valuations under the BC approach CE(BC), and CE valuations under the DV approach CE(DV).

The PR lotteries in this experiment are a P-bet with an 80% chance of winning £12 and £0 otherwise, and a \$-bet with a 25% chance of winning £50 and £0 otherwise. The DC was presented to subjects eight times, separated by other choices. When eliciting CE(BC), subjects faced binary choices between a bet and a series of changing sure payments. Each binary choice of CE valuation was randomly presented to subjects four times. Although both Shi et al. (2024) and Loomes and Pogrebna (2017) elicited CE through BC approach, there are some important differences: 1) The sure payments in the former study were adjusted using a bisection process, while the latter fixed the sure payments to the integer values within [4,11]. 2) In the former study, the binary choices of CE valuations were presented in sequence without interruption from other choices, while in the latter study, binary choices of CE valuations were presented randomly and were separated by other choices. The CE(BC) process in Loomes and Pogrebna (2017) is close to the DC process, with the only difference being that the alternative option is a sure payment. Under the DV approach of CE valuations, the CE valuations were elicited by asking subjects to state the amount Y at which they would prefer to play the lottery rather than receive the amount, and at Y+0.10 they would prefer to receive the amount rather than play the lottery (see Figure 6 in Loomes and Pogrebna (2017)).

As previously described, Loomes and Pogrebna (2017) has two CE valuation ap-

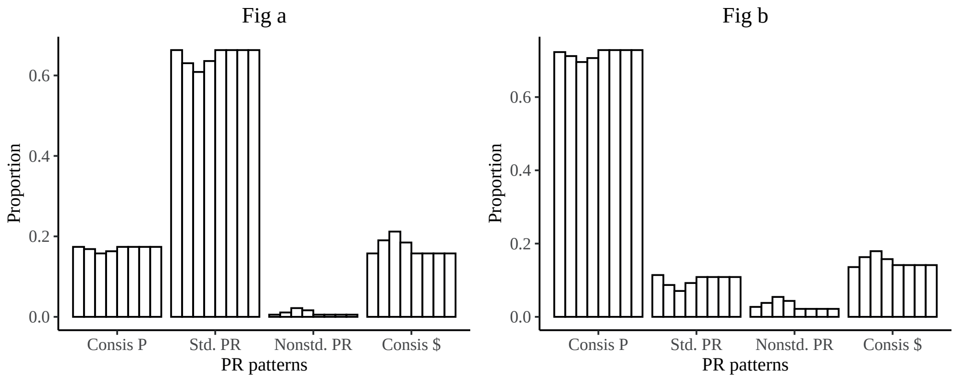


Figure 3.2: PR patterns categorized according to eight different DC in Loomes and Pogrebna (2017). In Fig a, PR patterns are classified by DC and CE(DV). In Fig b, PR patterns are classified by DC and CE(BC).

proaches, DV and BC. When we categorize the PR patterns by DC and CE(DV) as in most classical PR experiments, the proportions of PR patterns are illustrated in Fig a of Figure 3.2. The magnitude and asymmetry of PR are even larger compared to Shi et al. (2024). On average, 65.8% of subjects exhibited PR, and the asymmetry ratio of standard PR to non-standard PR is 91.88. Similar to Shi et al. (2024), the choice pattern differs significantly from the valuation pattern. 83.7% of subjects chose the P-bet at least four times while only 17.9% of subjects valued the P-bet as equal to or more than the $-bet in the CE(DV). The mean of CE_P and $CE_\$$ are 8.9 and 19.9, respectively. The one-sided Wilcoxon signed-rank test suggests that CE_P is significantly less than $CE_\$$ ($p < 0.01$). When we categorize the PR patterns by DC and CE(BC), substantially smaller proportions of PR are observed. As illustrated in Fig b of Figure 3.2, On average, only 13.1% of subjects exhibited PR, and the asymmetry ratio of standard PR to non-standard PR is 3.74. The choice pattern becomes more consistent with the valuation pattern. Specifically, 83.7% of subjects chose the P-bet at least four times and 75.0% of subjects valued the P-bet as equal to or more than the $-bet in CE(BC). The mean of CE_P and $CE_\$$ under the BC approach are 8.18 and 6.55, respectively. The one-sided Wilcoxon signed-rank test suggests that CE_P is significantly larger than $CE_\$$ ($p < 0.01$). As the PR patterns categorized by different DC are similar, we employed the PR proportions categorized by the first DC listed in the data file of Loomes and Pogrebna (2017) in the subsequent analysis.

Loomes and Pogrebna (2017) also find that a significant proportion of subjects exhibited stochastic choices, particularly in their CE valuation choices. In the experiment, 91.3% of subjects chose different options in at least one repeated choice. Specifically, 18.5% of subjects exhibited stochastic choice in DC and 90.2% of subjects exhibited stochastic choice in CE valuation. Given the high ratio of stochasticity in CE valuations, we perform a robustness analysis to examine whether stochastic choices account for the PR phenomenon after allowing for stochastic choices in both DC and CE tasks.

3.4 Results

In this section, we first report analysis results based on Shi et al. (2024) and then report results based on Loomes and Pogrebna (2017).

3.4.1 Estimating stochastic choice function

	The number of times choosing the P-bet									
Statistics	0	1	2	3	4	5	6	7	8	9
N	33	14	14	8	11	5	4	15	17	54
r	0.03	0.00	0.14	0.00	0.18	0.20	0.25	0.27	0.24	0.31

Table 3.1: Results of Nine Repeated Direct Choices: N represents the number of subjects choosing the P-bet a varying number of times (from 0 to 9) across nine repeated choices. r is the ratio of subjects evaluating the P-bet more based on the number of times they chose the P-bet.

Table 3.1 summarizes the changes in valuation patterns as the stochastic choice frequency varies in nine repeated direct choices. As the number of times choosing the P-bet increases from 0 to 9, we observe a generally increasing proportion of subjects valuing the P-bet higher, with some fluctuations, ranging from 0.03 to 0.31. However, the proportion of subjects valuing the P-bet higher remains below 0.5, even among those who always chose the P-bet in all nine repeated direct choices.

Below, for each subject, we estimate the stochastic choice function and the utility curvature from DC and CE(BC). We obtain the maximum likelihood estimators by maximizing the likelihood in equation (3.3) in Subsection 3.2.1. Table 3.2 reports the mean, standard deviation, median, and 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the estimated stochastic choice function parameter η and the estimated utility curvature parameter γ. As shown in M1, when using the Logit stochastic choice function, the mean estimated η is 2.11 (median = 0.86). The mean estimated γ is 0.78 (median = 0.78), indicating a generally risk-averse attitude. When using the stochastic choice function of Blavatskyy (2009), as we can see in M2, the mean

Estimation	Mean	Median	1%	5%	25%	75%	95%	99%
η								
M1	2.11 (2.88)	0.86	0.03	0.09	0.38	1.99	9.92	9.97
M2	1.94 (3.05)	0.55	0.01	0.02	0.12	2.06	9.93	9.99
M3	1.37 (2.70)	0.22	0.00	0.02	0.07	0.69	8.11	9.97
γ								
M1	0.78 (0.33)	0.78	0.04	0.28	0.58	0.97	1.42	1.77
M2	0.89 (0.31)	0.84	0.36	0.49	0.68	0.99	1.44	1.86
M3	0.83 (0.32)	0.81	0.19	0.37	0.63	0.98	1.32	1.87

Table 3.2: Statistics of estimated parameters from data in Shi et al. (2024). The statistics are mean, median, and the 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ through joint estimation from direct choices and CE valuation choices. The standard deviations are in parentheses. Under M1, we employ the Logit stochastic choice function. Under M2, we employ the stochastic choice function proposed by Blavatskyy (2009). Under M3, we consider the heteroscedastic random errors in the stochastic choice function in Blavatskyy (2009).

η is 1.94 (median = 0.55). The mean estimated γ is 0.89 (median = 0.84) in M2, suggesting a slight risk-averse attitude. After considering the heteroscedastic random errors in M3, the mean of η is 1.37 (median = 0.22) and the mean of γ for M3 is 0.83 (median = 0.81).

3.4.2 PR patterns

With the estimated stochastic functions, we can compute the direct choice probability $p(P, \$)$ and the CE valuation probability $p(CE_P \geq CE_\$)$. By combining these probabilities, we can calculate the individual-level estimated probabilities of PR patterns. Testing the estimated probabilities through one million simulations as described in Subsection 3.2.2, we obtain the means and the 95% confidence intervals of the simulated proportions as illustrated in Figure 3.3 (details in Table B.1 of Appendix B.1.1). It is shown that the simulated proportions fail to capture the ratios of PR patterns. Specifically, when we employ the Logit stochastic function in M1, the mean of simulated proportions of PR (standard PR and non-standard PR) is 26.1%, substantially less than the actual proportion of 47.5%. The mean asymmetry ratio of standard PR to non-standard PR is 1.58, much lower than the actual ratio of 9.33. When we employ the stochastic function in Blavatskyy (2009) and consider the homoscedastic random error in M2, the mean simulated proportions of PR is 28.8%, below the actual proportion of 47.5%. The mean asymmetry ratio of standard PR to non-standard PR is 1.48, lower than the actual ratio of

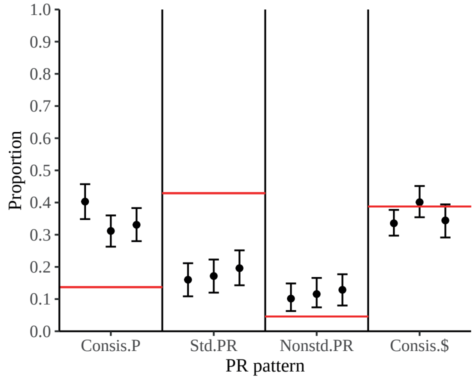


Figure 3.3: Comparison between simulated and actual proportions of CE PR patterns in Shi et al. (2024). For each CE PR pattern, the simulation results displayed from left to right correspond to simulated proportions using stochastic models M1 to M3. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Black dots are means of simulated proportions. Black bars denote [5%, 95%] intervals of simulated proportions. Red lines depict actual proportions in the experiment.

9.33. When we employ the stochastic function in Blavatskyy (2009) and consider the heteroscedastic random error in M3, the mean simulated proportions of PR is 32.5%, still lower than the actual proportion of 47.5%. Meanwhile, the mean asymmetry ratio of standard PR to non-standard PR is 1.52, much lower than the actual ratio of 9.33. The differences between the simulated and actual proportions of consistent preferences are significant for all models M1 to M3 ($p < 0.01$ according to Wilcoxon signed-rank test). Similarly, the differences between the simulated and actual asymmetry ratio are significant for all models M1 to M3 ($p < 0.01$ according to Wilcoxon signed-rank test).

The above results suggest that after allowing for stochastic choice and assuming each individual follows a consistent stochastic function in both direct choices and valuation choices, the aggregate-level simulated PR proportions are different from the observed proportions. Specifically, the simulated PR proportions underestimate the magnitude of PR and the asymmetry between standard PR and non-standard PR. To understand the source of the differences between the simulated and actual PR patterns, we examine the stochastic choices in DC and CE individually in the following analysis.

First, we compare the estimated and the actual choice patterns. Sub-table a) of

Sub-table a): DC pattern

Model	Act: $p_{DC} \geq 0.5$	Est: $p_{DC} \geq 0.5$	Act: $p_{DC} < 0.5$	Est: $p_{CE} < 0.5$
M1	95 (0.54)	116 (0.66)	80 (0.46)	59 (0.34)
M2	95 (0.54)	92 (0.53)	80 (0.46)	83 (0.47)
M3	95 (0.54)	100 (0.57)	80 (0.46)	75 (0.43)

Sub-table b): DC probability

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
	mean	0.56	0.56	0.48	0.53	0.00	-0.08	-0.04
$p(P, \$)$		(0.40)	(0.36)	(0.34)	(0.30)	(0.17)	(0.15)	(0.23)
	median	0.67	0.67	0.52	0.55	0.00	-0.04	-0.01

Table 3.3: The comparison between the actual and the estimated choice patterns in the experiment of Shi et al. (2024). M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Sub table a: the number of subjects choosing the P-bet more ($p_{DC} \geq 0.5$) and choosing the \$-bet more ($p_{DC} < 0.5$) based on the actual and the estimated data. The proportions are in parentheses. Sub table b: mean and median of the actual choosing probability, the estimated choosing probability, and the difference between the actual and the estimated choosing probability. The standard deviations are in parentheses.

Table 3.3 summarizes the comparison of the estimated and the actual numbers and proportions of two choice patterns in DC: $p(P, \$) \geq 0.5$ and $p(P, \$) < 0.5$. Under the stochastic choice model M1, the estimated proportion is 0.66, significantly larger than the actual proportion of 0.54 (proportion test, $p < 0.05$). Under M2 and M3, the estimated proportions are 0.53 and 0.57, which are close to the actual proportion of 0.54 (proportion tests, $p > 0.10$ for both M2 and M3). We also compare the estimated DC pattern with the actual DC pattern at the individual level. The results suggest that the estimated DC pattern under M1 significantly differs from the actual pattern (McNemar's chi-squared tests, $p < 0.01$), while the differences between estimated DC patterns under M2 and M3 and the actual pattern are not significant (McNemar's chi-squared tests, $p > 0.10$ for both M2 and M3). The statistics of the estimated DC probabilities and their differences from the actual DC probabilities are reported in Sub-table b) of Table 3.3. The estimated $p(P, \$)$ under M1 is close to the observed frequency (Wilcoxon signed-rank test, $p > 0.10$ for M1), while the estimated $p(P, \$)$ under M2 and M3 are significantly less than the actual frequency (Wilcoxon signed-rank test, $p < 0.01$ for M2, $p < 0.05$ for M3).

Although some differences exist between the estimated and the observed $p(P, \$)$, the mean estimates under M1 and M3, as well as the median estimates under M1, M2 and M3, exceed 50%. This suggests a higher frequency of choosing the P-bet in DC at the aggregate level, which is consistent with the observed data.

Sub-table a): CE pattern

Model	Act: $CE_P \geq CE_\$$	Est: $CE_P \geq CE_\$$	Act: $CE_P < CE_\$$	Est: $CE_P < CE_\$$
M1	32 (0.18)	93 (0.53)	143 (0.82)	82 (0.47)
M2	32 (0.18)	70 (0.40)	143 (0.82)	105 (0.60)
M3	32 (0.18)	71 (0.41)	143 (0.82)	104 (0.59)

Sub-table b): CE valuations

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
	mean	14.64	14.32	15.46	14.81	-0.42	0.82	0.17
CE_P		(4.11)	(3.51)	(2.44)	(3.07)	(4.97)	(3.31)	(2.63)
	median	14.25	15.16	15.71	15.43	0.57	1.07	0.48
	mean	21.89	13.42	16.24	14.72	-8.47	-5.65	-7.17
$CE_\$$		(10.83)	(8.42)	(7.97)	(8.23)	(12.94)	(9.34)	(9.04)
	median	19.38	13.45	15.39	14.34	-3.73	-1.93	-5.34

Table 3.4: The comparison between the actual and the estimated CE valuation patterns in the experiment of Shi et al. (2024). Sub-table a): the number of subjects valuing the P-bet more and valuing the \$-bet more based on the actual and the estimated data. The proportions are in parentheses. The estimated choice patterns based on stochastic choice functions are categorized according to the estimated CE valuation probability $p(CE_P \geq CE_\$) \geq 0.5$ or $p(CE_P \geq CE_\$) < 0.5$. Sub-table b): mean and median of CE valuations, the estimated CE valuations, and the difference between the actual and the estimated CE valuations. The standard deviations are in parentheses. M1 is the Logit stochastic choice model. M2 is the Blavatskyy (2009) choice model with homoscedastic random errors. M3 is the Blavatskyy (2009) choice model with heteroscedastic random errors.

Then we compare the estimated and the actual CE valuations. Sub-table a) of Table 3.4 summarizes the comparison of the estimated and the actual numbers and proportions of two CE patterns: $CE_P \geq CE_\$$ and $CE_P < CE_\$$. For the estimated CE patterns under stochastic functions, we classify $CE_P \geq CE_\$$ by the CE valuation probability $p(CE_P \geq CE_\$) \geq 0.5$ and $CE_P < CE_\$$ by the CE valuation probability $p(CE_P \geq CE_\$) < 0.5$. All stochastic models M1, M2 and M3 estimate significantly larger proportions of $p(CE_P \geq CE_\$)$ compared to the actual proportion (proportion tests, $p < 0.01$ for all stochastic choice models). At the individual level, the estimated CE valuation patterns differ significantly from the actual CE valuation pattern (McNemar's chi-squared tests, $p < 0.01$ for all stochastic choice models). Sub-table b) of Table 3.4 further compares the estimated and the actual CE valuations. Following Loomes and Pogrebna (2017), we use the stochastic indifference points as the CE valuations under the stochastic choices.[4] The results show that the estimated CE_P are similar to the observed CE_P under all stochastic models M1 to M3 (Wilcoxon signed-rank test, $p > 0.10$), while the estimated $CE_\$$ are significantly less than the actual $CE_\$$ (Wilcoxon signed-rank test, $p < 0.01$ for all stochastic choice models). Regarding the relationship between CE_P and $CE_\$$, the estimated CE valuations underestimate the extent to which $CE_\$$ exceeds CE_P. In the experiment, the CE_P is significantly less than the $CE_\$$ (mean: 14.64 vs. 21.89, median: 14.25 vs. 19.38, $p < 0.01$ according to Wilcoxon signed-rank test). However, the estimated CE_P is not significantly less than the observed $CE_\$$ (Wilcoxon signed-rank test, $p > 0.10$ for all stochastic choice models), and CE_P is even significantly larger than $CE_\$$ under M1 (Wilcoxon signed-rank test, $p < 0.01$).

After comparing the estimated and the actual direct choice and the CE valuations, we find that the difference between the simulated PR proportion and the actual PR proportion is mainly caused by the estimation bias for $CE_\$$. The stochastic functions underestimate $CE_\$$, thereby underestimating both the asymmetry ratio and the magnitude of CE PR. For robustness, we conduct a check using the CARA utility function in Appendix B.1.2. The result is consistent with our main analysis using the CRRA utility function that stochastic choices are insufficient to explain the PR. Although the stochastic choice functions based on the CARA utility function and the Logit choice model estimate a relatively large proportion of PR, they fail to capture the actual behaviors as they significantly underestimate the CE valuations (Wilcoxon one-sided signed-rank test, $p < 0.01$ for both CE_P and $CE_\$$).

In addition to analyzing the CE PR patterns, we perform a similar analysis for PE PR. Generally, PE PR is better explained after allowing for stochastic choice. While differences still exist, the simulated PE PR proportions are closer to the actual PE PR proportions. The analysis of PE PR is reported in Appendix B.2.

3.4.3 Allowing for stochastic valuation

Shi et al. (2024) did not repeat the valuation choices as they employed a novel method to elicit valuation randomization intervals, which served their research pur-

[4]The stochastic indifference point is the sure payment at which the stochastic choice probability between the bet and the sure payment is 50%-50%.

pose well. Since this paper focuses on stochastic choice in repeat choices, we perform a robustness analysis on Loomes and Pogrebna (2017) as they repeat the direct choice eight times and each valuation choice four times. They conducted two experiments in their paper. Our analysis focuses on their second experiment, where they used not only binary choices (the BC approach) but also direct valuation (the DV approach) to elicit CE valuations.

Following section 3.2.1, we estimate the stochastic choice function and utility curvature from all binary choices using the MLE by maximizing the likelihood in equation (3.3). As before, we consider the three stochastic choice models: the Logit model, the model proposed in Blavatskyy (2009) with homoscedastic random errors, and the model proposed in Blavatskyy (2009) with heteroscedastic random errors. Table 3.5 summarizes the mean, standard deviation, median, and 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ.

The means of η under M1 and M2 are 2.86 and 2.33 (medians: 1.99 and 1.76), indicating a great sensitivity to the utility difference in choice probability. When considering heteroscedastic random errors in M3, the mean of η decreases to 0.95 (median = 0.49). The means of estimated γ in M1, M2, and M3 are from 0.65 to 0.67, indicating generally risk-averse utility curvatures. After obtaining the stochastic choice function for each subject, we can calculate the probability of valuing the P-bet more $p(CE_P \geq CE_\$)$ and the probability of choosing the P-bet $p(P, \$)$. [5] By combining the probabilities, we obtain the estimated probability of each PR pattern for each subject. After running one million simulations as described in Subsection 3.2.2, we obtain the simulated proportion of each PR pattern. Figure 3.4 (details refer to Table B.2 in Appendix B.1.1) illustrates the mean and [5%, 95%] intervals of the simulated proportions.

When classifying the PR patterns with DC and CE(DV) as most PR experiments, the actual proportions of standard PR and non-standard PR fall outside the [5%, 95%] of all simulated proportions. Similar to previous findings, the simulated proportions underestimate both the magnitude of PR and the asymmetry ratio between

[5] Under the DV approach, CE valuations follow continuous distributions within the dominance range. We calculate the probability that the CE valuation of the P-bet is higher than that of the \$-bet using the function described by Blavatskyy (2009): $p(CE_P \geq CE_\$) = \int p(x, \$) dp(P, x)$. Under the BC approach, sure payments are integers and each choice was repeated four times. The stochastic indifference points of CE_P and $CE_\$$ follow discrete distributions with steps of 0.25. We approximate the probability density function of the continuous distribution and calculate the density for discrete values with intervals of 0.25.

Sub-table a

η	Mean	Median	1%	5%	25%	75%	95%	99%
M1	2.86 (2.71)	1.99	0.00	0.17	1.16	3.34	9.88	9.97
M2	2.33 (1.96)	1.76	0.06	0.17	1.03	3.06	6.54	8.56
M3	0.95 (1.43)	0.49	0.02	0.08	0.29	0.90	4.28	7.27

Sub-table b

γ	Mean	Median	1%	5%	25%	75%	95%	99%
M1	0.66 (0.38)	0.63	0.01	0.15	0.52	0.78	1.03	2.59
M2	0.67 (0.38)	0.63	0.03	0.19	0.51	0.78	1.04	2.59
M3	0.65 (0.46)	0.56	0.00	0.10	0.42	0.79	1.32	2.63

Table 3.5: Statistics of estimated parameters from data in Loomes and Pogrebna (2017). The statistics are mean, median, and the 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ through joint estimation from direct choices and CE valuation choices. The standard deviations are in parentheses. Under M1, we employ the Logit stochastic choice function. Under M2, we employ the stochastic choice function proposed by Blavatskyy (2009). Under M3, we consider the heteroscedastic random errors in the stochastic choice function in Blavatskyy (2009).

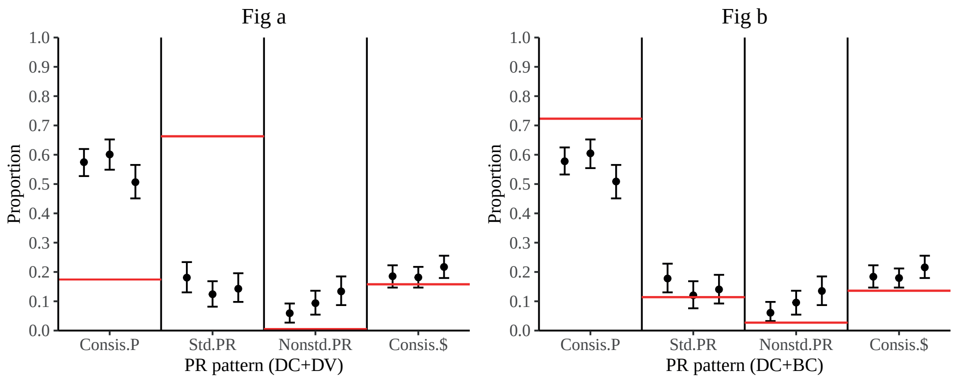


Figure 3.4: Comparison between simulated and actual proportions of CE PR patterns in Loomes and Pogrebna (2017). For each PR pattern, the three simulation results displayed from left to right correspond to simulated proportions using stochastic models M1 to M3. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Black dots are means of simulated proportions. Black bars denote [5%, 95%] intervals of simulated proportions. Red lines depict actual proportions in the experiment. In Fig a, the actual PR patterns are classified by DC and the CE(DV). In Fig b, the actual PR patterns are classified by DC and the CE(BC).

standard PR and non-standard PR. As shown in the left figure of Figure 3.4, the mean simulated proportion of PR (including both standard PR and non-standard PR) ranges from 21.8% to 27.7% under M1 to M3, significantly less than the actual ratio of 69.5% (Wilcoxon signed-rank test, $p < 0.01$ for all). The simulated asymmetry ratio ranges from 1.1 to 3.1, significantly lower than the actual asymmetry ratio of 132.6 (Wilcoxon signed-rank test, $p < 0.01$ for all). Under the BC approach, where valuations are elicited through binary choices that are mixed with other choices and presented randomly, the difference in framing between DC and CE is minimized. When classifying the PR patterns with DC and CE(BC), the proportions of PR decrease substantially, and the differences between the simulated proportions and the actual proportions diminish rapidly. As shown in Fig a of Figure 3.4, the actual proportions of standard PR and non-standard PR are either inside or close to the [5%, 95%] of the simulated proportions. The mean simulated proportion of PR (including both standard PR and non-standard PR) ranges from 21.6% to 27.5%, larger than the actual proportion of 14.1%. The mean asymmetry ratio of standard PR to non-standard PR (including both standard PR and non-standard PR) ranges from 1.0 to 2.9, less than the actual ratio of 4.2. Although differences remain, they are considerably reduced compared to the PR patterns classified by DC and CE(DV).

Following the previous analysis, we also examine direct choice and CE valuation separately by comparing the estimated results with the actual data. As reported in Table B.3 and Table B.4 in Appendix B.1.1, the estimated proportions of choosing the P-bet more are close to the actual proportion (proportion test, $p > 0.10$ for all stochastic choice models). The estimated proportions of subjects valuing the P-bet higher are much more than the actual proportion when using the conventional CE(DV) approach (proportion test, $p < 0.01$ for all stochastic choice models). When using the CE(BC) approach, the estimated proportions are closer to the actual proportion (proportion test, $p < 0.05$ for M1, $p > 0.10$ for M2 and M3). When we calculate the difference between the estimated and the observed $CE_\$$, the estimated $CE_\$$ is substantially less than the observed $CE_\$(DV)$, with the mean differences ranging from -13.72 to -13.44 under M1 to M3. However, when comparing with the observed $CE_\$(BC)$, the estimated values are much closer, with mean differences ranging from -0.41 to -0.14 under M1 to M3.

Generally, when classifying PR with DC and CE(DV), the analysis results of Loomes and Pogrebna (2017) are consistent with Shi et al. (2024). The stochastic choice functions underestimate both the magnitude and the asymmetry of PR. After the contextual differences between choice and valuation are concealed by using CE(BC), the PR phenomenon becomes less pronounced, and the differences between the estimated PR and the actual PR decrease rapidly.

3.5 Conclusion

In this paper, we study whether PR can be explained by stochastic choices. Building on existing literature, we mainly focus on the stochastic choice driven by decision noise and base our analysis on random utility models. In the analysis, we use two independent data sets, Shi et al. (2024) and Loomes and Pogrebna (2017). For each subject, we estimate a consistent stochastic choice function for each subject from the direct choices and the valuation choices. We then use it to compute the probabilities of PR patterns. The analysis involving a wide range of random utility models shows that the simulated PR proportions are not comparable with the actual PR proportions in terms of magnitude and asymmetry between standard and non-standard PR. Additionally, we separately compare the estimated direct choices and valuation choices with the actual behaviors observed in the experiment, the results suggest the stochastic functions significantly underestimate $CE_\$$.

This paper has two main implications: First, our analysis suggests that PR cannot be fully explained by stochastic choices under random utility models. By comparing the actual proportions with the simulated proportions of PR patterns computed based on a wide range of random utility models, we find that a consistent stochastic function cannot capture the behaviors in the PR experiment under any of the models. Specifically, the simulated proportions underestimate the magnitude and the asymmetry of PR. Second, we provide evidence that preferences are sensitive to different elicitation methods even after allowing for stochastic choice. Our results suggest that the PR phenomenon, which reveals the systematic disparity between choice and valuation, cannot be fully explained by stochastic choice. Additionally, when changes in elicitation methods are less noticeable, less PR is observed in the experiment, and the stochastic function provides a better estimate of the exhibited behaviors. For instance, both Loomes and Pogrebna (2017) and Shi et al. (2024) employ binary choices to elicit valuations. Loomes and Pogrebna (2017) reduces the differences in the framing between the choice task and the valuation task by concealing the iterative process in valuation elicitation. To do this, they separate the valuation choices with other choices and present the valuation choices in random order. A substantially low proportion of PR was found when categorizing PR patterns with DC and CE(BC), and the stochastic choice functions perform much better in estimating the PR proportions. This finding aligns with Bostic et al. (1990). A typical theory explaining why preferences are sensitive to different elicitation methods is scale compatibility(Tversky et al., 1990a). Our findings that stochastic functions underestimate $CE_{\$}$ and perform better in estimating PE PR are consistent with the idea of scale compatibility. Therefore, we suggest that the PR should be explained by a combination of stochastic choices and context-sensitive decision theories, such as scale compatibility.

Chapter 4

Intertemporal valuation uncertainty and present bias

While there is no uncertainty in the present value of an immediate payment, one may be uncertain about the present value of a delayed reward. We demonstrate theoretically that uncertainty in valuing delayed rewards, coupled with caution toward this uncertainty, could result in present bias. In two pre-registered experiments, we estimated subjects' present bias and measured their valuation uncertainty toward timed rewards. Consistent with our theoretical analysis, when facing monetary timed rewards, subjects with higher valuation uncertainty exhibited significantly stronger present bias. When the timed rewards were non-monetary, subjects revealed valuation uncertainty about the present monetary *equivalents of both the immediate and delayed rewards, and they exhibited substantially weaker present bias. Finally, higher valuation uncertainty related significantly to stronger front-end delay effects, the canonical signature of present bias.*

4.1 Introduction

What is the present equivalent of €100 received in four weeks? When faced with this question, many individuals may consider a range of possible values (e.g., between €92 and €99) instead of a single, precise value. Despite this potential uncertainty in valuation, experiments and surveys routinely ask individuals for a single present equivalent of the delayed rewards, which are then used to estimate discount factors. The elicitation of the present equivalent of a delayed reward involves individuals comparing the delayed reward with immediate payments explicitly (as in a price list) or implicitly (as in the Becker–DeGroot–Marschak mechanism, Becker et al., 1964). In those comparisons, since the valuation of an immediate payment is certain (the present equivalent of €100 received immediately is €100) while the valuation of the delayed reward is uncertain, individuals may select a value in the lower end of the range of possible values out of caution as the single present equivalent of the delayed reward. This leads to "extra discounting" of the delayed reward. Consequently, individuals exhibit a disproportional preference for immediate rewards over even slightly delayed rewards. In this paper, we demonstrate theoretically and experimentally that individuals' valuation uncertainty regarding delayed rewards, coupled with their cautious attitudes when faced with this uncertainty, is an important reason for present bias.

Determining the present equivalent of a delayed reward is a complex process. Consequences of the delayed reward can be hard to foresee, and its utility is difficult to estimate (von Böhm-Bawerk, 1890; Pigou, 1920; Gabaix and Laibson, 2017; Chakraborty, 2021). Even if individuals are able to estimate their utility from the delayed reward, determining its present equivalent requires them to make intricate trade-offs between the reward and the delay. Trade-offs across dimensions are cognitively demanding and prone to errors (Enke et al., 2023), which is evident from the inconsistency and probabilistic process of such choices (Tversky, 1972; Tversky et al., 1990b). Experiments on discounting are further complicated by subjects' belief that they may receive the delayed reward on a date different from the promised date, or not receive it at all (Halevy, 2008). Due to these complexities, individuals are rarely fully certain about their valuation of the delayed reward (Enke et al., 2023). Consequently, individuals' value of the delayed reward can be influenced by visceral factors (Frederick et al., 2002), and they often behave as if they have random discounting attitudes (Lu and Saito, 2018). When individuals face valuation uncertainty but need to determine a single value nonetheless, Cerreia-Vioglio et al. (2024) show theoretically in decision-making under risk that individuals may adopt a conservative criterion and act cautiously.

Building on these studies, we formalize a behavioral model of intertemporal valuation, in which valuation uncertainty and caution lead to an extra discount of the delayed reward, however short the delay is. The behavioral model is consistent with a heuristic of making decisions cautiously when the individual is unsure of what to do (Cerreia-Vioglio et al., 2024) and a preference representation model extended from Klibanoff et al. (2005), Cerreia-Vioglio et al. (2015), and Chakraborty (2021). We show that present bias can arise from the discontinuous jump in valuation uncertainty as soon as a delay is introduced.

Motivated by the theoretical analysis, we conducted two pre-registered experiments to investigate the role of valuation uncertainty in present bias. The experiments mainly consisted of two parts. The first part elicited the monetary equivalent of timed rewards in terms of the monetary payment received at an earlier date via price lists. In each price list, subjects chose between the fixed timed reward and the varying monetary payment received at an earlier date by indicating one switching point (monetary equivalents hereafter). In the second part, using the same price lists, we elicited the range of earlier monetary payments for which subjects strictly randomized between receiving the fixed reward and receiving the earlier monetary payment (R-range hereafter). We had three treatments. The baseline treatment involved eliciting present monetary equivalents and R-ranges of two monetary timed rewards: a monetary reward of €10 received in 4 weeks and 24 weeks. In the coupon treatment, we considered the present monetary equivalents and R-ranges of three non-monetary timed rewards: a €10 Amazon coupon received today, in 4 weeks, and in 24 weeks. In the front-end delay treatment, we asked subjects for the present monetary equivalent of €10 received in 4 weeks, the 4-week monetary equivalent of €10 received in 8 weeks, and the corresponding R-ranges.

Following recent studies on incomplete or imprecise preferences (Cettolin and Riedl, 2019; Agranov and Ortoleva, ress; Halevy et al., 2023; Arts et al., 2024), we use the R-range as a measure of valuation uncertainty. An analysis based on our preference representation model provides additional theoretical support for this measurement. We show that subjects perceiving valuation uncertainty about a timed reward hedge this uncertainty by strictly randomizing between the timed reward and a range of immediate monetary payments, and the R-range captures valuation uncertainty. Furthermore, valuation uncertainty affects the minimal payment of the R-range but not the maximal payment (lower bound and upper bound of the R-range hereafter, respectively). This leads to our first hypothesis that, in the baseline treatment, subjects with larger R-ranges exhibit stronger present bias. In addition, subjects exhibit greater present bias when their discount factors are estimated from the lower bounds of the R-range than the upper bounds.

We further demonstrate that, in the coupon treatment, subjects will display significant valuation uncertainty even when non-monetary rewards are paid out immediately. This is because, just like the difficulty in determining the present equivalent of a delayed reward, non-monetary rewards are difficult to price in *monetary terms*. Since valuation uncertainty is always present for non-monetary rewards, whether they are delayed or not, the impact of valuation uncertainty on present bias is weaker on non-monetary rewards than on monetary rewards. This leads to our second hypothesis that subjects have a positive R-range even when the non-monetary reward is immediate, and they exhibit significantly weaker present bias when facing non-monetary timed rewards. Further, subjects exhibit similar levels of present bias regardless of whether the upper bounds or the lower bounds are used to estimate their discount factors.

Finally, we show that valuation uncertainty can also explain the front-end delay effect, which is often considered the canonical signature of present bias. The front-end delay effect describes the phenomenon that subjects show greater impatience when they compare a delayed reward with an immediate payment relative to when a common delay is added to both options. This effect can be attributed to valuation uncertainty and caution, as subjects experience valuation uncertainty with the delayed reward but not with the immediate reward in the first comparison, whereas they experience valuation uncertainty with both rewards in the second comparison. This leads to a larger additional discount of the later reward in the first comparison than in the second comparison. Base on this reasoning, our third hypothesis states that, depending on how the individual perceives the earlier reward in the second comparison (as its present equivalent value or as a valuation currency), the R-range of the delayed reward in the comparison of immediate vs. delayed rewards or the difference of the R-range in the two comparisons are correlated with the front-end delay effect.

Our experimental results support our pre-registered hypotheses.[1] First, when the delayed rewards were monetary, subjects with a positive R-range exhibited signif-

[1] https://aspredicted.org/ka8ic.pdf and https://aspredicted.org/tr29a.pdf. The experimental analysis paid more attention than the pre-registered analysis to the relationship between the difference in the R-range in the two comparisons and the front-end delay effect.

icantly stronger present bias than those with no R-range. The median estimated present bias parameter β in the quasi-hyperbolic $\beta - \delta$ model (Laibson, 1997) was 0.85 when the R-range is positive vs. 0.98 when the R-range is zero. The relationship between present bias and R-range was significant and stable in a series of regressions and robustness checks. Subjects exhibited significantly weaker present bias when the estimation was based on the upper bound of the R-range than the lower bound (median: $\beta = 0.97$ vs. $\beta = 0.85$).

Second, when the timed rewards were the €10 Amazon coupon, more than half (60%, 175 subjects out of 291) of the subjects had a positive R-range over the non-monetary reward even when it was paid out immediately. Subjects exhibited significantly weaker present bias than with delayed monetary rewards (median: $\beta = 0.98$ for Amazon coupons vs. $\beta = 0.91$ for the monetary rewards). Furthermore, the difference between β estimated from the upper bound and the lower bound was much smaller for Amazon coupons (the upper bound vs. the lower bound: median $\beta = 1.0$ vs. 0.96 for Amazon coupons and 0.98 vs. 0.85 for monetary rewards).

Third, subjects in the front-end delay treatment exhibited a significant front-end delay effect and substantial R-ranges. More importantly, subjects without a positive R-range had a significantly weaker front-end delay effect than those with at least one positive R-range (mean: 0.984 vs. 0.945). Further, the front-end delay was significantly and negatively related with the R-range of the delayed reward in the first comparison (Spearman's $\rho = -0.198$) and the difference of the R-ranges in the two comparisons (Spearman's $\rho = -0.261$).

Overall, our findings suggest the crucial role of valuation uncertainty and caution in present bias. Together with other studies (e.g., Gabaix and Laibson, 2017; Vieider, 2023; Enke et al., 2023), they suggest that present bias may reflect more than non-standard time preferences such as temptation or myopic short-termism. Instead, non-motivational factors like valuation uncertainty also play an important role.

Our study builds directly on the literature that unites intertemporal preferences and preferences under risk and uncertainty (see e.g., Sozou, 1998; Dasgupta and Maskin, 2005). Keren and Roelofsma (1995) and Weber and Chapman (2005) find that present bias weakens when an explicit objective risk is introduced to both the present and future rewards. Consistent with these two studies, Halevy (2008) proposes that the future is inherently risky, and individuals discount rewards disproportionately when they are delayed, just as they discount rewards disproportionately when rewards change from certain to risky. Following Halevy (2008), several experimental studies connect present bias with the certainty effect using non-linear probability weighting in rank-dependent expected utility theory (Epper et al., 2011; Miao and Zhong, 2015; Epper and Fehr-Duda, 2015, 2023; Diecidue et al., 2023). Theoretically, Kochov (2015) provides a Savage-style axiomatization to the maxmin expected utility of Gilboa and Schmeidler (1989) by giving it an intertemporal structure, in which individuals behave cautiously toward the future. Chakraborty (2021) replaces stationarity with weak present bias and shows that present-biased preferences can be represented as those of the individual who has a set of utility functions for the timed reward and evaluates it based on the most pessimistic one.

Building on these ideas, our study demonstrates the link between present bias and valuation uncertainty. We measure valuation uncertainty with the R-range and manipulate it by varying the types of rewards. While non-linear probability weighting can account for some of our results, the differences in the results between monetary and non-monetary rewards support the role of valuation uncertainty, which does not necessarily involve risk, and caution in present bias.[2] Overall, our results highlight the importance of considering valuation uncertainty when examining present bias and its underlying mechanisms.

Our study is also related to the growing body of literature on Bayesian cognitive noise models in which individuals combine a noisy mentally simulated signal with a prior (Khaw et al., 2021, 2022; Enke and Graeber, 2023; Frydman and Jin, 2023). Focusing on intertemporal choices, Vieider (2023) proposes a Bayesian cognitive noise model of time perception that jointly accounts for subadditivity and present bias, and finds supporting evidence by manipulating the level of noise in time perception and the decision environment where the prior is formed. Via a series of experiments, Enke et al. (2023) provide evidence that hyperbolic discounting reflects mistakes that are driven by the complexity of evaluating delayed rewards. Gabaix and Laibson (2017) demonstrate theoretically that a Bayesian individual who is perfectly patient but perceives future utilities with noise may appear to have hyperbolic time preferences. Based on Gabaix and Laibson (2017), Gershman and Bhui (2020) find that cognitive noise helps to explain the magnitude effect in

[2] Additionally, valuation uncertainty and caution better accommodate the positive R-range, present bias, and their relationship simultaneously compared to the non-linear probability weighting approach. This is because the existence of the positive R-range, present bias, and their relationship require different properties of the probability weighting function that may be difficult to satisfy at the same time. See Appendix C.2 for more details.

intertemporal discounting. Some of our results are surprisingly similar to the predictions in these studies, suggesting the crucial role of valuation uncertainty/cognitive noise in intertemporal choices.

Our study differs from these studies in three important ways. First, our approach does not involve Bayesian updating, and our focus is on explaining present bias. The focus of Bayesian cognitive noise models lies in explaining how insufficient updating leads to the insensitivity of discounting to time delays. Second, in our approach, the individual is aware of valuation uncertainty, and she deliberately chooses stochastically to hedge it. This allows us to measure valuation uncertainty with the R-range and test different predictions regarding the two bounds of the R-range. In Bayesian cognitive noise models, stochastic choices arise from random realization of signals and are thus unconscious, and cognitive noise is measured by the variability in choices made when facing the same decision problem repeatedly. A final difference is that we included non-monetary rewards and asked for their *monetary* equivalents. This allows us to test our hypothesis that valuation uncertainty arises from the complexity in transforming timed rewards into present monetary equivalents, rather than from noisy time perception. Most previous studies consider monetary rewards or the *non-monetary equivalent* of the delayed non-monetary reward (Attema et al., 2010; Augenblick et al., 2015; Bleichrodt et al., 2016; Cohen et al., 2020; Cheung et al., 2022; Enke et al., 2023), and they generally find similar or stronger present bias toward non-monetary rewards than monetary rewards (Cheung et al., 2021).[3]

The findings that valuation uncertainty or cognitive noise plays an important role in intertemporal choices have several important implications. First, it affects the choice of methods for eliciting intertemporal preferences. Our findings suggest that discount factors estimated from present monetary equivalents of delayed rewards (or future equivalents of immediate payments) are strongly affected by valuation uncertainty. To reduce this impact, one could consider estimating discount factors from intertemporal trade-offs involving only delayed rewards. Second, policies must take valuation uncertainty into account when interpreting people's intertemporal choices. For example, households often fail to save sufficiently for their pension. This shortfall may not necessarily stem from household impatience but rather from their inability to fully imagine their future utilities (Gabaix and Graeber, 2024). Policies that help to reduce valuation uncertainty rather than changing people's

[3]An exception is Cubitt et al. (2018), where subjects stated monetary compensation that made them indifferent between timed rewards of different delays and types.

preferences may thus be more effective in encouraging pension savings (Imas et al., 2022). Third, our findings can reconcile some important puzzles. For example, valuation uncertainty explains why subjects discount money excessively despite the possibility to borrow and lend money in financial markets at substantially lower interest rates (Cohen et al., 2020). Since the effect of valuation uncertainty extends beyond timed rewards (e.g., also to lotteries, non-monetary rewards), it may also explain why there exist significant associations between estimated discount factors and real-life outcomes such as wealth (Epper et al., 2020), even though such discounting should provide little information about time preferences (Cubitt and Read, 2007).

Our paper proceeds as follows. Section 4.2 introduces the theoretical model. Section 4.3 explains the experimental design and develops the hypotheses. The experimental results are reported in Section 4.4. Section 4.5 concludes and discusses possible future research.

4.2 Theoretical analysis

Let X_t (receiving X at time t, $t = 0, 1, 2, \ldots$) denote a timed reward, which can be monetary or non-monetary. To capture the individual's valuation uncertainty about the present monetary equivalent of X_t, we assume that the individual considers a set of present monetary equivalents. Let $\mathbb{ME}_{0,t} \subset \mathbb{R}$ denote this set, $\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}$ be an element. We further assume there exists a subjective cumulative distribution F_t over the set. Valuation uncertainty and caution can be captured as follows:

$$me_{0,t} = E_{F_t}(\mathrm{ME}_{0,t}) - \kappa VU_{0,t}, \tag{4.1}$$

where $me_{0,t}$ is the present monetary equivalent of X_t, $E_{F_t}(\mathrm{ME}_{0,t})$ is the mean of $\mathrm{ME}_{0,t}$ with respect to the belief distribution F_t, $VU_{0,t}$ is a measure of valuation uncertainty about X_t (e.g., the standard deviation of F_t), and κ captures the individual's attitude toward valuation uncertainty. This formulation can be applied to general settings in which the individual faces complexity in evaluating choice objects. Thus, valuation uncertainty is not unique to evaluations of timed rewards.

There are different ways to motivate Equation 4.1. For example, it can be viewed as "a heuristic adopted when agents are unsure of what to do" (Cerreia-Vioglio et al., 2024, p.31). A formal approach is to assume the following preference:

$$me_{0,t} = \phi^{-1}\left[\int_{\mathrm{ME}_{0,t}\in\mathbb{ME}_{0,t}} \phi\left(\mathrm{ME}_{0,t}\right) dF_t\right], \tag{4.2}$$

where $\phi(\cdot)$ is concave, capturing a cautious attitude toward valuation uncertainty.[4] This preference is adapted from the smooth model of ambiguity (Klibanoff et al., 2005) to the current setting where the uncertainty is about the valuation of the timed reward. It can be considered a smooth version of the cautious expected utility model (Cerreia-Vioglio et al., 2015) or the minimum present equivalent representation (Chakraborty, 2021), with the attitude toward valuation uncertainty captured by $\phi(\cdot)$ instead of the minimum operation. Derivations in Appendix C.1 demonstrate that $me_{0,t} \approx E_{F_t}(\mathrm{ME}_{0,t}) - \kappa VU_{0,t}$, where $VU_{0,t} = \frac{1}{2}\sigma^2_{F_t}$ and $\kappa = -\frac{\phi''(\mathrm{ME}_{0,t})}{\phi'(\mathrm{ME}_{0,t})}$. The parameter $-\phi''/\phi'$ captures attitude toward valuation uncertainty, just like the utility curvature $-u''/u'$ captures attitude toward risk, and $\sigma^2_{F_t}$ is the variance of F_t, capturing the variability of $\mathrm{ME}_{0,t}$.

To illustrate the relationship between valuation uncertainty and the individual's discounting, we start with the standard benchmark, where the individual behaves according to the exponential discounting model with the per period discount factor δ. The individual perceives no valuation uncertainty (F_t is degenerated) or is neutral toward it ($\kappa = 0$), and thus $me_{0,t} = E_{F_t}(\mathrm{ME}_{0,t})$. Intertemporally, the individual discounts X_t exponentially: $me_{0,t} = E_{F_t}(\mathrm{ME}_{0,t}) = \delta^t$, where we standardize the monetary equivalent of X_0 to 1. This holds both for monetary and non-monetary timed rewards.

Now consider the individual perceives valuation uncertainty and is averse toward it. Since there is no valuation uncertainty about the immediate monetary reward, $me_0 = \delta^0 - 0 = 1$. For a delayed monetary reward, we have $me_{0,t} = E_{F_t}(\mathrm{ME}_{0,t}) - \kappa VU_{0,t} = \delta^t - \kappa VU_{0,t}$, where $E_{F_t}(\mathrm{ME}_{0,t}) = \delta^t$ as above. The discount factors from time 0 to 1 and from t to $t+1$ when $t \geq 1$, respectively, are as follows:

$$\begin{aligned}
\delta_{0,1} &= me_{0,1}/1 = \delta - \kappa VU_{0,1}, \\
\delta_{t,t+1} &= (\delta^{t+1} - \kappa VU_{0,t+1})/(\delta^t - \kappa VU_{0,t}) = \delta - \frac{\kappa VU_{0,t+1} - \kappa\delta VU_{0,t}}{\delta^t - \kappa VU_{0,t}}, \quad t \geq 1
\end{aligned}$$

The present bias parameter, β_t, measuring the relative change of the discount factors

[4] We implicitly assume linear utility over intertemporal payments. There is evidence that the utility curvature over intertemporal payments is close to linearity (Andreoni and Sprenger, 2012a).

involving the present (from 0 to 1) and a future delay (from t to $t+1$, $t \geq 1$), can be expressed as: $\beta_t = \delta_{0,1}/\delta_{t,t+1} = \frac{\delta - \kappa VU_{0,1}}{1-0} \times \frac{\delta^t - \kappa VU_{0,t}}{\delta^{t+1} - \kappa VU_{0,t+1}}$. This suggests that β_t depends on the change in valuation uncertainty from time 0 to 1 relative to the change from time t to $t+1$. For example, the quasi-hyperbolic discounting model holds as a special case when $VU_{0,t} = VU_{0,t+1}/\delta$, $t \geq 1$, which gives the discount rate of $\delta_{t,t+1} = \delta$ and the present bias parameter $\beta = \delta_{0,1}/\delta_{t,t+1} = 1 - \kappa VU_{0,1}/\delta < 1$.

Since valuation uncertainty in our framework comes from the complexity in transforming delayed rewards into their present monetary equivalents, this complexity arises once the individual faces a delayed reward, regardless of how short the delay is. For further future delays, we assume that the marginal change in valuation uncertainty is small ($VU_{0,1} - 0 \gg VU_{0,t+1} - VU_{0,t}, t > 0$). This is because the complexity of this transformation is unlikely to change substantially when the delay increases. Our assumption about the discontinuous change in valuation uncertainty differs from Gabaix and Laibson (2017), where valuation uncertainty increases smoothly in delay. With this assumption, $VU_{0,t+1} - \delta VU_{0,t} \approx 0$ and $\delta_{t,t+1} \approx \delta$, which gives:

$$\beta_t \approx \beta = 1 - \kappa VU_{0,1}/\delta. \tag{4.3}$$

Thus, while the individual's core time preference is exponential, she may behave as if she is presently biased when she perceives valuation uncertainty about delayed rewards and behaves cautiously.

When the timed reward is non-monetary, the individual may be uncertain about its monetary equivalent even when it is paid out immediately, and thus $me_0 = 1 - \kappa VU_0$, with $VU_0 > 0$. The discount factors from 0 to 1 and from t to $t+1$ when $t \geq 1$, respectively, are as follows:

$$\begin{aligned} \delta_{0,1} &= (\delta - \kappa VU_{0,1})/(1 - \kappa VU_0) \\ \delta_{t,t+1} &= (\delta^{t+1} - \kappa VU_{0,t+1})/(\delta^t - \kappa VU_{0,t}), \quad t \geq 1. \end{aligned}$$

Since there is no substantial change in valuation uncertainty when the individual moves from 0 to 1 versus from t to $t+1$ with $t \geq 1$, unlike monetary rewards, valuation uncertainty is likely to have a substantially weaker impact on present bias.

The analysis so far always considers the present monetary equivalent of a timed reward. In experiments about the front-end delay effect, the individual compares a delayed payment with an immediate payment (X_t vs. x) and compares these two payments when they are equally delayed (X_{t+1} vs. x_1). The derivation in Appendix C.1.4 demonstrates that, depending on how the individual perceives x_1, the front-end delay can be calculated as follows:

$$\beta_{FE} \begin{cases} \approx 1 - \kappa VU_{0,t}/\delta^t, & \text{Approach 1} \\ = 1 - \frac{\kappa VU_{0,t} - \kappa' VU_{1,t+1}}{\delta^t - \kappa' VU_{1,t+1}}. & \text{Approach 2} \end{cases} \tag{4.4}$$

Approach 1 follows from the classical view that intertemporal comparisons are based on present monetary equivalents. Valuation uncertainty and caution lead to the front-end delay effect in the same way they lead to present bias. Approach 2 assumes that the individual treats x_1 as the *valuation currency* when reporting the monetary equivalent of X_{t+1} in terms of x_1. In Equation 4.4, $VU_{1,t+1}$ is the valuation uncertainty about the monetary equivalent of X_{t+1} in terms of x_1, and κ' is the cautious attitude toward $VU_{1,t+1}$, which could differ from κ. The front-end delay effect arises when $\kappa' VU_{1,t+1} < \kappa VU_{0,t}$. Intuitively, caution implies that the individual favors the immediate payment over the delayed reward, but she may not similarly favor x_1 over X_{t+1} because she experiences valuation uncertainty with both rewards. In both approaches, the front-end delay effect is directly related to valuation uncertainty.

4.3 Experimental Design

The experiment consisted of two main parts. The first part elicited the $t1(\leq t2)$ *monetary* equivalent of a timed reward X_{t2}. The second part elicited a measure of valuation uncertainty. We describe the experimental procedures in detail below.

4.3.1 Eliciting the monetary equivalents with price lists

Subjects saw price lists, each comprising a fixed timed reward and a varying earlier reward that increased monotonically from €0.5 to €10 in increments of €0.5. For each price list, subjects were asked to select the switching row in which they preferred the earlier reward to the timed reward for the first time. Once the switching row was selected, the price list was automatically completed and highlighted in bold and larger fonts to reflect a choice of the timed reward for all rows above the switching row and a choice of the earlier reward for all rows below and including the switching row. Subjects could change their choices freely until they finalized their choices by clicking on the "confirm" button.

#	Option A	Option B	Your choice
1	**€10.00 in four weeks**	€0.50 today	Switch here
2	**€10.00 in four weeks**	€1.00 today	Switch here
3	**€10.00 in four weeks**	€1.50 today	Switch here
4	**€10.00 in four weeks**	€2.00 today	Switch here
5	**€10.00 in four weeks**	€2.50 today	Switch here
6	**€10.00 in four weeks**	€3.00 today	Switch here
7	**€10.00 in four weeks**	€3.50 today	Switch here
8	**€10.00 in four weeks**	€4.00 today	Switch here
9	**€10.00 in four weeks**	€4.50 today	Switch here
10	**€10.00 in four weeks**	€5.00 today	Switch here
11	**€10.00 in four weeks**	€5.50 today	Switch here
12	**€10.00 in four weeks**	€6.00 today	Switch here
13	€10.00 in four weeks	**€6.50 today**	**Switch here**
14	€10.00 in four weeks	**€7.00 today**	Switch here
15	€10.00 in four weeks	**€7.50 today**	Switch here
16	€10.00 in four weeks	**€8.00 today**	Switch here
17	€10.00 in four weeks	**€8.50 today**	Switch here
18	€10.00 in four weeks	**€9.00 today**	Switch here
19	€10.00 in four weeks	**€9.50 today**	Switch here
20	€10.00 in four weeks	**€10.00 today**	Switch here

Figure 4.1: An example screenshot of price lists to elicit the monetary equivalent of a timed reward.

We compute the monetary equivalent of the timed reward at the earlier date by taking the average of the earlier reward in the switching row and the row before it. For example, Figure 4.1 shows that the present monetary equivalent of the timed reward of €10 in four weeks is €6.25.

4.3.2 Measuring valuation uncertainty with R-range

Figure 4.2 shows the screenshot of a modified price list used to elicit the valuation uncertainty of the monetary reward of €10 in four weeks. The modified price lists had the same payment options as the price lists described earlier. The difference is that, instead of selecting the switching row, subjects had to indicate the randomization probabilities they would like to receive the timed reward and the earlier

#	Option A	Your choice		Option B
1	€10 in four weeks	100% A	0% B	€0.50 today
2	€10 in four weeks	100% A	0% B	€1.00 today
3	€10 in four weeks	100% A	0% B	€1.50 today
4	€10 in four weeks	100% A	0% B	€2.00 today
5	€10 in four weeks	100% A	0% B	€2.50 today
6	€10 in four weeks	100% A	0% B	€3.00 today
7	€10 in four weeks	100% A	0% B	€3.50 today
8	€10 in four weeks	100% A	0% B	€4.00 today
9	€10 in four weeks	100% A	0% B	€4.50 today
10	€10 in four weeks	100% A	0% B	€5.00 today
11	€10 in four weeks	100% A	0% B	€5.50 today
12	€10 in four weeks	100% A	0% B	€6.00 today
13	€10 in four weeks	100% A	0% B	€6.50 today
14	€10 in four weeks	100% A	0% B	€7.00 today
15	€10 in four weeks	100% A	0% B	€7.50 today
16	€10 in four weeks	80% A	20% B	€8.00 today
17	€10 in four weeks	70% A	30% B	€8.50 today
18	€10 in four weeks	50% A	50% B	€9.00 today
19	€10 in four weeks	0% A	100% B	€9.50 today
20	€10 in four weeks	0% A	100% B	€10.00 today

Figure 4.2: An example screenshot to elicit the R-range.

reward on a slider in each row of the price list. The sliders had no pre-selected values. Subjects had to click on each slider and adjust its position to their desired probabilities. Moving the slider toward the timed reward would increase the probability of receiving the timed reward by 5% and decreased the probability of receiving the earlier reward by 5%. In each price list, subjects had to make twenty randomization probability choices.

We obtain a lower bound and an upper bound value from subjects' randomization choices in each price list. The lower bound is the average of the minimum earlier reward for which subjects selected a randomization probability that was less than 100% for the timed reward and the earlier reward in the previous row. The upper bound is the average of the maximum earlier reward for which subjects selected a randomization probability that was more than 0% to the timed reward and the earlier reward in the following row. The R-range is obtained by taking the difference between the upper bound and the lower bound. For example, in Figure 4.2, the lower bound is €7.75, the upper bound is €9.25, and the R-range is €1.50. We standardize the R-range by dividing it by 10 and report the standardized R-range in the results section to maintain comparability with our theoretical analysis.

The use of R-range to capture valuation uncertainty is consistent with recent studies (Cettolin and Riedl, 2019; Agranov and Ortoleva, ress; Halevy et al., 2023; Arts et al., 2024). In Appendix C.1, we rely on equation 4.2 and show that the lower bound, the upper bound, and the R-range are calculated as follows:

$$\underline{x}_{0,t} = E_F(ME_{0,t}) - 2\kappa VU_{0,t}, \quad \bar{x}_{0,t} = E_F(ME_{0,t}), \quad \text{R-range}_{0,t} = \bar{x}_{0,t} - \underline{x}_{0,t} = 2\kappa VU_{0,t}.$$

These derivations provide a theoretical rationale for using R-range as a measure of valuation uncertainty. They further show that the upper bound of the R-range ($\bar{x}$) is free from valuation uncertainty, while the lower bound ($\underline{x}_{0,t}$) is adjusted downward for valuation uncertainty. Note that the R-range is not equivalent to the range of possible present equivalent values that the individual considers, and the upper bound of the R-range is the mean of the possible values.

Apart from the R-range, with some parametric assumptions about $\phi(\cdot)$ and the belief distribution function F in Equation 4.2, one can also use the randomization probabilities at different values of x (not just the two bounds) to estimate VU and κ. We discuss this in Appendix C.1.3.

4.3.3 Treatments

Table 4.1 details the key designs, tasks, and main estimators of our treatments. The first treatment is the baseline treatment of monetary rewards. We elicited the subjects' present monetary equivalent and R-range for each of two timed rewards: receiving €10 in 4 weeks and in 24 weeks. The timed rewards were described in weeks rather than days to avoid weekday effects.

The second treatment is the coupon treatment. It is identical to the baseline treatment, except that the timed rewards were Amazon coupons rather than monetary payments. We elicited the present monetary equivalent and the R-range of €10 Amazon coupon receiving today, in 4 weeks, and in 24 weeks. These two treatments were within-subject.

The third treatment is the front-end delay treatment. We elicited the present monetary equivalent of €10 in 4 weeks, the 4-week equivalent of €10 in 8 weeks, the present monetary equivalent of €10 in 8 weeks, and the corresponding R-ranges in these comparisons.

Expt.	Treatments	Timed rewards	Measures	Present bias estimators
1	Baseline (Money)	€10 in 4 weeks	PME &R-range	Structural estimation
		€10 in 24 weeks	PME &R-range	
	Coupon	€10 coupon today	PME &R-range	
		€10 coupon in 4 weeks	PME &R-range	
		€10 coupon in 24 weeks	PME &R-range	
2	Front-end delay	€10 in 4 weeks	PME &R-range	Front-end delay effect
		€10 in 8 weeks	PME &R-range 4-week ME & R-range	

Table 4.1: Overview of the experiments. PME stands for present monetary equivalent, and 4-week ME stands for the monetary equivalent in terms of payments received in 4 weeks.

4.3.4 Risk attitudes, ambiguity attitudes, and beliefs about payments

As the curvature of utility functions could affect the estimation of discount factors, we measured risk attitude with a separate price list. In each row of the price list, subjects faced two options: a lottery that pays €15 or €5 with equal probability and a sure payment of €X, where X varies from €5 to €15 with increments of €0.5. Subjects had to indicate the switching row in which they would prefer the sure payment over the lottery for the first time. We note that the utility curvature over intertemporal payments can differ from that over risky payoffs (Andreoni and Sprenger, 2012b; Miao and Zhong, 2015). Further, when the individual has valuation uncertainty, it is unclear how to estimate the utility function.

There is no established method for eliciting attitudes toward valuation uncertainty. As an exploratory investigation, we elicited attitudes toward belief ambiguity. Subjects faced a known box with a clearly described composition of colored balls and an unknown box with an ambiguous composition of colored balls. We administered three levels of beliefs in the unknown box: one winning color out of 10 possible colors, one out of two possible colors, and nine out of 10 possible colors. We elicited the matching probability of these three unknown boxes with price lists by asking for the switching row. Following Dimmock et al. (2016), we use the three matching probabilities to estimate the ambiguity aversion parameter b (larger b means more aversion) and the insensitivity parameter a (larger a means less sensitive to probability changes).

In another exploration, we asked subjects about their subjective belief of receiving the payment *on time* at the end-of-experiment questionnaire. In Experiment 1, we asked for subjects' general belief about receiving the experimental payment on time. In Experiment 2, we asked for their belief about receiving the reward today, in 4 weeks, and in 8 weeks separately. The elicitation of these beliefs was not incentivized.[5]

4.3.5 Sample and experimental procedure

Subjects were recruited from the Radboud University Individual Decision-making Lab subject pool. Experiment 1 was conducted between October 25th and November 15th, 2022, and Experiment 2 was conducted between 12th and 27th in June, 2023. A total of 291 subjects participated in Experiment 1 and 150 subjects in Experiment 2.

The experiments were conducted online through Qualtrics. Subjects had to correctly answer two comprehension questions about the incentives in the price lists before they could proceed to the main tasks. An incorrect answer would reduce the final experimental payment by €0.50. Subjects received the correct answer if they answered incorrectly. Experiment 1 included an additional set of comprehension questions that tested the subjects' understanding of the R-range task (referred to as the R-range task control). This set of questions required subjects to indicate their desired probability of receiving a fixed immediate payment of €10 and the probability of receiving an immediate payment of €9.90, €9.75, €9.50, €9.25, or €9.00. Subjects who understood the R-range task should always assign a probability of 100% to the fixed present payment of €10. This task was not incentivized.

We randomized the order of eliciting the monetary equivalent and R-range. The order of the tasks was either monetary equivalents -> the ambiguity task -> R-range -> the risk task, or R-range -> the ambiguity task -> monetary equivalents -> the risk task. Apart from the order of the tasks, we randomized the order of the timed rewards for each subject. At the end of the experiment, a single choice was randomly selected by the computer for each participant. If a choice in the R-range task was selected, the computer randomly generated an integer between 1 and 100.

[5] Halevy (2008) models the future as a random process that has a positive probability of stopping at any given period. The stopping probability can arise from the hazard of mortality or the experimenter failing to keep the promise. Obtaining beliefs about this stopping probability require asking subjects about their belief of receiving the reward in each period *conditional* on not receiving the reward in an earlier period. While our beliefs do not correspond directly to the stopping probability, one may expect a positive correspondence between the two.

Subjects received the timed reward if the randomly generated number was smaller than the randomization probability assigned to it (in %), and they received the earlier payment otherwise. Subjects were informed of the selected choice and the corresponding payment on the last page of the experiment. The experiment took place online and lasted an average of 15 minutes. The average payoff was €9.56 across the experiments.

As subjects' trust in when and how they would receive their payment may affect their choices in intertemporal choice experiments, we took three steps to assure them that payments would be made on time. First, subjects were informed in the invitation email that they could only participate in the experiment between 8:00 and 21:00 so that payments that must be made on the day of the experiment could be processed before 23:59. Operationalizing the present as the day of the choice is consistent with most empirical research on intertemporal choices (see, e.g., Andreoni and Sprenger, 2012a). When delayed rewards were selected for payment, subjects were informed of the date they would receive the payment and were reminded to save the payment date on their calendar. Second, we informed subjects that they would receive an additional compensation of €5 if they were not paid on time. Finally, we made all payments online to mitigate the concern that differential transaction costs affect time preferences. Subjects received their monetary payment via an online bank transfer, and received the Amazon coupon via an email.

4.3.6 Hypotheses

Hypothesis 1 (**Monetary rewards**). *In the baseline treatment with monetary rewards:*
a) Subjects randomize strictly over a range of immediate monetary payments for each timed reward. Further, discount factors estimated from the present monetary equivalents of two timed rewards decrease with the difference in the R-ranges during the time interval.
b) Subjects with a positive R-range exhibit stronger present bias than those without a positive R-range. Present bias relates negatively to R-range$_{0,4}$*. Subjects exhibit significantly weaker present bias when present bias is estimated from the upper bounds of the R-ranges compared to when it is estimated from the lower bounds.*

Hypothesis 2 (**Non-monetary rewards**). *The hypotheses pertaining to the non-monetary rewards are similar to those regarding the monetary rewards, with three exceptions:*

a) Subjects randomize strictly over a range of immediate monetary payments even when the non-monetary reward is immediate.
b) Subjects exhibit weaker present bias toward non-monetary rewards than monetary rewards.
c) Subjects exhibit similar extent of present bias regardless of whether present bias is estimated from the upper bound or the lower bound of the randomization range.

Hypothesis 3 (The front-end delay effect). *When the rewards are monetary, the front-end delay effect relates negatively to R-range*$_{0,4}$ *or R-range*$_{0,4}$$-$*R-range*$_{4,8}$*, depending on the theoretical approach.*

4.4 Experimental results

In reporting the experimental results, we consider the full sample for all non-parametric statistical analyses. To mitigate the impact of outliers (see the summary statistics of the key variables in Table C.5 and Table C.8), we restrict the values of the dependent variables to the range between their 5th and 95th percentiles for parametric analyses such as comparisons of means and standard deviations, and OLS regressions. This restriction also removes most of the subjects who failed the R-range task control question (see Table C.10 in Appendix C.3.1).

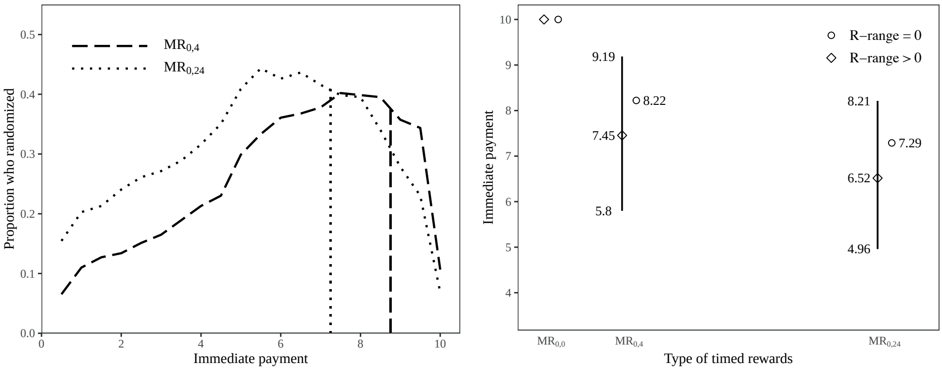
Our analyses below use the R-range as a measure of valuation uncertainty. We show in Appendix C.4 that the R-range is highly correlated with the structurally estimated measure of valuation uncertainty as in Appendix C.1.3 (Spearman's ρ between 0.75 to 0.88), and the associations between the structurally estimated valuation uncertainty and present bias are comparable to those using the R-range.

4.4.1 Baseline: Monetary rewards

Our first result summarizes subjects' randomization behavior by reporting the R-ranges obtained for each timed reward and the relationship between the R-range and the discount factor.

Result 1.6 (Analysis 6). *Consistent with Hypothesis 1a, the majority of subjects randomized more than once in at least one price list, and they randomized over a substantial range. Their discount factors were negatively related to the changes in the R-ranges during the time interval.*

Figure 4.3(a) illustrates the randomization behavior. More than two-thirds of sub-



(a) Proportion of subjects who randomized

(b) R-range

Figure 4.3: (a) The proportion of subjects who randomized between a fixed monetary timed reward (MR) and a varying immediate monetary reward. The vertical lines indicate the present equivalents of the timed rewards. (b) The spikes show the mean upper bound and lower bound values of the R-range of each timed reward. The diamonds represent the mean present monetary equivalent of subjects reporting a positive R-range in one of the price lists, and the circles represent the mean present monetary equivalent of subjects who did not randomize in any price list.

jects randomized once or more in each of the price lists (delayed rewards of 4 weeks: 68% and 24 weeks: 71%). More subjects randomized when the value of the immediate payment was close to the present equivalents of the delayed rewards. Figure 4.3b shows the mean upper bound value and lower bound value for each timed reward. The mean R-range was €3.25 (median: 1.5) when the delay was 4 weeks, and it increased significantly to €3.82 (median: 2.25) when the delay was 24 weeks (Wilcoxon signed-rank test: $p < 0.01$). This suggests valuation uncertainty is stronger for rewards that are more distant in the future. In addition, the difference in the R-range between 0 and 4 weeks (mean and median R-range$_{0,4}$: e3.25 and 1.5) was substantially larger than the difference in the R-range between 4 weeks and 24 weeks (mean and median R-range$_{0,24}$-R-range$_{0,4}$: e0.45 and €0), even though the latter has a longer time interval. This supports the approximation in Equation 4.3. Figure 4.3(b) also shows that subjects with at least one positive R-range had significantly lower present monetary equivalents than those without, suggesting that higher valuation uncertainty is associated with stronger discounting (medians: 8.25 vs. 9.75 for 4 weeks, and 6.75 vs. 9.25 for 24 weeks, two-sample Wilcoxon rank tests, $p < 0.01$ in both comparisons. See Table C.11 in Appendix C.3.2 for more details).

Table 4.2 reports the OLS regression estimates that examine the relationship between the annualized discount factor and the R-range (see Table C.6 for paired correlations among all main variables).[6] Consistent with Hypothesis 1a), discount factors were significantly and negatively associated with the change in R-range ($p < 0.01$). This relationship was robust to the inclusion of controls. More ambiguity aversion was weakly significantly associated with less discounting, which is inconsistent with the impact of caution. Insensitivity toward belief uncertainty was not significantly related to discount factors. These findings suggest that attitudes toward belief uncertainty may be qualitatively different from the attitudes toward valuation uncertainty. Belief about payment also did not correlate significantly with discount factors.

	Discount factors ($\delta_{0,4}$ and $\delta_{4,24}$)				
	(1)	(2)	(3)	(4)	(5)
ΔR-range$_{t,t+1}$	-3.47^{***}	-3.00^{***}	-2.92^{***}	-2.86^{***}	
	(0.55)	(0.56)	(0.55)	(0.57)	
Present		-0.112^{***}	-0.061^{***}	-0.062^{***}	-0.157^{***}
		(0.024)	(0.031)	(0.032)	(0.023)
$\delta^{\bar{x}}_{t,t+1}$			0.213^{**}	0.213^{**}	
			(0.094)	(0.096)	
Aversion (b)				0.233^{*}	0.298^{*}
				(0.144)	(0.167)
Insensitivity (a)				-0.064	-0.040
				(0.082)	(0.100)
Belief about				-0.003	0.068
payment				(0.071)	(0.079)
Intercept	0.502^{***}	0.558^{***}	0.396^{***}	0.403^{***}	0.472^{***}
	(0.016)	(0.020)	(0.065)	(0.084)	(0.069)
Observations	450	450	450	450	450
R^2	0.116	0.148	0.219	0.224	0.078

Table 4.2: OLS regressions to explain annualized discount factors ($\delta_{0,4}$ and $\delta_{4,24}$) when rewards are monetary. $\Delta\text{R-range}_{t,t+1} = \left(\text{R-range}_{t+1}/\bar{x}_{t+1} - \text{R-range}_t/\bar{x}_t\right)$ and $\delta^{\bar{x}}_{t,t+1} = \frac{\bar{x}_{t+1}}{\bar{x}_t}$. We restrict the sample to (5%, 95%) of $\delta_{0,4}$ and $\delta_{4,24}$. *p<0.1; **p<0.05; ***p<0.01.

The comparison between Columns (4) and (5) further suggests a link between val-

[6]The regression model follows from

$$\delta_{t,t+1} = \delta - \frac{\kappa VU_{0,t+1} - \delta\kappa VU_{0,t}}{\delta^t - \kappa VU_{0,t}} = \delta^{\bar{x}}_{t,t+1} - \frac{\bar{x}_{t+1}}{\bar{x}_t + \underline{x}_t} \times \left(\text{R-range}_{t+1}/\bar{x}_{t+1} - \text{R-range}_t/\bar{x}_t\right).$$

In the second equality, we use the observations that $\kappa VU_{0,t} = {}^1\!/\!{}_2\text{R-range}_t$, $\delta^t - \kappa VU_{0,t} = ME_{0,t} = \frac{1}{2}[\bar{x}_t + \underline{x}_t]$, and $\delta = \delta^{\bar{x}}_{t,t+1} = \frac{\bar{x}_{t+1}}{\bar{x}_t}$. See Table C.5 in Appendix C.3.1 for statistics on each independent variable.

uation uncertainty and present bias: accounting for valuation uncertainty reduces present bias by over 50%. We examine this link more closely below.

Result 1.7 (Analysis 7). *Consistent with Hypothesis 1b), present bias was significantly related with valuation uncertainty: (1) subjects with a positive R-range exhibited significantly stronger present bias than those without R-range; (2) subjects exhibited substantially weaker present bias when discounting was estimated from the upper bound than the lower bound of the R-range; (3) the relationship between present bias and R-range was robust to the inclusion of some control variables.*

To examine the relationship between valuation uncertainty and present bias, we computed the present-bias parameter β and the exponential discount parameter δ for each subject based on the quasi-hyperbolic discount model. We calculated β and δ using the present monetary equivalents as well as the upper bound and the lower bound of the R-range.

	β			$\bar{\beta}$			$\underline{\beta}$		
	Full	Test	(5,95)	Full	Test	(5,95)	Full	Test	(5,95)
Subjects: All									
Median	0.91	0.92	0.91	0.98	0.98	0.98	0.85	0.86	0.85
Mean	0.85	0.86	0.86	0.94	0.94	0.95	0.72	0.78	0.73
	(0.22)	(0.20)	(0.15)	(0.16)	(0.16)	(0.07)	(0.38)	(0.35)	(0.28)
Subjects: R-range>0									
Median	0.85	0.85	0.85	0.98	0.98	0.98	0.70	0.75	0.71
Mean	0.81	0.83	0.83	0.94	0.94	0.94	0.65	0.71	0.65
	(0.22)	(0.19)	(0.15)	(0.15)	(0.13)	(0.08)	(0.40)	(0.37)	(0.29)
Subjects: R-range=0									
Median	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Mean	0.95	0.95	0.94	0.95	0.95	0.95	0.95	0.95	0.94
	(0.20)	(0.20)	(0.11)	(0.20)	(0.20)	(0.06)	(0.20)	(0.20)	(0.10)

Table 4.3: Median, mean, and SD (in parentheses) of β (present bias) for the monetary reward in different samples. The Test sample includes subjects who responded correctly in the R-range control task, and (5,95) includes subjects with β values within the range of their 5th to 95th percentiles. The present bias values β, $\bar{\beta}$, and $\underline{\beta}$ are calculated from present monetary equivalents, the upper bound, and the lower bound of the R-range, respectively. The parameters $\bar{\beta}$ and $\underline{\beta}$ are not always equivalent among subjects with R-range=0 because the (5%,95%) samples for $\bar{\beta}$ and $\underline{\beta}$ are not identical.

Table 4.3 reports the mean and median estimated β for the monetary rewards with different sample restrictions imposed. Subjects with a positive R-range had a significantly lower β than those without a positive R-range (mean and median β: 0.85 vs. 0.98, a two-sample Wilcoxon test, $p < 0.01$). This is consistent with a statistically significant negative correlation between β and R-range$_{0,4}$ (Spearman's $\rho = -0.44$, $p < 0.01$). Further, present bias was significantly weaker when β was estimated from the upper bound than when it was estimated from the lower bound of the R-range (mean and median β: 0.95 and 0.98 from the upper bound vs. 0.86 and 0.85 from the lower bound, Wilcoxon rank sum test: $p < 0.01$). When we restricted the sample to subjects who had at least one positive R-range, which is the focus of our theoretical analysis, the difference in the present bias parameters increased (mean and median β: 0.94 and 0.98 from the upper bound vs. 0.65 and 0.70 from the lower bound, Wilcoxon rank sum test: $p < 0.01$). Note that this difference cannot be explained by the fact that the upper bound is above the lower bound, because present bias depends on the *relative* change of the bounds.

Since utility curvature is known to affect the estimates of β and δ, we followed previous studies and used the separately elicited risk attitude to adjust the estimates (e.g., Andersen et al., 2008). Results remained broadly similar, with one notable difference that the adjusted β is closer to one (see Table C.12 in Appendix C.3.2), which is consistent with the findings in the literature. In light of our theoretical analysis, one reason for this difference could be that adjusting for the estimated risk attitude removes some of the impact of valuation uncertainty on present bias because the estimated risk attitude also incorporates valuation uncertainty about the risky lottery and caution.

Next, we utilized Equation 4.3 to examine the relationship between valuation uncertainty and present bias with a series of OLS regressions. Table 4.4 reports the regression results. Consistent with Hypothesis 1b, the increase in valuation uncertainty from the present to a future date as measured by R-range$_{0,4}$ was significantly related to present bias, with larger R-ranges associated with stronger present bias. Table C.14 in Appendix C.3.2 includes the change in valuation uncertainty from 4 weeks to 24 weeks as an additional control variable. The impact of R-range$_{0,4}$ on present bias remains the same, supporting the approximation used in the main regression model.

In addition, Table 4.4 shows a statistically significant relationship between R-range$_{0,4}$ and present bias that was estimated from the lower bounds of the R-ranges ($\underline{\beta}$) but not between R-range$_{0,4}$ and present bias that was estimated from the upper

	$1-\beta$		$1-\bar{\beta}$		$1-\underline{\beta}$	
	(1)	(2)	(3)	(4)	(5)	(6)
R-range$_{0,4}$	0.20***	0.19***	0.01	0.01	0.98***	0.98***
	(0.03)	(0.04)	(0.01)	(0.01)	(0.02)	(0.02)
Aversion (b)		-0.075		0.005		-0.043
		(0.101)		(0.055)		(0.088)
Insensitivity (a)		-0.022		-0.013		-0.042
		(0.064)		(0.034)		(0.053)
Belief about		-0.050		0.007		0.074
payment		(0.071)		(0.025)		(0.051)
Intercept	0.088***	0.137**	0.054***	0.051**	0.054***	0.001
	(0.011)	(0.066)	(0.006)	(0.024)	(0.011)	(0.049)
Observations	260	260	263	263	256	256
R^2	0.137	0.152	0.001	0.002	0.792	0.797

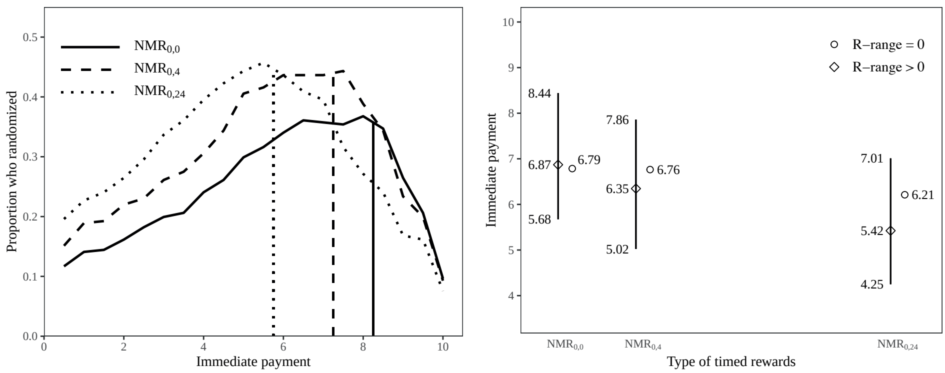
Table 4.4: OLS regressions on present bias estimated from the present monetary equivalents (β) as well as from the upper bounds ($\bar{\beta}$) and the lower bounds ($\underline{\beta}$) of the R-ranges. The sample is restricted to $[5\%, 95\%]$ of the dependent variable. *p<0.1; **p<0.05; ***p<0.01.

bounds ($\bar{\beta}$). The coefficient of R-range$_{0,4}$ was also significantly larger when the present bias was estimated from the lower bounds ($\underline{\beta}$) of the R-ranges compared to when the present bias is estimated from the present value equivalents (0.98 vs. 0.19, $p < 0.01$). These results are consistent with the theoretical analyses, which showed that the lower bound of the R-range was affected by valuation uncertainty but not the upper bound and that valuation uncertainty had a stronger effect on the lower bound than the present monetary equivalents.

4.4.2 Non-monetary rewards (Amazon coupons)

We now proceed to examine subjects' discounting toward the non-monetary timed rewards of Amazon coupon.

Result 2. *(a)The majority of subjects randomized over a substantial range, even when the non-monetary reward was paid immediately. The change in the R-ranges was negatively related to discount factors. (b) Subjects exhibited significantly weaker present bias toward the non-monetary rewards compared to the monetary rewards. Furthermore, while present bias estimated from the upper bound of the R-range differed significantly from that estimated from the lower bound, this difference was significantly smaller than with the monetary rewards.*



(a) Proportion of subjects randomized (b) R-range

Figure 4.4: (a) The proportion of subjects who randomized between a fixed non-monetary timed reward (NMR) and a varying immediate monetary reward. The vertical lines indicate the present equivalents of the timed reward. (b) The spikes show the mean upper bound and lower bound values of the R-range of each timed reward. The diamonds represent the mean present monetary equivalent of subjects reporting a positive R-range in one of the price lists, and the circles represent the mean present monetary equivalent of subjects who did not randomize in any price list.

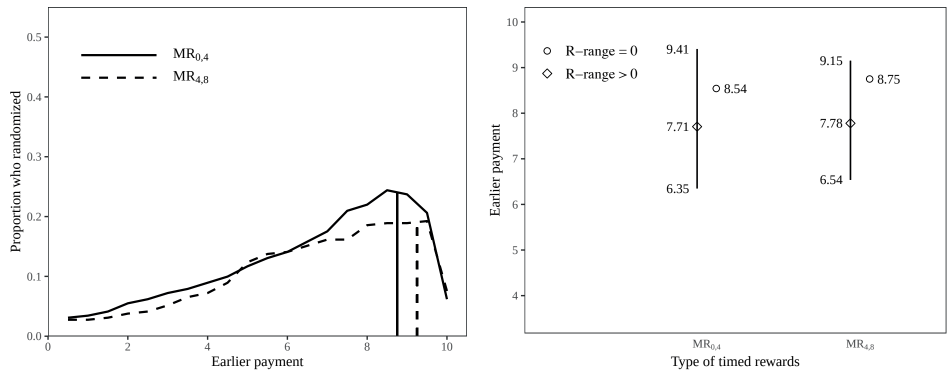
Figure 4.4(a) shows that randomization was also common in price lists involving non-monetary timed rewards. Around 60% of the subjects randomized once or more when the non-monetary reward was paid immediately, and 53% of the subjects randomized at least twice. Hence, many subjects faced valuation uncertainty toward the non-monetary reward even when it was paid out immediately. When the non-monetary rewards were delayed (received in 4 or 24 weeks), 67% of the subjects randomized at least twice, which is only slightly more than when the non-monetary reward was paid out immediately. Figure 4.4(b) shows that the R-ranges were substantial for all three payment dates. The mean (median) R-range was 2.47 (1) when the payment date was immediate, 2.98 (2) in $t = 4$ weeks, and 3.05 (2.5) in $t = 24$ weeks. Thus, similar to monetary rewards, the R-range tended to increase when the payment date was more distant in the future. The increase was statistically significant from today to 4 weeks ($p < 0.01$) but insignificant from 4 to 24 weeks ($p = 0.1538$).

We compare the association between discount factors and the change in R-ranges between monetary and non-monetary rewards in Table 4.5 (see Table C.18 in Appendix C.3.3 for the complete table). The fully interacted model shows that the coefficient of the change in R-ranges was negative and statistically significant, but not the coefficient of the interaction term ΔR-range$_{t,t+1} \times D_{coupon}$. These results suggest that discount factors vary negatively with the change in R-ranges, and this relationship does not differ significantly between the monetary and non-monetary rewards.

	Discount factors ($\delta_{0,4}$ and $\delta_{4,24}$)			
	(1)	(2)	(3)	(4)
ΔR-range$_{t,t+1}$	-4.68^{***}	-3.43^{***}	-3.18^{***}	-3.17^{***}
	(0.57)	(0.57)	(0.54)	(0.55)
D_{coupon}	0.004	-0.021	0.009	-0.051
	(0.022)	(0.023)	(0.058)	(0.130)
ΔR-range$_{t,t+1} \times D_{coupon}$	1.26	0.32	-0.32	-0.28
	(1.09)	(1.06)	(1.02)	(1.03)
Present		-0.246^{***}	-0.166^{***}	-0.165^{***}
		(0.025)	(0.028)	(0.028)
$D_{coupon} \times$Present		-0.072^{*}	-0.048	-0.047
		(0.039)	(0.042)	(0.043)
$\delta^{\bar{x}}_{t,t+1}$			0.342^{***}	0.349^{***}
			(0.047)	(0.046)
$D_{coupon} \times \delta^{\bar{x}}_{t,t+1}$			-0.001	-0.003
			(0.069)	(0.069)
Intercept	0.625^{***}	0.721^{***}	0.446^{***}	0.507^{***}
	(0.016)	(0.019)	(0.043)	(0.080)
Control for ambiguity attitudes and beliefs	No	No	No	Yes
Observations	951	951	951	951
R^2 (clustered SE)	0.090	0.172	0.256	0.261

Table 4.5: OLS regressions to explain discount factors ($\delta_{0,4}$ and $\delta_{4,24}$). The sample is restricted to (5%,95%) of $\delta_{0,4}$, $\delta_{4,24}$, and $\delta^{\bar{x}}_{t,t+1}$. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

Next, we compared the β of the monetary and non-monetary rewards. Subjects exhibited weaker present bias toward non-monetary rewards than monetary rewards. The mean (median) β of non-monetary reward estimated from the present monetary equivalents was 0.94 (0.98), which is significantly larger than the β of the monetary rewards of 0.86 (0.91) (Wilcoxon rank sum test, $p < 0.01$). Further, valuation uncertainty exhibited a weaker association with present bias for non-monetary rewards than monetary rewards. Subjects with a positive R-range in at least one of the price lists with non-monetary rewards had a mean (median) β of 0.93 (0.97), while those without a positive R-range had a mean (median) β of 0.97 (1.00). This difference was substantially smaller than that of the monetary rewards (mean: 0.83 vs. 0.94, and median: 0.85 vs. 0.97). Likewise, the β of non-monetary rewards



(a) Proportion of subjects randomized (b) R-range

Figure 4.5: (a) The proportion of subjects randomized between a monetary timed reward (MR) and an earlier monetary reward. The vertical lines indicate the monetary equivalents of the timed rewards in terms of the earlier payment. (b) The spikes show the mean upper bound and lower bound values of the R-range of each timed reward. The diamonds represent the mean monetary equivalent of subjects reporting a positive R-range in one of the price lists, and the circles represent the mean monetary equivalent of subjects who did not randomize in any price list.

estimated from the upper bounds and the lower bounds of the R-ranges were also significantly closer to each other than those of monetary rewards ($\bar{\beta}$ vs. $\underline{\beta}$ in mean: 0.97 vs. 0.89, in median: 1.0 vs. 0.96 for non-monetary rewards; in mean: 0.95 vs. 0.73, in median: 0.97 vs. 0.85 for monetary rewards. One-sided Wilcoxon signed rank test, $p < 0.01$). This smaller difference between the β estimated from the upper bounds and the lower bounds of the R-ranges also held among subjects with a positive R-range. For more details, please refer to Table C.16 in Appendix C.3.3 for β estimates across various samples and Figure C.3 for a comparison of the distribution of β between non-monetary and monetary rewards in violin plots.

4.4.3 The front-end delay treatment

The following result summarizes the findings for the front-end delay treatment:

Result 3. *On average subjects exhibited significant front-end delay effect. Subjects with at least one positive R-range exhibited significantly stronger front-end delay effect than those without R-range. The front-end delay effect was significantly and negatively related to R-range$_{0,4}$ and R-range$_{0,4}$−R-range$_{4,8}$.*

Figure 4.5 illustrates subjects' randomization behavior, R-ranges, and the two monetary equivalents. Like Figure 4.3(a), Figure 4.5(a) shows that randomization was not uncommon, and more subjects randomized around the monetary equivalent of each reward. Further, Figure 4.5(b) shows that R-range$_{0,4}$ (mean: 2.89, median: 2.00) was significantly larger than R-range$_{4,8}$ (mean: 2.78, median: 2.00; two-sided Wilcoxon signed rank test, $p < 0.01$).

Moreover, the 4-week monetary equivalent of €10 in 8 weeks was significantly larger than the present monetary equivalent of €10 in 4 weeks ($ME_{4,8}$ vs. $ME_{0,4}$ in mean: 8.49 vs. 8.12, in median: 9.25 vs. 8.75, Wilcoxon rank sum test, $p < 0.01$), consistent with the front-end delay effect. Formally, we measured the front-end delay effect using the elicited discount factors $\delta_{0,4}$ and $\delta_{4,8}$ with the parameter $\beta_{FE} = \delta_{0,4}/\delta_{4,8}$. We found that β_{FE} was significantly smaller than 1.0 (two-sided Wilcoxon signed rank test, $p < 0.01$), with a mean of 0.97 (median of 1.0), suggesting the presence of the front-end delay effect. The weak front-end delay effect in our study relative to earlier studies could be due to the short length of the front-end delay in our study (Laury et al., 2012; Jang and Urminsky, 2023).

Relating valuation uncertainty to the front-end delay, we found that subjects without a positve R-range had a significantly larger β_{FE} (mean: 0.984) than those with at least one positive R-range (mean: 0.945, two-sided two sample Wilcoxon test, $p < 0.05$).[7] Further, consistent with the theoretical analysis, β_{FE} estimated from the upper bound of the R-range was not significantly different from 1 (Wilcoxon rank sum test, $p > 0.10$), while β_{FE} estimated from the lower bound was significantly smaller than 1 (Wilcoxon rank sum test, $p < 0.05$).

Our theoretical analysis suggests that how the front-end delay effect relates to valuation uncertainty depends on whether subjects behaved according to Approach 1 or 2. Examining the correlation between β_{FE} and R-range$_{0,4}$ (Approach 1) as well as between β_{FE} and ΔR-range=R-range$_{0,4}$ -R-range$_{4,8}$ (Approach 2), both correlations were negative and statistically significant (Spearman's $\rho = -0.198$ between β_{FE} and R-range$_{0,4}$, $p < 0.05$; $\rho = -0.261$ between β_{FE} and ΔR-range, $p < 0.01$). These results suggest that higher valuation uncertainty is associated with stronger front-end delay effects, which corroborates with earlier comparison of β_{FE} between subjects with and without a positive R-range.

[7] There were subjects who did not have a positive R-range but switched at values different from those in the binary price list. In light of Vieider (2023), these subjects also had valuation uncertainty, although they did not show it via randomization (e.g., they could have randomized subjectively in their head). If we group these subjects with those with positive R-ranges, this difference is similar and slightly stronger (mean 0.993 vs. 0.946, $p < 0.01$).

	Approach 1			Approach 2		
	$1-\beta_{FE}$	$1-\bar{\beta}_{FE}$	$1-\underline{\beta}_{FE}$	$1-\beta_{FE}$	$1-\bar{\beta}_{FE}$	$1-\underline{\beta}_{FE}$
	(1)	(2)	(3)	(4)	(5)	(6)
R-range$_{0,4}$	0.079**	-0.002	0.104*			
	(0.033)	(0.017)	(0.055)			
ΔR-range				0.458***	-0.004	1.212***
				(0.123)	(0.063)	(0.137)
Aversion (b)	0.091	0.169***	0.071	0.079	0.164***	0.193
	(0.104)	(0.052)	(0.154)	(0.107)	(0.052)	(0.117)
Insensitivity (a)	0.028	-0.035	0.059	0.070	-0.009	0.024
	(0.059)	(0.031)	(0.091)	(0.060)	(0.031)	(0.067)
Belief about	-0.024	0.010	-0.054	-0.059	0.004	-0.065
payment	(0.039)	(0.020)	(0.060)	(0.038)	(0.020)	(0.044)
Intercept	0.035	-0.000	0.042	0.053*	-0.004	0.050
	(0.033)	(0.016)	(0.05)	(0.031)	(0.016)	(0.035)
Observations	134	134	134	122	122	122
Adjusted R^2	0.029	0.053	0.016	0.103	0.052	0.374

Table 4.6: OLS regressions to explain the front-end delay effect (β_{FE}). We restrict to observations with $(5\%, 95\%)$ of β_{FE} in regressions of Approach 1, and further restrict to observations with $(5\%, 95\%)$ of ΔR-range in regressions of Approach 2. *p<0.1; **p<0.05; ***p<0.01.

We also conducted OLS regressions on β_{FE} estimated from the monetary equivalents, the upper bounds, and the lower bounds of R-ranges based on Equation 4.4. Table 4.6 summarizes the regression results based on Approach 1 in Columns (1) to (3) and the regression results based on Approach 2 in Columns (4) to (6).[8] Both approaches revealed a significant relation between front-end delay effect and valuation uncertainty. Using β_{FE} estimated from the monetary equivalents, Column (1) shows that R-range$_{0,4}$ was significantly and positively associated with the severity of the front-end delay effect $(1-\beta_{FE})$, consistent with Approach 1. Column (4) shows that ΔR-range also relates significantly and positively with the severity of the front-end delay effect (0.47, $p < 0.01$), consistent with Approach 2.

Focusing on the β_{FE} estimated from the upper bounds (Columns (2) and (5)) and

[8]In Columns (4) to (6), we further exclude subjects whose ΔR-range falls outside the $(5\%, 95\%)$ range to reduce noise in the data. Figure C.4 in Appendix C.3.4 shows that the additional restriction of $(5\%, 95\%)$ for ΔR-range removes the outliers of ΔR-range. The non-parametric correlation between β_{FE} and ΔR-range remains significant and slightly stronger in this sample (Spearman's $\rho = -0.349$, $p < 0.01$ as opposed to $\rho = -0.261$ in the full sample).

the lower bounds of the R-ranges (Columns (3) and (6)), we found that, in both approaches, valuation uncertainty relates significantly with β_{FE} estimated from lower bounds but not from upper bounds, consistent with the theoretical result that upper bounds are not affected by valuation uncertainty. Overall, the results are consistent with our theoretical analysis of the role of valuation uncertainty in the front-end delay effect.

Apart from β_{FE}, we also obtained present bias ($\beta = \delta_{0,4}^2/\delta_{0,8}$) using structural estimation. Consistent with the literature (Cohen et al., 2020), β is significantly smaller than β_{FE} (mean: 0.89 vs. 0.97, median: 0.93 vs. 1.0, Wilcoxon signed rank test, $p < 0.01$), although the two were significantly and positively correlated (Spearman's $\rho = 0.47$, $p < 0.01$). To assess whether valuation uncertainty could account for the systematically larger β_{FE} compared to β, we predicted β_{FE} using β and the R-ranges based on our theoretical analysis in Appendix C.1.5. The calculation indicated that the median predicted β_{FE} was 1.09 based on Approach 1 and 1.0 based on Approach 2, both of which were significantly greater than β (paired Wilcoxon-rank sum tests, $p < 0.01$ in both tests).[9] The observed β_{FE} significantly differed from the predicted β_{FE} as per Approach 1 (Wilcoxon signed rank test, $p < 0.01$) but not Approach 2 (Wilcoxon signed rank test, $p > 0.10$). Therefore, valuation uncertainty has some potential to explain why the front-end delay effect was systematically weaker than the structurally estimated present bias.

4.5 Concluding remarks

Present bias is commonly associated with non-standard intertemporal preferences (Gul and Pesendorfer, 2001; Fudenberg and Levine, 2006). In this paper, we proposed that present bias could arise from individuals' valuation uncertainty toward delayed rewards and their cautious attitudes when facing this uncertainty. We illustrated this relationship theoretically and provided supporting evidence from two pre-registered experiments.

There is now accumulating evidence for the role of valuation uncertainty in a wide range of anomalies such as the willingness to accept (WTA)−willingness to pay (WTP) gap (Dubourg et al., 1994; Bayrak and Hey, 2020a; Cerreia-Vioglio et al.,

[9] Since the predicted $\hat{\beta}_{FE}$ depends on the theoretical approach, by comparing $\hat{\beta}_{FE}$ with the actual β_{FE}, we can assess, among the subjects who have at least one positive R-range (101 out of 150 subjects), the number of subjects who behaved closer to Approach 1 or 2. We found that 42 subjects behaved closer to Approach 1, 57 closer to Approach 2, and two subjects were equally close.

2024), preference reversals (Butler and Loomes, 2007), stochastic choices (Agranov and Ortoleva, 2017, ress; Shi et al., 2024), insensitivity to variation in probabilities (Enke and Graeber, 2023; Oprea, 2022), and small-stakes risk aversion (Khaw et al., 2021). However, the research on valuation uncertainty faces several significant challenges. First, it is difficult to quantify valuation uncertainty theoretically (Oprea, 2020; Gabaix and Graeber, 2024) and measure it behaviourally. Our study follows recent studies like Cettolin and Riedl (2019), Agranov and Ortoleva (ress), and Halevy et al. (2023) in measuring valuation uncertainty with subjects' randomization behavior in price lists. Other measures related to valuation uncertainty include self-reported confidence (Butler and Loomes, 2007), choice deferral (Danan and Ziegelmeyer, 2006; Gerasimou, 2017; Costa-Gomes et al., 2022), and choice inconsistency (Vieider, 2023; Enke et al., 2023). However, there is no consensus on the best method to measure valuation uncertainty. Apart from measuring valuation uncertainty, it can also be important to specifically measure attitudes toward valuation uncertainty. Our finding that ambiguity attitudes were not strongly correlated with R-range suggests that attitudes toward ambiguity in beliefs may not effectively capture attitudes toward valuation uncertainty, probably because valuation uncertainty arises mainly from uncertainty in tastes rather than uncertainty in beliefs (Nielsen and Rigotti, 2022). Hence, it is important for future research to develop better methods to capture attitudes toward uncertainty in tastes.

Second, distinguishing valuation uncertainty from noise is not straightforward, as both factors may be influenced by the strength of preferences (Butler et al., 2014b). For example, Oprea (2022) suggests that probability weighting results from the noisy process of valuing lotteries. McGranaghan et al. (2024) show that common ratio effects arise from systematically biased choices due to noise. In our study, there was significantly more noise when the rewards were non-monetary than monetary, due to the complexity of transforming non-monetary rewards to monetary equivalents. However, we believe that noise is unlikely to be the reason that subjects exhibited significantly weaker present bias with non-monetary rewards than with monetary payments. This is because the randomization behavior was systematic and exhibited similar patterns with both types of rewards. More importantly, our comparison between the two types of rewards focused on the mean and median differences, and pure noise should not affect average behavior differently across rewards. In general, differentiating valuation uncertainty from noise remains a crucial area for future research.

Chapter 5

Conclusion

5.1 Overall conclusion

The main objective of this dissertation is to expand the understanding of uncertainty in preferences, with a particular focus on two economic anomalies: preference reversals and present bias. To address the challenges in measuring preference uncertainty in an incentivized way, we propose novel experimental methods. Using these methods, we studied individuals' preference uncertainty when faced with conflicting objectives such as those between risk and return, and between time and reward. Our findings indicate a strong uncertainty over preferences in both the preference reversal experiment and the inter-temporal choice experiment. More importantly, our results suggest that this uncertainty plays a crucial role in explaining both preference reversals and present bias. By expanding the understanding of uncertainty in preferences, this dissertation not only supplements conventional economic theories on non-deterministic preferences but also provides insights that help individuals better understand their own preferences, thereby enabling them to make decisions that can improve their welfare.

In Chapter 2, we employ a novel preference uncertainty elicitation method to distinguish between conscious and unconscious stochastic choices. Using this method, we elicit the range where subjects choose the randomization option and exhibit conscious preference uncertainty. We employed this method in a preference reversal experiment. This method, together with the repeated direct choices, allows us to estimate stochastic functions from direct choices and indirect valuations. In the analysis, we mainly consider the three major models of conscious stochastic choice models: preference incompleteness, preference imprecision, and hedging of preference uncertainty. In the analysis, we compare the stochastic choice functions estimated from different elicitation procedures and evaluate quantitatively the explanatory power of stochastic choice for preference reversals. The experimental results suggest that stochastic choice is prevalent, but the estimated stochastic functions depend on elicitation procedures. Moreover, we find that a consistent stochastic function estimated from direct choices or valuation choices fails to capture the preference reversal pattern in the experiment. Consistent with Chapter 2, we conclude that both stochastic choice and procedure-dependent preferences are important in explaining preference reversals.

Chapter 3 builds on Chapter 2. There, we focus on the preference uncertainty arising from random shocks in preferences and employ it in explaining the preference reversal phenomenon. We investigate whether the preference reversal phenomenon could be fully explained after allowing for the stochastic choices under random utility models. Using two independent data sets from Shi et al. (2024) and Loomes and Pogrebna (2017), we estimate a stochastic function for each subject and compute the probabilities of preference reversal patterns based on the function. Our results suggest that, although a considerable proportion of subjects exhibited stochastic behaviors in repeated choices, the stochastic choice alone is insufficient for explaining the preference reversal phenomenon.

In Chapter 4, we employ our novel experiment method to elicit the uncertainty in valuing the timed rewards and investigate whether this uncertainty, coupled with caution toward this uncertainty, could result in present bias. In this chapter, we formalize a theoretical model of intertemporal valuation with preference uncertainty, in which valuation uncertainty and caution lead to an extra discount of the delayed reward. Following this theoretical framework, we conducted two pre-registered experiments. In the experiments, we elicit the monetary equivalent of timed rewards in terms of the monetary payment received at an earlier date and the valuation uncertainty in formulating this monetary equivalent. Besides the baseline treatment of studying the classical present bias effect using monetary rewards, we also compare the difference in the present bias effect of monetary and non-monetary rewards. Additionally, we implemented a front-end delay treatment to study the role of valuation uncertainty in the front-end delay effect. The experiment results align with our theoretical analysis and pre-registered hypotheses, demonstrating that the valuation uncertainty regarding the delayed rewards and corresponding cautious attitudes are important in explaining the present bias, a disproportional preference for immediate rewards over even slightly delayed rewards.

5.2 Implications

5.2.1 Theoretical Implications

This dissertation contributes to the measurement of preference uncertainty and its application in explaining various economic anomalies. In this dissertation, we propose two incentivized methods to directly measure preference uncertainty and validate the effectiveness of this method in two experiments. The direct measurement of the uncertainty over preferences allows us to assess and differentiate prominent theoretical models, including random utility, preference imprecision, preference incompleteness, and hedging of preference uncertainty. Furthermore, we perform analyses on these theoretical preference uncertainty models and demonstrate their potential to explain the two economic anomalies: preference reversal and present bias. Specifically, in Chapters 2 and 3, we focus on preference uncertainty arising from conflicting objectives between risk and return. In Chapter 2, we investigate conscious stochastic choice, basing our analysis on models of preference imprecision, preference incompleteness, and hedging of preference uncertainty. In Chapter 3, we focus on the stochastic choice caused by random shocks and use the random utility models in our analysis. Integrating the findings from Chapters 2 and 3, we conclude that both conscious and unconscious stochastic choices alone are insufficient to explain the preference reversal phenomenon. Instead, the preference reversal phenomenon can be better explained by combining stochastic choices with procedure-dependent preferences. In Chapter 4, we elicit the preference uncertainty resulting from conflicting objectives between time and reward, employing preference uncertainty and corresponding cautious attitudes to explain present bias. Our findings suggest a significant link between present bias and valuation uncertainty of timed rewards.

5.2.2 Methodological Implications

In this dissertation, we propose incentivized methods to measure preference uncertainty, addressing one of the main challenges in empirical studies of preference uncertainty. In one method, instead of forcing subjects to choose a preferred option, we allow them to randomize between two options in a binary choice. In the other method, we embed a randomization slider in each row of a multiple price list. The valuation uncertainty is measured by the range where subjects select a randomization probability that is neither 0% nor 100%. These methods can be used for future research under preference uncertainty to measure the preference uncertainty in both choices and valuations.

5.2.3 Policy Implications

The study of preference uncertainty has significant implications for policy making. Many policies need information about preferences of the public. However, when the public face uncertainty in their preferences and policy makers ignore it, wrong policies may be proposed or unintended consequences may follow. We have demonstrated this implication in one study in which we demonstrate the key role of preference uncertainty in present bias. There are many more fields that future research can investigate. For example, when policymakers simply rely on binary yes-or-no answers in polls, potential issues may arise due to preference uncertainty. This preference uncertainty can lead to inaccurate representations of public opinion, as individuals' choices may be stochastic and may change quickly. Recognizing and measuring the uncertainty in individuals' preferences can help policymakers develop more nuanced and effective policies that better maximize social welfare.

Additionally, long-term investments like healthcare, financial planning, and pension funds typically involve strong preference uncertainty, as predicting future consequences can be challenging. By understanding preference uncertainty in such areas, policymakers can create more flexible and adaptive regulations that better address people's diverse and evolving needs.

Furthermore, implementing a randomization mechanism instead of forcing a definitive choice when preferences are potentially uncertain may help reduce inequality and promote fairness. For instance, universities and employers often evaluate candidates based on multiple dimensions, which may lead to preference uncertainty among several applicants. According to social-psychological research, feelings of uncertainty in decision-making could evoke a social identification with an ingroup, inducing more reliance on stereotypic perceptions and prejudices, and hence more discrimination against an outgroup, e.g., women, colored races, or foreigners (Mullin and Hogg, 1998; Vendrik and Schwieren, 2010). Therefore, allowing for randomization in situations of high preference uncertainty among applicants has the potential to reduce discrimination and enhance fairness for minorities.

5.3 Limitations

While this dissertation offers insights into decision-making under preference uncertainty, it is important to acknowledge three limitations. First, all the studies in this dissertation focus on theoretical anomalies, with less emphasis on the practical applications of these findings in real-world scenarios. In the real world, multiple conflicting objectives are involved in many decisions, such as financial investments and policymaking. By recognizing and measuring preference uncertainty in these decisions, individuals could better understand their own preferences and make decisions that better reflect their true preferences.

The second limitation is that this dissertation mainly relies on experimental methods with low monetary incentives. Experiments enable us to create an idealized and controlled environment to study individuals' decision-making. However, this controlled setting may exclude other factors that could influence decision-making in real-world scenarios. Further, while monetary incentives are a key objective in decision-making, they do not encompass other important aspects, such as social responsibility and self-identity, which can also influence people's decisions. Consequently, the findings in this dissertation may not fully capture the complexity of real-world decision-making processes.

The third limitation is that the studies in this dissertation primarily employ the research paradigm of economics, with a lack of interaction with other disciplines. Individual decision-making is a research topic involving multiple disciplines, such as psychology, sociology, neuroscience, and economics. In addition to economic experiments, methods like eye-tracking, functional magnetic resonance imaging (fMRI), and electroencephalography (EEG) can directly reflect changes in people's biological traits during their decision-making, offering deeper insights into preference uncertainty. By integrating perspectives and methodologies from diverse disciplines, future research could achieve a more comprehensive understanding of the decision-making processes under preference uncertainty.

Apart from the three general limitations, each project also has some specific limitations. For instance, in Chapter 3, our analysis was confined to random utility models. While random utility models are widely studied, we should employ more models to bolster the robustness of our findings. In Chapters 2 and 4, we only recruit student participants in the experiments. In order to enhance the representativeness and robustness of our findings, we should recruit participants from broader demographics in future studies.

5.4 Future studies

This dissertation provides evidence for the existence of uncertainty over individuals' preferences and suggests that preference uncertainty has the potential to explain two economic anomalies: preference reversal and present bias. Given the aforementioned limitations, future research could build on our findings and address more practical questions by recognizing and measuring preference uncertainty in real-life scenarios, such as consumer behavior, financial decision-making, or policy-making. Additionally, future research could incorporate methodologies from other disciplines to investigate decision-making under preference uncertainty.

In addition to the general directions, we are currently working on a project that elicits individuals' beliefs about future stochastic choices. Understanding expectations about future decisions is crucial, as expectations can influence current decisions. To address this question, we are developing a transparent method to elicit beliefs that accommodate hesitation and potential stochastic choices among multiple options. To demonstrate the effectiveness of this method, we will conduct an experiment to compare the beliefs and the actual choices to be made several days after the belief elicitation. Additionally, we will conduct a follow-up experiment in which each choice is repeated multiple times to further compare the choice frequencies in repeated choices with the reported beliefs. This project aims to provide insights into eliciting beliefs allowing for stochastic choice in an incentive-compatible way. This project has the potential to offer valuable information for policymakers to collect public expectations regarding future actions.

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Appendices

A Appendices to Chapter 2

A.1 Comparison with results in Butler and Loomes (2007)

Sub-table a)

	$CE_P > CE_\$$	$CE_P \leq CE_\$$	$PE_P > PE_\$$	$PE_P \leq PE_\$$
Choose P-bet	13.7% (11.2%)	42.9% (57.3%)	47.4% (64.0%)	9.1% (4.5%)
Choose \$-bet	4.6% (1.1%)	38.8% (30.3%)	20.0% (20.2%)	23.4% (11.2%)

Notes: The categorization of choosing the P-bet or the \$-bet is based on the first direct choice.

Sub-table b)

	$CE_P > CE_\$$	$CE_P \leq CE_\$$	$PE_P > PE_\$$	$PE_P \leq PE_\$$
Choose P-bet	15.3% (11.2%)	38.6% (57.3%)	49.7% (64.0%)	4.6% (4.5%)
Choose \$-bet	2.8% (1.1%)	42.6% (30.3%)	17.7% (20.2%)	28.0% (11.2%)

Notes: The categorization of choosing the P-bet or the \$-bet is based on the more frequent (5 times or more) choice in the nine repeated direct choices.

Table A.1: The percentages of subjects in each choice pattern. The classification of the four choice patterns in sub-table a) is based on subjects' decisions in the first direct choice and the indirect valuations, as in Butler and Loomes (2007). The classification of the four choice patterns in sub-table b) is based on subjects' more frequent (5 times or more) choice in the nine repeated direct choices and the indirect valuations. In both tables, numbers in parentheses are from Butler and Loomes (2007).

Result A.1 . *Our experiment replicates the results in Butler and Loomes (2007). When the valuation is based on CE, a substantial proportion (47.5%) of subjects exhibited (standard or non-standard) preference reversals, of which majority are standard reversals (42.9% versus 4.6%). When the valuation is based on PE, a less but non-negligible proportion (29.1%) of subjects exhibited (standard or non-standard) preference reversals, of which majority are non-standard reversals (20% versus 9.1%).*

In the experiment, subjects faced the direct choice between the P-bet and the \$-bet nine times, and 50.3% of them did not always choose the same bet. Among

Choice		$CE_P > CE_\$$	$CE_P \leq CE_\$$	$PE_P > PE_\$$	$PE_P \leq PE_\$$
#1	Chose P	13.7%	42.9%	47.4%	9.1%
	Chose $	4.6%	38.9%	20.0%	23.4%
#2	Chose P	13.7%	44.6%	49.7%	8.6%
	Chose $	4.6%	37.1%	17.7%	24.0%
#3	Chose P	16.6%	40.6%	50.3%	6.9%
	Chose $	1.7%	41.1%	17.1%	25.7%
#4	Chose P	13.7%	40.6%	47.4%	6.9%
	Chose $	4.6%	41.1%	20.0%	25.7%
#5	Chose P	14.9%	44.0%	51.4%	7.4%
	Chose $	3.4%	37.7%	16.0%	25.1%
#6	Chose P	15.4%	38.9%	47.4%	6.9%
	Chose $	2.9%	42.9%	20.0%	25.7%
#7	Chose P	16.0%	39.4%	49.1%	6.3%
	Chose $	2.3%	42.3%	18.3%	26.3%
#8	Chose P	15.4%	40.6%	50.3%	5.7%
	Chose $	2.9%	41.1%	17.1%	26.9%
#9	Chose P	15.4%	40.0%	49.7%	5.7%
	Chose $	2.9%	41.7%	17.7	26.9%
Aggregate	Chose P	15.3%	38.6%	49.7%	4.6%
	Chose $	2.8%	42.6%	17.7%	28.0%

Table A.2: The choice patterns based on each of the direct choice between the P-bet and the $-bet. In the last row about the aggregate pattern, subjects who choose the P-bet no less than 5 times are defined as choosing the P-bet and as choosing the $-bet otherwise.

the nine repeated choices, the average ratio of choosing the P-bet ($-bet) is 56.6% (43.4%).[1] We classify subjects' choice patterns in two ways: following Butler and Loomes (2007) and using subjects' decision in the first direct choice or classifying choice patterns according to subjects' more frequent choices in the nine repeated direct choices. Table A.1 shows the proportions of subjects in each of the four choice patterns.

In sub-table a) of Table A.1, the four choice patterns are classified via subjects' decisions in the first direct choice and the indirect valuation choices. A substantial proportion of subjects exhibited (standard or non-standard) PR (47.5% when the

[1] Butler and Loomes (2007) repeated the direct choice 3 times in their experiment. The average ratio of choosing the P-bet in their experiment is 72.3%.

valuation is based on CE and 29.1% when the valuation is based on PE). More importantly, there is a clear asymmetry, with the majority of subjects exhibiting standard PR rather than non-standard PR when the valuation is based on CE (42.9% vs. 4.6%), while more subjects exhibiting non-standard PR than standard PR when the valuation is based on PE (9.1% vs. 20%).

We also classified subjects' choice patterns according to the more frequent choices in the nine repeated direct choices and the indirect valuations. More specifically, subjects are regarded as choosing the P-bet over the $-bet if they selected the P-bet five times or more, and they are categorized as choosing the $-bet over the P-bet otherwise. According to sub-table b) of Table A.1, there remain non-negligible proportions of subjects exhibiting (standard and non-standard) preference reversals (41.4% when the valuation is based on CE and 22.3% when the valuation is based on PE). Likewise, the majority of subjects exhibited standard PR rather than non-standard PR when the valuation is based on CE (38.6% vs. 2.8%), and more subjects exhibited non-standard PR than standard PR (17.7% vs. 4.6%) when the valuation is based on PE. Findings from both classifications indicate that the overall choice patterns of our experiment are qualitatively similar to those of Butler and Loomes (2007). For the reader's convenience, Table A.2 also reports the choice patterns classified according to subjects' decisions in each of the nine repeated direct choices and the indirect valuation choices. Qualitatively similar results hold.

We compare the experimental results in the two studies more closely by looking at the distributions of the elicited CE and PE. The average CE of the P-bet and the $-bet in our experiment are 14.6 and 21.9 separately, close to the corresponding values of 14 and 21.8 in Butler and Loomes (2007). The upper panel in Figure A.1 displays two violin plots of the distributions of CE in our experiment and those in Butler and Loomes (2007). It shows that, in both studies, the CE distribution of the $-bet is right-skewed, while the CE distribution of the P-bet is roughly symmetric around the mean value. The distributions of PE in the two studies are less comparable due to the use of different reference lotteries. Our reference lottery consisted of paying ¥100 with probability p and 0 otherwise, whereas the reference lottery in Butler and Loomes (2007) offered subjects $160 with probability p and 0 otherwise. The average PE for the P-bet and the $-bet are 26.5% and 18.6% in our experiment, while they are 32.4% and 15.5% in Butler and Loomes (2007). Nevertheless, the lower panel in Figure A.1 demonstrates that the distribution of PE in the two studies share similar features: both are left-skewed for the $-bet while close to symmetry for the P-bet. Note additionally that changing the payment amount from 160 to 100 in

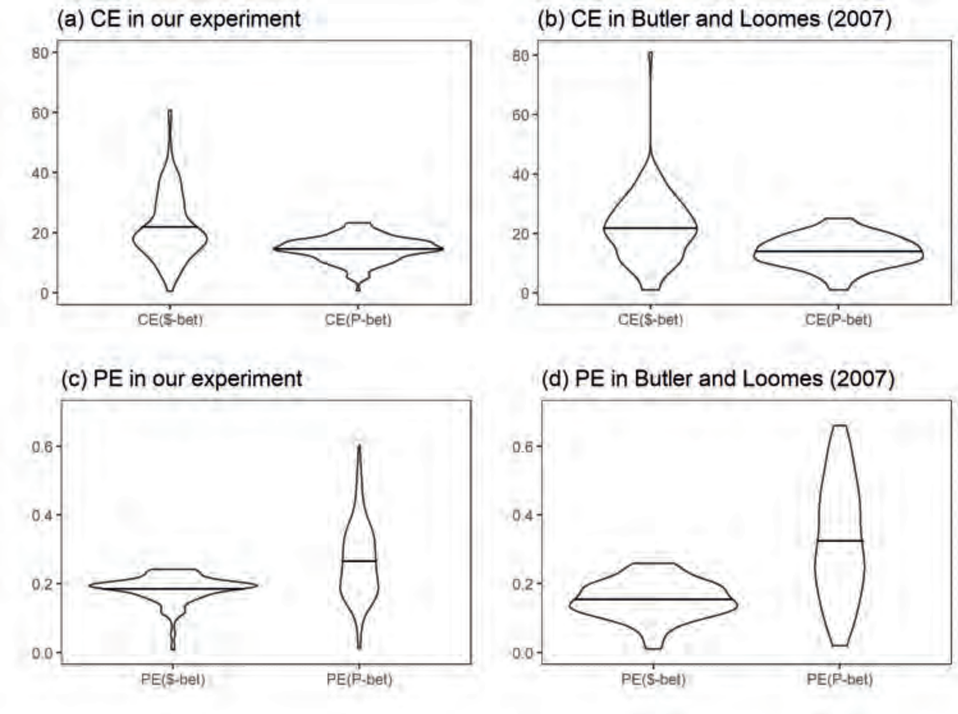


Figure A.1: The comparison of the elicited CE and PE between our study and Butler and Loomes (2007). The lines in the violins are the mean equivalents.

the reference lottery should lead to a smaller asymmetry that favors non-standard reversals according to Butler and Loomes (2007), which is exactly what we find.

Result A.2 . *There is no evidence that the bisection method caused any significant distortion in our experimental results.*

Butler and Loomes (2007) used the incentive compatible incremental choice method. The comparability of our results with theirs thus suggests that our subjects were unlikely to behave strategically in the bisection method. We further checked the potential incentive incompatibility of the bisection method by comparing the CEs elicited in our experiment with the resulting CEs if subjects knew the complete procedure of the bisection method and behaved strategically to maximize their expected payoffs. We find that the average CE in our experiment is 14.6 for the P-bet and 21.9 for the $-bet, which are significantly smaller than the corresponding optimal CE of 18.75 and 61.625, respectively (one-sided Wilcoxon rank sum tests, $p < 0.01$ for both tests).

A.2 Additional figures and tables

	The number of times (percentages) choosing the bet in 9 direct choices									
	0	1	2	3	4	5	6	7	8	9
The P-bet	33	14	14	8	11	5	4	15	17	54
	(18.9%)	(8.0%)	(8.0%)	(4.6%)	(6.3%)	(2.9%)	(2.3%)	(8.6%)	(9.7%)	(30.9%)
The \$-bet	54	17	15	4	5	11	8	14	14	33
	(30.9%)	(9.7%)	(8.6%)	(2.3%)	(2.9%)	(6.3%)	(4.6%)	(8.0%)	(8.0%)	(18.9%)

Table A.3: The number (and percentage in parentheses) of subjects who chose the P-bet or the \$-bet 0, 1, ..., 9 times in the nine repeated direct choices.

		Risk measure 1			
		Averse	Neutral	Seeking	Aggregate
Risk measure 2	Averse	138	2	0	140
	Neutral	20	5	3	28
	Seeking	5	0	2	7
	Aggregate	163	7	5	175

Table A.4: The overview of subjects' risk attitudes according to two risk measures. Risk measure 1 is estimated from the risk attitude measurement task where we vary the winning amount of money in the reference lottery. Risk measure 2 is estimated from the measurement task where we vary the winning probability in the reference lottery.

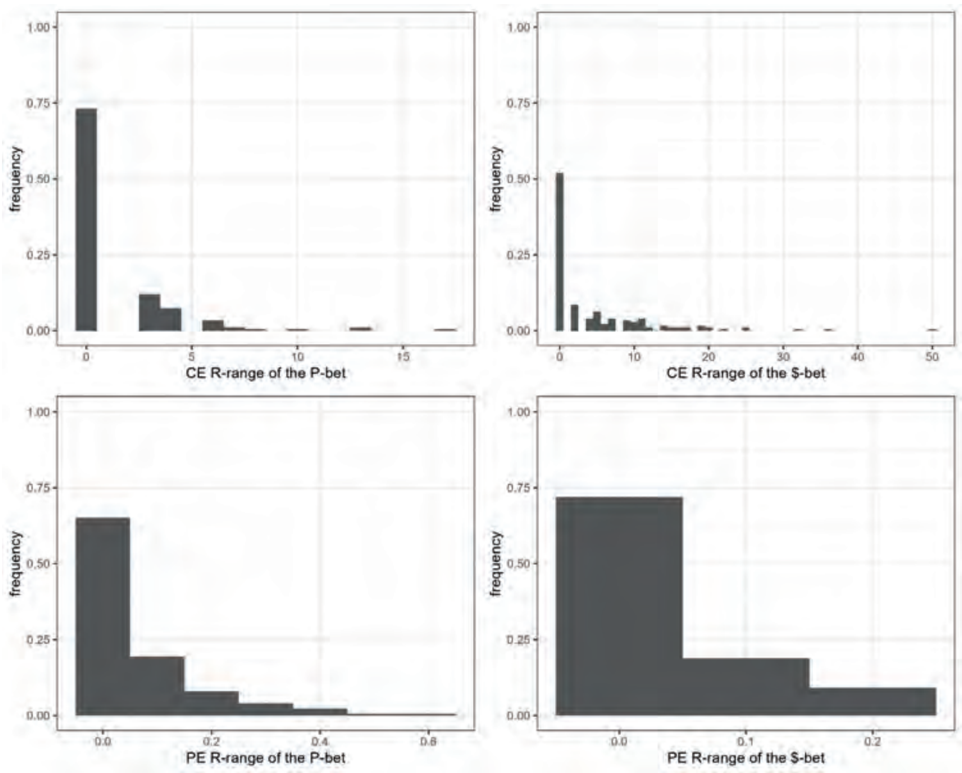


Figure A.2: The histogram of the size of the CE and PE R-range of the P-bet and the $-bet. The average size of the CE R-range is 1.35 for the P-bet and 4.47 for the $-bet. The average size of the PE R-range is 0.06 for the P-bet and 0.04 for the $-bet.

A.3 Additional results about models of incomplete/imprecise preferences

The CE PR patterns

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	13.7%	42.9%	4.6%	38.8%
M1: Direct Choices	43.6% [39.4%, 48.0%]	12.7% [8.6%, 17.1%]	12.4% [8.6%, 16.6%]	31.3% [26.9%, 35.4%]
M2: CE R-ranges 1 (50%:50%)	25.5% [22.3%, 28.6%]	10.0% [6.3%, 13.7%]	6.0% [2.9%, 9.1%]	58.5% [54.9%, 62.3%]
M3: CE R-ranges 2 (valuation)	26.5% [24.0%, 29.1%]	4.9% [2.3%, 8.0%]	4.9% [2.3%, 8.0%]	63.6% [60.6%, 66.9%]
M4: Two stochastic functions 1 (50%:50% from choices)	22.2% [18.9%, 25.7%]	33.8% [28.6%, 38.9%]	9.3% [6.3%, 12.6%]	34.7% [30.3%, 39.4%]
M5: Two stochastic functions 2 (actual ratios)	23.9% [20.6%, 27.4%]	32.4% [28.0%, 37.1%]	7.6% [5.1%, 10.3%]	36.1% [32.0%, 40.0%]

Table A.5: Means and 95% confidence intervals (in square brackets) of simulated proportions of CE PR patterns according to models of preference incompleteness/imprecision. In M1: Direct choices, we assume the probability of valuing the P-bet higher than the $-bet is 50% if subjects chose different bets in the nine-repeated direct choices and it is 1 (or 0) if subjects chose the P-bet (or the $-bet) for nine times. In M2: CE R-ranges 1, we assume subjects choose randomly (50%-50%) when their CE R-ranges for the P-bet and the $-bet overlap. In M3: CE R-ranges 2, we calculate the choice probability from CE R-ranges by assuming that subjects' choices are determined by randomly sampling the potential values of the bet in the CE R-range. In M4 and M5: Two stochastic functions 1 and 2, we calculate the probability of choosing the P-bet in the direct choice from the nine-repeated direct choices and use CE R-ranges to calculate the probability of valuing the P-bet higher than the $-bet. More specifically, in M4: Two stochastic functions 1, we assume random choice (50%:50%) if subjects chose differently in the nine-repeated direct choices. In M5: Two stochastic functions 2, we use the actual choice ratio in the nine-repeated direct choices as the probability of choosing the P-bet in the direct choice.

Results about PE PR patterns

The following result summarizes the findings about PE PR patterns:

Result A.3. *While preference incompleteness/imprecision explains the PE PR patterns better than CE PR patterns, in general, results on PE PR patterns are similar to those in CE PR patterns.*

In general, results on PE PR patterns are similar to those in CE PR patterns. A sizeable proportion of subjects (37.4%) have PE outside their PE R-range in at least one bet. Among subjects who had no overlapping PE R-ranges, 49.6% chose differently in the nine-repeated choices; among those who chose differently in the nine-repeated choices, 79.5% had no overlapping PE R-ranges. For more details please refer to Table A.6. Table A.8 suggests that the simulated proportions capture the general PE PR patterns better than the CE PR patterns by correctly estimating the PE PR pattern of choosing the P-bet consistently as the most frequent and choosing the $-bet consistently as the second most frequent. However, like the simulations in CE PR patterns, the simulated proportions overestimate the proportion of consistent choices and fail to capture the magnitude as well as the asymmetry of PE PR. Assuming those who chose differently in the nine-repeated direct choices randomly select between the two bets(50%:50%) and use the PE R-ranges to calculate the probability of valuing the P-bet higher than the $-bet, the simulated proportions are then close to the actual proportions. They capture the ranking of the four PE PR patterns as well as the asymmetry of the non-standard PR versus the standard PR. Moreover, for all PE PR patterns the actual proportions fall in the 95% confidence intervals of the simulated proportions. The same result holds when we use the actual ratio of choosing the P-bet in the nine-repeated choices as the probability of choosing the P-bet in the direct choice.

Sub-table a)

	No R-range	In the R-range	Outside the R-range	
			larger than the upper bound	lower than the lower bound
PE_P	106 (60.6%)	43 (24.6%)	6 (3.4%)	20 (11.4%)
$PE_\$$	99 (56.6%)	65 (37.1%)	4 (2.3%)	7 (4.2%)
PE aggregate	84 (48.0%)	57 (32.6%)	34 (19.4%)	

Sub-table b)

	In the nine-repeated direct choices	
	Chose consistently	Chose differently
Overlapping PE R-ranges	16	18
Non-overlapping PE R-ranges	71	70

Table A.6: Sub-table a) reports the number of subjects who had no R-range, who had positive R-ranges with PE falling in the R-range, and who had positive R-ranges but with PE outside the R-range. The group "No R-range" includes subjects who had no positive ranges in any of the tasks. The group "In the R-range" includes subjects who had positive R-ranges in at least one task, and their elicited PE were in the R-range. The group "Outside the R-range" includes subjects who had positive R-ranges in at least one task and their elicited PE were outside the R-range in at least one task. Sub-table b) reports the number of subjects who had overlapping PE R-ranges, who had non-overlapping PE R-ranges as well as those who chose consistently or who chose differently in the nine-repeated direct choices.

Sub-table a)

	In the nine-repeated direct choices	
	Chose consistently	Chose differently
Overlapping CE R-ranges	40	16
Non-overlapping CE R-ranges	78	41

Notes: Subjects who chose the same bet in eight of the nine repeated direct choices were also included in the “chose consistently” group.

Sub-table b)

	In the nine-repeated direct choices	
	Chose consistently	Chose differently
Overlapping PE R-ranges	22	12
Non-overlapping PE R-ranges	96	45

Notes: Subjects who chose the same bet in eight of the nine repeated direct choices were also included in the “chose consistently” group.

Table A.7: Sub-table a) shows the number of subjects who had overlapping CE R-ranges or non-overlapping CE R-ranges, as well as those who chose consistently or differently in the nine-repeated direct choices. Sub-table b) displays the number of subjects who had overlapping PE R-ranges or non-overlapping PE R-ranges, as well as those who chose consistently or differently in the nine-repeated direct choices.

The PE PR patterns

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	47.4%	9.1%	20.0%	23.4%
M1: Direct choices	43.6% [39.4%, 48.0%]	12.7% [8.6%, 17.1%]	12.4% [8.6%, 16.6%]	31.3% [26.9%, 35.4%]
M2: PE R-range 1 (50%:50%)	63.3% [60.6%, 66.3%]	3.0% [1.1%, 5.1%]	6.7% [4.0%, 9.7%]	27.0% [25.1%, 29.1%]
M3: PE R-range 2 (valuation)	67.2% [64.6%, 69.7%]	2.8% [0.6%, 5.1%]	2.8% [0.6%, 5.1%]	27.3% [25.7%, 29.1%]
M4: Two stochastic functions 1 (50%:50% from choices)	46.5% [41.7%, 51.4%]	9.5% [6.3%, 12.6%]	23.4% [18.9%, 28.0%]	20.6% [17.7%, 24.0%]
M5: Two stochastic functions 2 (actual ratios)	48.8% [45.1%, 52.6%]	7.5% [5.1%, 10.3%]	21.1% [17.1%, 25.1%]	22.6% [20.0%, 25.7%]

Table A.8: The comparison of means and 95% confidence intervals (in square brackets) of simulated proportions of PE PR patterns according to models of preference incompleteness/imprecision. In M1: Direct choices, we assume the probability of valuing the P-bet higher than the $-bet is 50% if subjects chose different bets in the nine repeated direct choices and it is 1 (or 0) if subjects chose the P-bet (or the $-bet) for nine times. In M2: PE R-ranges 1, we assume subjects choose randomly (50%-50%) when their R-ranges for the P-bet and the $-bet overlap. In M3: PE R-ranges 2, we calculate the choice probability from PE R-ranges by assuming that subjects' choices are determined by randomly sampling the potential values of the bet in the PE R-range. In M4 and M5: Two stochastic functions 1 and 2, we calculate the probability of choosing the P-bet from the nine-repeated direct choices and use the PE R-ranges to calculate the probability of valuing the P-bet higher than the $-bet. In M4: Two stochastic functions 1, we assume random choice (50%:50%) if subjects chose differently in the nine-repeated direct choices. In M5: Two stochastic functions 2, we use the actual ratio of choosing the P-bet in the nine repeated direct choices as the probability of choosing the P-bet in the direct choice.

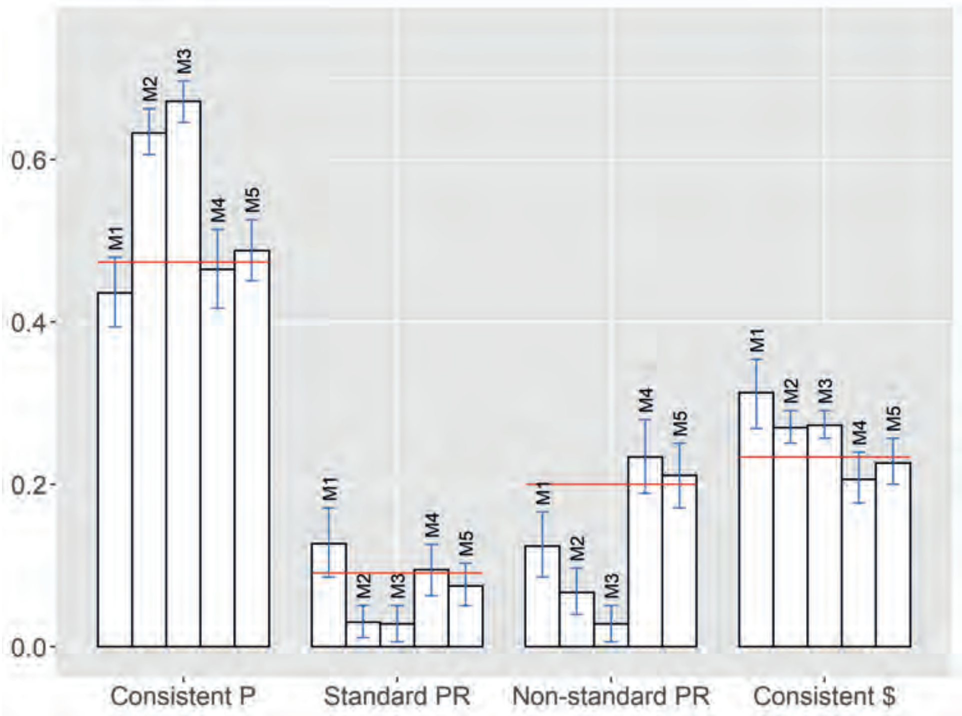


Figure A.3: The comparison of means and 95% confidence intervals of simulated proportions of PE PR patterns according to models of preference incompleteness/imprecision across different simulation methods. The error bars are the 95% confidence intervals of the simulated proportions. The red horizontal lines are the actual proportions in the experiment. The horizontal red line is the actual proportion in the experiment. In M1, we assume the probability of valuing the P-bet higher than the $-bet is 50% if subjects chose different bets in nine repeated direct choices and it is 1 (or 0) if subjects chose the P-bet (or the $-bet) for nine times. In M2, we assume subjects choose randomly (50%-50%) when their R-ranges for the P-bet and the $-bet overlap. In M3, we calculate the choice probability from PE R-ranges by assuming that subjects' choices are determined by randomly sampling the potential values of the bet in the PE R-range. In M4 and M5, we calculate the probability of choosing the P-bet in the direct choice from the nine-repeated direct choices and use PE R-ranges to calculate the probability of valuing the P-bet higher than the $-bet. In M4, we assume random choice (50%:50%) if subjects chose differently in the nine-repeated direct choices. In M5, we use the actual choice ratio in the nine repeated direct choices as the probability of choosing the P-bet in the direct choice.

A.4 Additional results about hedging of preference uncertainty (Fudenberg et al., 2015)

	The CE PR patterns			
	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	13.7%	42.9%	4.6%	38.8%
M1: Direct choice	35.6% [30.9%, 40.0%]	19.4% [14.3%, 25.1%]	6.3% [2.9%, 9.7%]	38.7% [34.3%, 42.9%]
M2: CE R-range P-bet	25.7% [22.9%, 28.6%]	6.2% [2.9%, 9.7%]	6.3% [2.9%, 9.7%]	61.9% [58.3%, 65.1%]
M3: CE R-range $-bet	22.6% [18.9%, 26.3%]	10.9% [6.9%, 15.4%]	8.5% [4.6%, 12.6%]	58.0% [53.7%, 62.3%]
M4: CE R-range mean	25.5% [22.3%, 28.6%]	6.7% [3.4%, 10.3%]	6.8% [3.4%, 10.3%]	61.0% [57.1%, 64.6%]
M5: Two stochastic functions (P-bet)	22.6% [18.3%, 26.9%]	32.5% [27.4%, 37.7%]	9.4% [6.3%, 12.6%]	35.6% [30.9%, 40.6%]
M6: Two stochastic functions ($-bet)	21.8% [17.1%, 26.3%]	33.2% [27.4%, 38.9%]	9.2% [5.7%, 13.1%]	35.7% [30.9%, 40.6%]
M7: Two stochastic functions (mean)	22.8% [18.9%, 26.9%]	32.2% [26.9%, 37.7%]	9.4% [6.3%, 13.1%]	35.5% [30.9%, 40.0%]

Table A.9: Means and 95% confidence intervals (in square brackets) of the simulated proportions of the CE PR patterns under the deliberately stochastic model of Fudenberg et al. (2015). The simulations M1 to M4 use a consistent stochastic function estimated from the nine-repeated direct choices, from the CE R-ranges of the P-bet, the CE-Ranges of the $-bet, and the mean η across the P-bet and the $-bet, respectively. The simulations M5 to M7 use the stochastic function estimated from the nine-repeated direct choices for the direct choice and the stochastic function estimated from the CE R-ranges of the P-bet, the CE R-ranges of the $-bet, and the mean η across the P-bet and the $-bet for valuation choices, respectively.

	CE R-range			PE R-range		
	(1)	(2)	(3)	(4)	(5)	(6)
Amb. aversion	0.12		0.08	0.01^{*}		0.005
	(0.20)		(0.20)	(0.00)		(0.00)
Less control		0.08^{**}	0.08^{**}		0.002^{***}	0.002^{***}
		(0.03)	(0.03)		(0.00)	(0.00)
Risk aversion	0.03	−0.07	−0.04	0.01^{***}	0.004^{*}	0.01^{**}
	(0.18)	(0.16)	(0.18)	(0.00)	(0.00)	(0.00)
Intercept	2.00	5.83^{***}	5.12^{**}	−0.04	0.08^{***}	0.04
	(2.11)	(1.71)	(2.47)	(0.03)	(0.03)	(0.04)
Observations	280	280	280	280	280	280
Adj. R^2	−0.01	0.01	0.01	0.02	0.07	0.07

Table A.10: Regression results of CE R-range and PE R-range on ambiguity attitude and the desirability of control, with standard errors in parentheses. * denotes $p < 0.10$, ** denotes $p < 0.05$, *** denotes $p < 0.01$.

Results about PE PR patterns

Result A.4. *In the deliberately stochastic model of Fudenberg et al. (2015), results on PE PR patterns are in general similar to those in CE PR patterns.*

The median η estimated from the PE of the P-bet is 4.37 (mean=7.41) and from the PE of the \$-bet is 4.43 (mean=9.81). The median η estimated from the nine-repeated direct choices is -0.24 (mean=3.80). Wilcoxon signed rank tests that compare η estimated from the direct choices and η estimated from the PE for both the P-bet and the \$-bet suggest that the difference is significant in both comparisons ($p < 0.01$). See Table A.11 for more details. Table A.12 reports the simulated proportions of the PE PR patterns, indicating that the simulated proportions capture the general PE PR patterns. When we use the actual choice ratio from the nine-repeated direct choice as the choice probability of choosing the P-bet in the direct choice and the probabilities calculated from the PE R-ranges as valuation probabilities, the simulated proportions are quite close to the actual proportions. They capture the ranking of the four PE PR patterns as well as the asymmetry of the non-standard PR versus standard PR. Furthermore, the actual proportions fall in the 95% confidence intervals of the simulated proportions.

	The estimated η		
	from the direct choices	from the PE R-ranges	
		P-bet	\$-bet
Mean	-0.24	7.41***	9.81***
	(3.80)	(8.59)	(11.59)
Median	0.28	4.37	4.43

Table A.11: The fitted η in the deliberately stochastic models. Standard deviations are in parentheses. The η of the stochastic function from the direct choices is estimated from the equation $P(P - bet \succ \$ - bet) = \frac{e^{\eta U(P)}}{e^{\eta U(P)} + e^{\eta U(\$)}}$. The η of the stochastic function from the PE R-ranges is estimated from the equation $\eta_X = \frac{2.55}{(\overline{p} - \underline{p})u(100)}$, where X is the P-bet or the \$-bet. Wilcoxon signed rank tests check the difference in the estimated η from the direct choices and the PE R-ranges. * denotes $p < 0.10$, ** denotes $p < 0.05$, *** denotes $p < 0.01$.

	The PE PR patterns			
	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	47.4%	9.1%	20.0%	23.4%
M1: Direct choice	51.4% [46.3%, 56.6%]	3.6% [1.1%, 6.3%]	34.8% [29.1%, 40.6%]	10.2% [6.9%, 13.7%]
M2: PE R-range P-bet	62.6% [58.9%, 66.3%]	5.4% [2.3%, 8.6%]	10.3% [6.9%, 14.3%]	21.6% [18.3%, 24.6%]
M3: PE R-range $-bet	65.6% [62.3%, 68.6%]	4.2% [1.7%, 6.9%]	7.8% [4.6%, 10.9%]	22.5% [19.4%, 25.1%]
M4: PE R-range mean	64.9% [61.7%, 68.0%]	4.6% [1.7%, 7.4%]	8.3% [5.1%, 12.0%]	22.2% [19.4%, 25.1%]
M5: Two stochastic functions (P-bet)	47.8% [42.9%, 52.6%]	7.3% [4.0%, 10.9%]	25.2% [20.0%, 30.3%]	19.8% [16.0%, 23.4%]
M6: Two stochastic functions ($-bet)	48.3% [43.4%, 53.1%]	6.8% [4.0%, 10.3%]	25.1% [20.0%, 30.3%]	19.9% [16.0%, 23.4%]
M7: Two stochastic functions (mean)	48.1% [43.4%, 53.1%]	6.9% [4.0%, 10.3%]	25.1% [20.0%, 30.3%]	19.9% [16.0%, 23.4%]

Table A.12: Means and 95% confidence intervals (in square brackets) of simulated proportions of the PE PR patterns under the deliberately stochastic model of Fudenberg et al. (2015). The simulations M1 to M4 use a consistent stochastic function estimated from the nine-repeated direct choices, from the PE R-ranges of the P-bet, the PE R-ranges of the $-bet, and the mean η across the P-bet and the $-bet, respectively. The simulations M5 to M7 use the stochastic function estimated from the nine-repeated direct choices for the direct choice and the stochastic function estimated from the PE R-ranges of the P-bet, the PE R-ranges of the $-bet, and the mean η across the P-bet and the $-bet for valuation choices respectively.

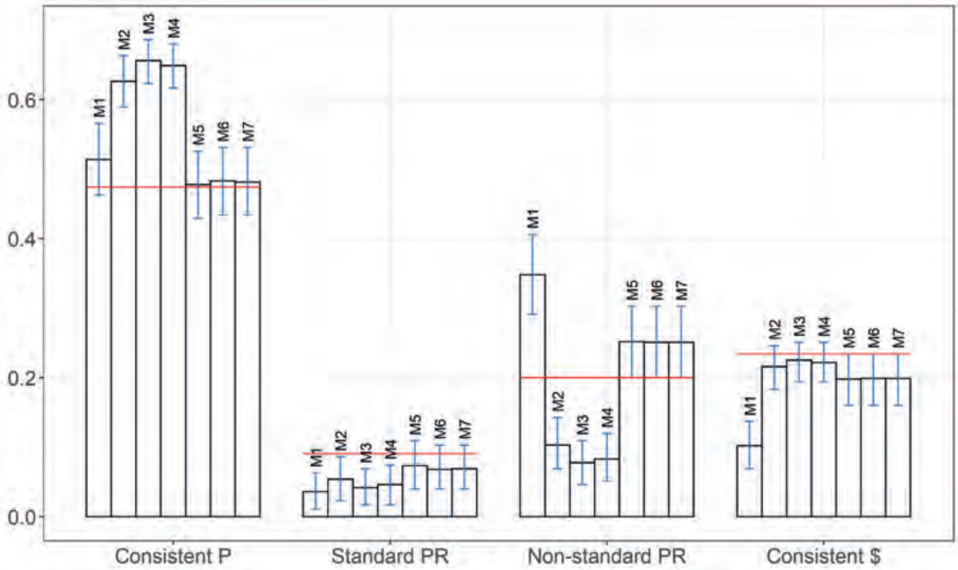


Figure A.4: The comparison of means and 95% confidence intervals of simulated proportions of PE PR patterns under the deliberately stochastic model of Fudenberg et al. (2015) across different simulation methods. The error bars are the 95% confidence intervals of the simulated proportions. The red horizontal lines are the actual proportions in the experiment. The simulations M1 to M4 use a consistent stochastic function estimated from the nine-repeated direct choices, from the PE R-ranges of the P-bet, the PE R-ranges of the $-bet, and the mean η across the P-bet and the $-bet, respectively. The simulations M5 to M7 use the stochastic function estimated from the nine-repeated direct choices for the direct choice and the stochastic function estimated from the PE R-ranges of the P-bet, the PE R-ranges of the $-bet, and the mean η across the P-bet and the $-bet for valuation choices respectively.

A.5 Experimental materials

A.5.1 Translated decision screens in the *CE valuation* and *PE valuation*

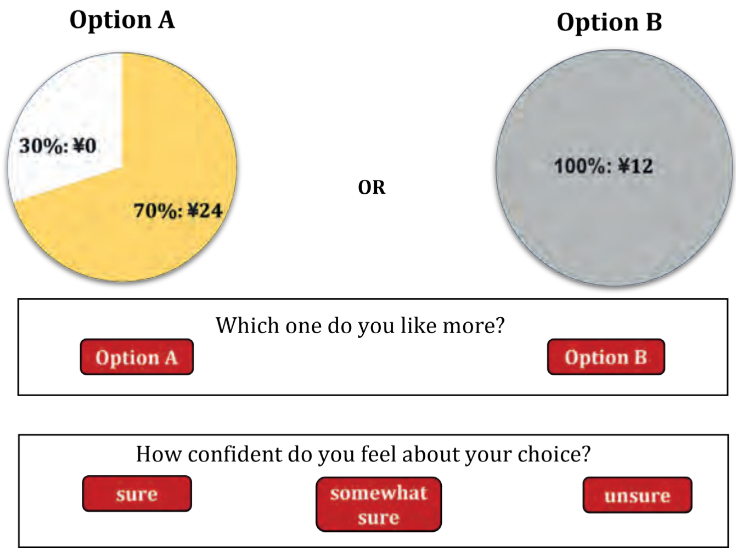


Figure A.5: Display of binary choice in CE valuation

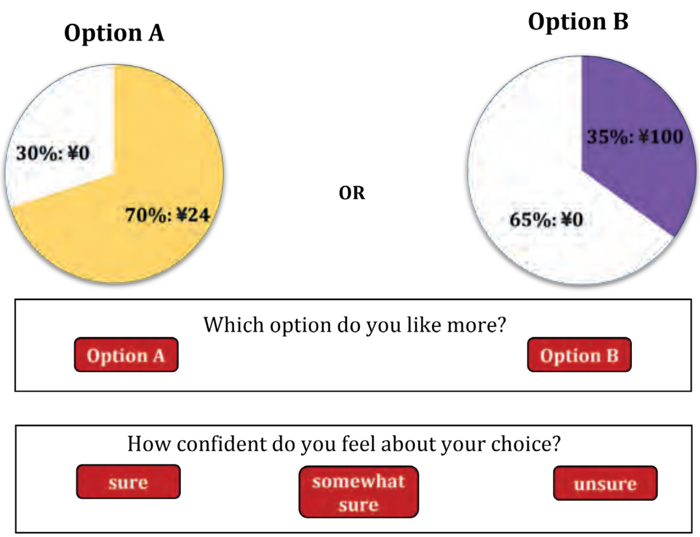


Figure A.6: Display of binary choice in PE valuation

A.5.2 Risk and ambiguity attitudes

In the elicitation of the risk attitudes, subjects face two multiple price lists (MPL). In each row of the lists, subjects face two options: a sure payment and a lottery. In the first MPL, subjects face a sure payment of ¥60 and a lottery of receiving ¥110 with probability 50% and ¥X otherwise. The amount of ¥X increases from ¥0 to ¥60 from the top to the bottom rows. In the second MPL, subjects face a sure payment of ¥50 and a lottery of receiving ¥90 with probability p% and ¥10 otherwise. Moving down the rows p% increases from 10% to 100%. Note that the first MPL is close to the CE valuation where we vary the monetary payoff across choices, while the second MPL is close to the PE valuation where we vary the winning probabilities across choices. We ask subjects to indicate the row they switch from one option to another. Table A.13 and Table A.14 illustrate the tasks.

The literature documents substantial order effects: the elicited risk attitude may depend on the presentation order (ascending or descending) of the reference option. However, using one order in MPL is unlikely to bias our results. This is because PR is about the disparity between valuations and choices, and we use the same risk attitude for valuations and choices in our estimation of stochastic functions. Consequently, even if the elicited risk attitude were influenced by the order, its impact on the difference between valuations and choices would be limited.

Situation	Option 1	Option 2	Your choice
1	100%: 60	50%: 110, 50%: 0	prefer option 2
2	100%: 60	50%: 110, 50%: 5	prefer option 2
3	100%: 60	50%: 110, 50%: 10	prefer option 2
4	100%: 60	50%: 110, 50%: 15	prefer option 2
5	100%: 60	50%: 110, 50%: 20	prefer option 2
6	100%: 60	50%: 110, 50%: 25	prefer option 2
7	100%: 60	50%: 110, 50%: 30	prefer option 2
8	100%: 60	50%: 110, 50%: 35	prefer option 2
9	100%: 60	50%: 110, 50%: 40	prefer option 2
10	100%: 60	50%: 110, 50%: 45	prefer option 2
11	100%: 60	50%: 110, 50%: 50	prefer option 2
12	100%: 60	50%: 110, 50%: 55	prefer option 2
13	100%: 60	50%: 110, 50%: 60	prefer option 2

Table A.13: Risk task 1 (varying the monetary amounts in the reference lottery)

We use the classical Ellsberg paradox to elicit ambiguity attitude. There are two boxes, box U and box K, each of which contained 100 balls of either red or black.

Situation	Option 1	Option 2	Your choice
1	100%: 50	10%: 90, 90%: 10	prefer option 2
2	100%: 50	20%: 90, 80%: 10	prefer option 2
3	100%: 50	30%: 90, 70%: 10	prefer option 2
4	100%: 50	40%: 90, 60%: 10	prefer option 2
5	100%: 50	50%: 90, 50%: 10	prefer option 2
6	100%: 50	60%: 90, 40%: 10	prefer option 2
7	100%: 50	70%: 90, 30%: 10	prefer option 2
8	100%: 50	80%: 90, 20%: 10	prefer option 2
9	100%: 50	90%: 90, 10%: 10	prefer option 2
10	100%: 50	100%: 90, 0%: 10	prefer option 2

Table A.14: Risk task 2 (varying the probabilities in the reference lottery)

The composition of red and black balls is known in box K while unknown in box U. Subjects first choose their preferred color – red or black – as the winning color. They then face an MPL with 11 rows. In each of them they chose to let the computer randomly draw a ball from box U or box K. Moving down the rows, box U remains the same while the proportion of winning colored balls in box K increases from 0% to 100%. We ask subjects to indicate the row they switch from drawing a ball from box U to drawing a ball from box K. For experimental payment, one out of the 11 rows will be randomly chosen and subjects receive ¥100 if the randomly drawn ball from their chosen box at that row had the same color as their winning color. We take the average of the proportions of winning color balls in box K at the switching row and the above row as the matching probability. Table A.15 illustrates the task.

Note: you chose color Red as your winning color.

Situation	Box U	Box K	Your choice
1	? red balls, (100-?) black balls	0 red balls, 100 black balls	prefer Box K
2	? red balls, (100-?) black balls	20 red balls, 80 black balls	prefer Box K
3	? red balls, (100-?) black balls	30 red balls, 70 black balls	prefer Box K
4	? red balls, (100-?) black balls	40 red balls, 60 black balls	prefer Box K
5	? red balls, (100-?) black balls	45 red balls, 55 black balls	prefer Box K
6	? red balls, (100-?) black balls	50 red balls, 50 black balls	prefer Box K
7	? red balls, (100-?) black balls	55 red balls, 45 black balls	prefer Box K
8	? red balls, (100-?) black balls	60 red balls, 40 black balls	prefer Box K
9	? red balls, (100-?) black balls	70 red balls, 30 black balls	prefer Box K
10	? red balls, (100-?) black balls	80 red balls, 20 black balls	prefer Box K
11	? red balls, (100-?) black balls	100 red balls, 0 black balls	prefer Box K

Table A.15: Ambiguity task

A.5.3 The bisection process

To illustrate, consider the elicitation of CE via the bisection process. For each choice, we start by presenting the mid-point of the possible range of CE as the sure payment option. Let Y denote the positive payment in the bet ($Y = 24$ for the P-bet and $Y = 80$ for the $-bet). The CE of a bet should be between 0 and Y considering first-order stochastic dominance. The sure payment in the first choice should be $Y/2$, which is the mid-point of the possible range $(0, Y)$. If a subject chooses the bet in the first choice, the possible range would be adjusted to $(Y/2, Y)$ and she will face the mid-point $3Y/4$ as the sure payment in the second choice. Instead, if she chooses the sure payment in the first choice, the possible range will be adjusted to $(0, Y/2)$ and she will face the mid-point $Y/4$ in the second choice. We continue this process four times for the P-bet and six times for the $-bet due to the larger payment amount in the $-bet.

Subjects may make mistakes, and these mistakes could have a large impact on CE or PE if they occur in the initial choices of the bisection process. To mitigate this impact, we add an additional choice after the second choice in the bisection process. For example, a subject that chooses the P-bet for $Y/2$ will not only face $3Y/4$, but also $Y/4$ as additional choice before continuing with the next step of the bisection process. If subjects' decisions in the first three choices are inconsistent, e.g., choosing the bet over $Y/2$ (implying the bet $\succ Y/2$) but choosing $Y/4$ over the bet (implying $Y/4 \succ$ the bet), they return to the initial choice with the sure payment of $Y/2$.

A.5.4 The elicitation of the randomization range (R-range)

We illustrate the elicitation of the R-range with the CE R-range task. In each choice subjects have three options: they can 1) pay a small cost (¥0.10) and choose the fixed bet, 2) pay a small cost (¥0.10) and choose the sure payment, or 3) choose a randomization option for free, according to which a computer randomly selects one of the two options. Across choices, the sure payment changes according to the bisection method. Let Y denote the positive payment of the bet, and y^r the sure payment for which subjects chose the randomization option for the first time. Let $\underline{y}$ and $\overline{y}$ denote the lower and the upper bound of the R-range separately. It is clear that $0 \leq \underline{y} \leq y^r$ and $y^r \leq \overline{y} \leq Y$. To find $\underline{y}$, we raise (lower) the sure payment in the next choice if subjects choose the bet (the randomization option) according to the bisection procedure with the same number of iterations as in CE valuation. Similarly, to find $\overline{y}$ we increase (decrease) the sure payment in the next choice

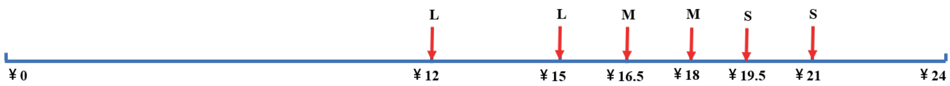


Figure A.7: An example of eliciting the CE R-range for the P-bet. S stands for choosing the sure payment, L stands for choosing the P-bet, and M stands for the randomization option. We use $\underline{y}$ and $\overline{y}$ to represent the lowest and the highest sure payments of the R-range separately. In the example, a subject chooses the P-bet when the sure payment is ¥12, and picks the randomization option when the sure payment increases to ¥18. We find $\underline{y}$ in the range [¥12, ¥18]. In the following choices, the subject chooses the P-bet when the sure payment is ¥15 and selects the randomization option when the sure payment is ¥16.5. We therefore obtain a $\underline{y}$ of ¥15.75 for the subject. Similarly, we find $\overline{y}$ in the range [¥18, ¥24]. In the choices of finding $\overline{y}$, the subject first selects the sure payment when it is ¥21 and picks the sure payment again when it is ¥19.5. We thus obtain a $\overline{y}$ of ¥18.75 for the subject. Finally, the elicited monetary randomization range for the subject is [¥15.75, ¥18.75].

if subjects choose the randomization option (the sure payment) according to the bisection procedure. If subjects never choose the randomization option, their CE R-range is zero. Figure A.7 provides an example decision screen for the elicitation of the CE R-range for the P-bet.

A.5.5 Measuring the desirability of control

The questions to elicit the control desirability in the questionnaire (Burger and Cooper, 1979; Gebhardt and Brosschot, 2002):

1. When I see a problem, I prefer to do something about it rather than sit by and let it continue.

2. I wish I could push many of life's daily decisions off on someone else.

3. There are many situations in which I would prefer only one choice rather than having to make a decision.

4. I like to wait and see if someone else is going to solve a problem so that I don't have to be bothered by it.

5. I prefer a job where I have a lot of control over what I do and when I do it.

6. I try to avoid situations where someone else tells me what to do.

7. Others usually know what is best for me.

8. I enjoy making my own decisions.

9. I enjoy having control over my own destiny.

10. I prefer to avoid situations where someone else has to tell me what it is I should be doing.

B Appendices to Chapter 3

B.1 Additional figures and tables

B.1.1 Additional tables for analysis in the main text

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	13.7%	42.9%	4.6%	38.8%
M1	40.3% [34.9%, 45.7%]	16.0% [10.9%, 21.1%]	10.1% [6.3%, 14.69%]	33.6% [29.7%, 37.7%]
M2	31.2% [26.3%, 36.0%]	17.2% [12.0%, 22.3%]	11.6% [7.4%, 16.6%]	40.1% [35.4%, 45.1%]
M3	33.1% [28.0%, 38.3%]	19.6% [14.3%, 25.1%]	12.9% [8.0%, 17.7%]	34.4% [29.1%, 39.4%]

Table B.1: Means and 95% confidence intervals [in brackets] of simulated proportions of CE PR patterns in Shi et al. (2024). The first row reports the actual PR proportions in the experiment. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively.

Sub-table a: PR proportions classified by DC and CE(DV)

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	17.4%	66.3%	0.5%	15.8%
M1	57.5% [52.7%, 62.0%]	18.1% [13.0%, 23.4%]	5.9% [2.7%, 9.2%]	18.5% [14.7%, 22.3%]
M2	60.1% [54.9%, 65.2%]	12.4% [8.2%, 16.8%]	9.3% [5.4%, 13.6%]	18.1% [14.7%, 21.7%]
M3	50.6% [45.1%, 56.5%]	14.3% [9.8%, 19.6%]	13.4% [8.7%, 18.5%]	21.7% [17.9%, 25.5%]

Sub-table b: PR proportions classified by DC and CE(BC)

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	72.3%	11.4%	2.7%	13.6%
M1	57.8% [51.6%, 62.0%]	17.8% [12.0%, 21.7%]	6.1% [4.3%, 11.4%]	18.4% [15.8%, 22.4%]
M2	60.5% [54.9%, 65.2%]	12.0% [7.6%, 16.8%]	9.6% [6.0%, 13.6%]	17.9% [15.2%, 21.7%]
M3	50.9% [41.3%, 53.3%]	14.0% [10.3%, 20.7%]	13.5% [10.3%, 20.1%]	21.6% [18.5%, 26.6%]

Table B.2: Means and 95% confidence intervals [in brackets] of simulated proportions of CE PR patterns in Loomes and Pogrebna (2017). The first row reports the actual PR proportions in the experiment. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. In the sub-table a), the actual PR patterns are classified by DC and CE(DV), and in the sub-table b), the actual PR patterns are classified by DC and CE(BC).

Sub-table a: DC pattern

Model	Act: $p(P,\$) \geq 0.5$	Est: $p(P,\$) \geq 0.5$	Act: $p(P,\$) < 0.5$	Est: $p(P,\$) < 0.5$
M1	154 (0.84)	148 (0.80)	30 (0.16)	36 (0.20)
M2	154 (0.84)	148 (0.80)	30 (0.16)	36 (0.20)
M3	154 (0.84)	146 (0.79)	30 (0.16)	38 (0.21)

Sub-table b: DC probability

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
$p(P,\$)$	mean	0.82	0.76	0.73	0.65	-0.06	-0.09	-0.17
		(0.34)	(0.31)	(0.32)	(0.31)	(0.19)	(0.18)	(0.24)
	median	1.00	0.91	0.85	0.73	-0.02	-0.05	-0.16

Table B.3: The comparison between the actual and the estimated choice patterns in the experiment of Loomes and Pogrebna (2017). M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Sub-table a: the number of subjects choosing the P-bet more ($p(P,\$) \geq 0.5$) and choosing the \$-bet more ($p(P,\$) < 0.5$) based on the actual and the estimated data. The proportions are in parentheses. Sub-table b: mean and median of the actual choosing probability, the estimated choosing probability, and the difference between the actual and the estimated choosing probability. The standard deviations are in parentheses.

Sub-table a): CE pattern

Model	Act: $CE_P \geq CE_\$$		Est: $CE_P \geq CE_\$$	Act: $CE_P < CE_\$$		Est: $CE_P < CE_\$$
	DV	BC		DV	BC	
M1	33 (0.18)	138 (0.75)	123 (0.67)	151 (0.82)	46 (0.25)	61 (0.33)
M2	33 (0.18)	138 (0.75)	145 (0.79)	151 (0.82)	46 (0.25)	39 (0.21)
M3	33 (0.18)	138 (0.75)	146 (0.79)	151 (0.82)	46 (0.25)	38 (0.21)

Sub-table b): CE valuations

	statistics	Act.	Est. M1	Est. M2	Est. M3	Diff. M1	Diff. M2	Diff. M3
$CE_P(DV)$	mean	8.93	7.97	7.99	7.76	-0.96	-0.94	-1.17
		(2.61)	(2.00)	(1.99)	(1.95)	(2.98)	(2.97)	(2.93)
	median	9.90	8.43	8.43	8.04	-1.34	-1.24	-1.29
$CE_\$$(DV)	mean	19.85	6.36	6.41	6.14	-13.49	-13.44	-13.72
		(12.97)	(4.99)	(5.07)	(6.20)	(13.27)	(13.28)	(13.91)
	median	15.10	5.58	5.58	4.16	-10.00	-10.03	-10.44
$CE_P(BC)$	mean	8.18	7.97	7.99	7.76	-0.22	-0.20	-0.42
		(2.05)	(2.00)	(1.99)	(1.95)	(1.46)	(1.45)	(1.18)
	median	8.25	8.43	8.43	8.04	-0.15	-0.16	-0.14
$CE_\$(BC)$	mean	6.55	6.36	6.41	6.14	-0.19	-0.14	-0.41
		(2.71)	(4.99)	(5.07)	(6.20)	(3.04)	(3.12)	(4.25)
	median	5.75	5.58	5.58	4.16	-0.25	-0.21	-1.40

Table B.4: The comparison between the actual and the estimated CE valuation patterns in the experiment of Loomes and Pogrebna (2017). Sub-table a: the number of subjects valuing the P-bet more and valuing the \$-bet more based on the actual and the estimated data. The proportions are in parentheses. The estimated choice patterns are categorized according to the estimated $p(CE_P \geq CE_\$) \geq 0.5$ or $p(CE_P \geq CE_\$) < 0.5$. Sub-table b: mean and median of CE valuations, the estimated CE valuations, and the difference between the actual and the estimated CE valuations. The actual CE valuations are computed in two approaches, the direct valuation (DV) approach and the binary choice (BC) approach. The standard deviations are in parentheses. M1 is the Logit stochastic choice model. M2 is the Blavatskyy (2009) choice model with homoscedastic random errors. M3 is the Blavatskyy (2009) choice model with heteroscedastic random errors.

B.1.2 Addition results for Shi et al. (2024) using CARA utility function

Estimation	Mean	Median	1%	5%	25%	75%	95%	99%
η								
M1	2.81 (3.03)	1.14	0.00	0.00	0.10	6.17	7.78	9.16
M2	1.17 (1.36)	0.84	0.03	0.07	0.35	1.36	3.39	6.55
M3	0.94 (1.19)	0.57	0.01	0.11	0.30	1.10	2.91	5.76
γ								
M1	3.21 (3.34)	2.48	-0.02	0.00	0.11	6.13	9.48	9.91
M2	0.18 (0.75)	0.11	-0.01	0.00	0.03	0.20	0.29	0.34
M3	0.20 (0.75)	0.12	0.00	0.01	0.04	0.20	0.38	0.68

Table B.5: Statistics of estimated parameters from data in Shi et al. (2024). The statistics are mean, median, and the 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ through joint estimation from direct choices and CE valuation choices. We employ the CARA utility function in the estimations. The standard deviations are in parentheses. Under M1, we employ the Logit stochastic choice function. Under M2, we employ the stochastic choice function proposed by Blavatskyy (2009). Under M3, we consider the heteroscedastic random errors in the stochastic choice function in Blavatskyy (2009).

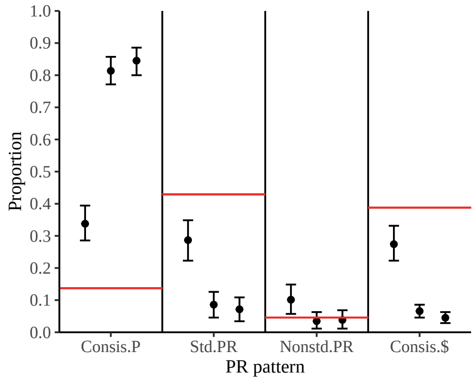


Figure B.1: Comparison between simulated and actual proportions of CE PR patterns in Shi et al. (2024). For each CE PR pattern, the simulation results displayed from left to right correspond to simulated proportions using stochastic models M1 to M3. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. We employ the CARA utility function. Black dots are means of simulated proportions. Black bars denote [5%, 95%] intervals of simulated proportions. Red lines depict actual proportions in the experiment.

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	13.7%	42.9%	4.6%	38.8%
M1	33.8% [28.6%, 39.4%]	28.7% [22.3%, 34.9%]	10.1% [5.7%, 14.9%]	27.4% [22.3%, 33.1%]
M2	81.4% [77.1%, 85.7%]	8.6% [4.6%, 12.6%]	3.5% [1.1%, 6.3%]	6.6% [4.6%, 8.6%]
M3	84.5% [80.0%, 88.6%]	7.1% [3.4%, 10.9%]	3.8% [1.1%, 6.9%]	4.5% [2.9%, 6.3%]

Table B.6: Means and 95% confidence intervals [in brackets] of simulated proportions of CE PR patterns in Shi et al. (2024). We employ the CARA utility function. The first row reports the actual PR proportions in the experiment. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively.

Sub-table a): DC pattern

Model	Act: $p(P,\$) \geq 0.5$	Est: $p(P,\$) \geq 0.5$	Act: $p(P,\$) < 0.5$	Est: $p(P,\$) < 0.5$
M1	95 (0.54)	151 (0.86)	80 (0.46)	24 (0.14)
M2	95 (0.54)	163 (0.93)	80 (0.46)	12 (0.07)
M3	95 (0.54)	165 (0.94)	80 (0.46)	10 (0.06)

Sub-table b): DC probability

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
$p(P,\$)$	mean	0.56	0.62	0.90	0.92	0.06	0.34	0.35
		(0.40)	(0.25)	(0.24)	(0.20)	(0.26)	(0.41)	(0.40)
	median	0.67	0.65	0.99	0.99	0.00	0.17	0.22

Table B.7: The comparison between the actual and the estimated choice patterns in the experiment of Shi et al. (2024). We employ the CARA utility function. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Sub-table a: the number of subjects choosing the P-bet more ($p(P,\$) \geq 0.5$) and choosing the \$-bet more ($p(P,\$) < 0.5$) based on the actual and the estimated data. The proportions are in parentheses. Sub-table b: mean and median of the actual choosing probability, the estimated choosing probability, and the difference between the actual and the estimated choosing probability. The standard deviations are in parentheses.

Sub-table a: CE pattern

Model	Act: $CE_P \geq CE_\$$	Est: $CE_P \geq CE_\$$	Act: $CE_P < CE_\$$	Est: $CE_P \geq CE_\$$
M1	32 (0.18)	92 (0.53)	143 (0.82)	83 (0.47)
M2	32 (0.18)	60 (0.34)	143 (0.82)	115 (0.66)
M3	32 (0.18)	59 (0.34)	143 (0.82)	116 (0.66)

Sub-table b: CE valuations

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
	mean	14.64	4.89	10.25	9.80	-9.76	-4.40	-4.85
CE_P		(4.11)	(6.46)	(4.56)	(4.56)	(7.88)	(5.59)	(5.69)
	median	14.25	0.48	9.68	9.12	-12.09	-4.11	-2.66
	mean	21.89	4.13	5.98	5.08	-17.76	-15.91	-16.80
$CE_\$$		(10.83)	(8.58)	(6.54)	(5.30)	(12.67)	(13.21)	(12.30)
	median	19.38	0.11	2.66	2.45	-17.92	-15.27	-14.46

Table B.8: The comparison between the actual and the estimated CE valuation patterns in the experiment of Shi et al. (2024). We employ the CARA utility function. Sub-table a: the number of subjects valuing the P-bet more and valuing the \$-bet more based on the actual and the estimated data. The proportions are in parentheses. The estimated choice patterns are categorized according to the estimated $p(CE_P \geq CE_\$) \geq 0.5$ or $p(CE_P \geq CE_\$) < 0.5$. Sub-table b: mean and median of CE valuations, the estimated CE valuations, and the difference between the actual and the estimated CE valuations. The standard deviations are in parentheses. M1 is the Logit stochastic choice model. M2 is the Blavatskyy (2009) choice model with homoscedastic random errors. M3 is the Blavatskyy (2009) choice model with heteroscedastic random errors.

B.2 Additional results for PE PR patterns

For the PE valuations, we estimate stochastic functions from direct choice and PE valuation choices. Similar to before, we consider three stochastic choice models, the Logit model, the model proposed in Blavatskyy (2009) with homoscedastic random errors, and the model proposed in Blavatskyy (2009) with heteroscedastic random errors. Table B.9 summarizes mean, standard deviation, median, and 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ.

Estimation	Mean	Std	Median	1%	5%	25%	75%	95%	99%
				η					
M1	3.91	3.77	1.83	0.05	0.30	0.86	7.39	9.97	9.99
M2	2.89	3.48	1.11	0.01	0.09	0.44	3.58	9.95	9.98
M3	2.79	3.50	0.91	0.06	0.11	0.40	3.34	9.98	9.99
				γ					
M1	0.71	0.30	0.75	0.17	0.28	0.49	0.90	1.08	1.28
M2	0.76	0.29	0.77	0.20	0.30	0.52	0.96	1.15	1.37
M3	0.75	0.26	0.79	0.14	0.29	0.51	0.94	1.08	1.35

Table B.9: Statistics of estimated parameters from data in Shi et al. (2024). The statistics are mean, median, and the 1%, 5%, 25%, 75%, 95%, and 99% percentiles of the stochastic choice function parameter η and the utility curvature parameter γ through joint estimation from direct choices and PE valuation choices. The standard deviations are in parentheses. Under M1, we employ the Logit stochastic choice function. Under M2, we employ the stochastic choice function proposed by Blavatskyy (2009). Under M3, we consider the heteroscedastic random errors in the stochastic choice function in Blavatskyy (2009).

With the estimated stochastic choice function, we can construct the probability density function of PE_P elicited through the bisection process as follows

$$PE_P(m) = \prod_{i=1}^{5} p(P, R_{m_i})^{I_i^{P'}} p(R_{m_i}, P)^{1-I_i^{P'}} \tag{5.1}$$

where R_{m_i} represents the ith reference lottery a subject faces in the binary choices under the bisection process. The outcomes of R_{m_i} are fixed and the winning probability is m_i. $I_i^{P'}$ is an indicator which equals to 1 if $m > m_i$ and 0 if $m < m_i$. The probability density function of $PE_\$$ under the bisection process can be calculated similarly:

$$PE_\$(n) = \prod_{i=1}^{4} p(\$, R_{n_i})^{I_i^{\$'}} p(R_{n_i}, \$)^{1-I_i^{\$'}} \tag{5.2}$$

where R_{n_i} represents the ith reference lottery a subject faces in the binary choices under the bisection process. The outcomes of R_{n_i} are fixed and the winning probability is n_i. $I_i^{\$'}$ is an indicator which equals to 1 if $n > n_i$ and 0 if $n < n_i$. Shi et al. (2024) elicited PE_P through five binary choices and $PE_\$$ through four choices. Based on equations 5.1 and 5.2, the certainty equivalent valuation of the P-bet is not less than that of the \$-bet is

$$p(PE_P \geq PE_\$) = \sum_{n \geq m, m \in M, n \in N} PE_P(m) PE_\$(n) \tag{5.3}$$

where M and N are the sets of all possible CE valuations of the P-bet and the \$-bet obtained through the bisection process.

Combining the PE valuation probability $p(PE_P \geq PE_\$)$ with the direct choice probability $p(P, \$)$, we can calculate the individual-level estimated probabilities of PE PR patterns. Testing the estimated probabilities with one million simulations as described in subsection 3.2.2, we obtain the means and the 95% confidence intervals of the simulated proportions as illustrated in Figure B.2 (details in Table B.10). As shown in Figure B.2, the stochastic choice functions perform much better in estimating the PE PR proportions compared to the CE PR pattern. The actual proportion of standard PR is inside the [5%, 95%] intervals of the simulated proportions for all stochastic models M1 to M3. Furthermore, the simulated magnitudes of PE PR are from 16.9% to 20.9%, close to the actual PE PR magnitude of 29.1%. The simulated asymmetry ratios are from 0.71 to 0.78, lower than the actual ratio of 0.46.

	Consis. P	Std. PR	Nonstd. PR	Consis. $
Actual	47.4%	9.1%	20.0%	23.4%
M1	54.2% [50.3%, 58.3%]	7.4% [4.0%, 10.9%]	9.5% [5.7%, 13.7%]	28.9% [25.7%, 32.0%]
M2	49.3% [45.1%, 53.7%]	8.4% [4.6%, 12.6%]	11.6% [7.4%, 16.0%]	30.7% [26.9%, 34.9%]
M3	48.8% [44.6%, 53.1%]	8.7% [5.1%, 12.6%]	12.2% [8.0%, 17.1%]	30.3% [26.3%, 34.3%]

Table B.10: Means and 95% confidence intervals [in brackets] of simulated proportions of PE PR patterns in Shi et al. (2024). The first row reports the actual PR proportions in the experiment. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively.

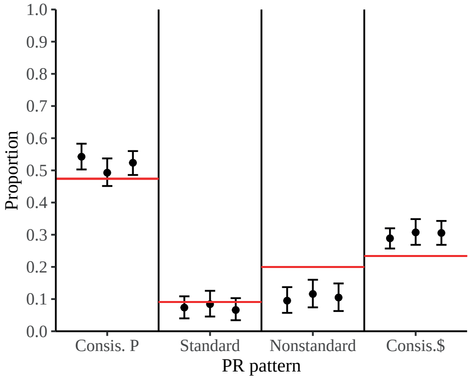


Figure B.2: The comparison between simulated and actual proportions of PE PR patterns in Shi et al. (2024). For each PE PR pattern, the simulation results displayed from left to right correspond to simulated proportions using stochastic models M1 to M3. M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Black dots are means of simulated proportions. Black bars denote [5%, 95%] intervals of simulated proportions. Red lines depict actual proportions in the experiment.

The above results suggest that after allowing for the stochastic choice and assuming individuals behave consistently following a consistent stochastic function in both direct choices and PE valuation choices, the simulated PE PR proportions are close to the actual PR proportions. Although differences still exist, the simulated PE PR magnitude and asymmetry ratio are closer to the actual data compared to those of the CE PR. In the following analysis, we discuss the choice pattern and the PE valuation separately by comparing the estimated results with the actual data.

First, we compare the estimated DC choice ratio. As shown in the Sub-table a of Table B.11, the estimated stochastic functions slightly overestimate the number of subjects choosing the P-bet more frequently in direct choices. The difference between the estimated and the actual proportions choosing the P-bet more frequently is significant under M1 (proportion test, $p < 0.01$), but not significant under M2 and M3 (proportion test, $p > 0.10$ for M2 and M3). Sub-table b of Table B.11 summarizes the estimated DC probabilities and their differences from the actual DC probabilities. The difference between the estimated and the actual DC probabilities is significant under M1 (Wilcoxon signed-rank test, $p < 0.01$), but not significant under M2 and M3 (Wilcoxon signed-rank test, $p > 0.10$ for M2 and M3).

Then we compare the estimated and the actual PE valuations. Sub-table a) of

Sub-table a: DC pattern

Model	Act: $p(P,\$) \geq 0.5$	Est: $p(P,\$) \geq 0.5$	Act: $p(P,\$) < 0.5$	Est: $p(P,\$) < 0.5$
M1	95 (0.54)	117 (0.67)	80 (0.46)	58 (0.33)
M2	95 (0.54)	101 (0.58)	80 (0.46)	74 (0.42)
M3	95 (0.54)	101 (0.58)	80 (0.46)	74 (0.42)

Sub-table b: DC probability

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
$p(P,\$)$	mean	0.56	0.62	0.58	0.58	0.05	0.01	0.01
		(0.40)	(0.39)	(0.38)	(0.36)	(0.13)	(0.09)	(0.11)
	median	0.67	0.76	0.65	0.61	0.00	0.00	0.00

Table B.11: The comparison between the actual and the estimated choice patterns in the experiment of Shi et al. (2024). M1 to M3 represent the Logit stochastic model, the stochastic model in Blavatskyy (2009) with homoscedastic random errors, and the stochastic model in Blavatskyy (2009) with heteroscedastic random errors, respectively. Sub-table a: the number of subjects choosing the P-bet more ($p(P,\$) \geq 0.5$) and choosing the \$-bet more ($p(P,\$) < 0.5$) based on the actual and the estimated data. The proportions are in parentheses. Sub-table b: mean and median of the actual choosing probability, the estimated choosing probability, and the difference between the actual and the estimated choosing probability. The standard deviations are in parentheses.

Table B.12 summarizes the comparison of the estimated and the actual numbers and proportions of two PE patterns: $PE_P \geq PE_\$$ and $PE_P < PE_\$$. [2] The difference between the estimated and the actual proportions valuing the P-bet higher is significant under M2 and M3 (proportion test, $p < 0.01$ for M2 and M3), but not significant under M1 (proportion test, $p > 0.10$). Meanwhile, the difference between the estimated PE valuation pattern and the actual PE valuation pattern is not significant under M1 (McNemar's chi-squared tests, $p > 0.10$), but significant under M2 and M3 (McNemar's chi-squared tests, $p < 0.01$ for M2 and M3). Furthermore, we compare the estimated PE valuations and the actual PE valuations in Sub-table b of Table B.12. For the PE_P, the estimated PE_P is significantly different from the actual PE_P (Wilcoxon signed-rank test, $p < 0.01$ for all stochastic models M1 to M3). Meanwhile, the estimated $PE_\$$ is significantly different from the $PE_\$$ (Wilcoxon signed-rank test, $p < 0.01$ for all stochastic models M1 to M3). Although the differences exist in the estimated PE valuations and the actual PE valuations,

[2]For the estimated PE patterns under stochastic functions, we classify $PE_P \geq PE_\$$ by the valuation probability $p(PE_P \geq PE_\$) \geq 0.5$ and $PE_P < PE_\$$ by the valuation probability $p(PE_P \geq PE_\$) < 0.5$.

Sub-table a: PE pattern

Model	Act: $PE_P \geq PE_\$$	Est: $p_{PE} \geq 0.5$	Act: $PE_P < PE_\$$	Est: $p_{PE} < 0.5$
M1	121 (0.69)	116 (0.66)	54 (0.31)	59 (0.33)
M2	121 (0.69)	103 (0.59)	54 (0.31)	72 (0.41)
M3	121 (0.69)	104 (0.59)	54 (0.31)	71 (0.41)

Sub-table b: PE valuation

		Act.	Est.			Diff.		
	statistics		M1	M2	M3	M1	M2	M3
PE_P	mean	0.27	0.27	0.26	0.26	0.01	-0.01	-0.01
		(0.11)	(0.11)	(0.10)	(0.10)	(0.06)	(0.05)	(0.05)
	median	0.25	0.24	0.23	0.23	0.00	0.00	0.00
$PE_\$$	mean	0.19	0.21	0.21	0.21	0.03	0.03	0.03
		(0.04)	(0.01)	(0.01)	(0.01)	(0.04)	(0.04)	(0.04)
	median	0.20	0.21	0.21	0.21	0.02	0.02	0.02

Table B.12: The comparison between the actual and the estimated PE valuation patterns in the experiment of Shi et al. (2024). Sub-table a: the number of subjects valuing the P-bet more and valuing the \$-bet more based on the actual and the estimated data. The proportions are in parentheses. The estimated choice patterns are categorized according to the estimated $p(PE_P \geq PE_\$) \geq 0.5$ or $p(PE_P \geq PE_\$) < 0.5$. Sub-table b: mean and median of PE valuations, the estimated PE valuations, and the difference between the actual and the estimated PE valuations. The standard deviations are in parentheses. M1 is the Logit stochastic choice model. M2 is the Blavatskyy (2009) choice model with homoscedastic random errors. M3 is the Blavatskyy (2009) choice model with heteroscedastic random errors.

the difference is less pronounced compared to the CE valuation. The mean difference between the estimated and the actual PE_P is from -0.1 to 0.1 for M1 to M3 (median = 0.00 for M1 to M3) and the mean difference between the estimated and the actual $PE_\$$ is around 0.03 for M1 to M3 (median = 0.02 for M1 to M3).

Generally, the stochastic choices perform better in explaining PE PR compared to CE PR. The simulated proportions of PE PR patterns are close to the actual proportions. Although differences still exist between the estimated and the actual PE valuations, the discrepancies are lower compared to the CE valuations.

B.3 Bisection process

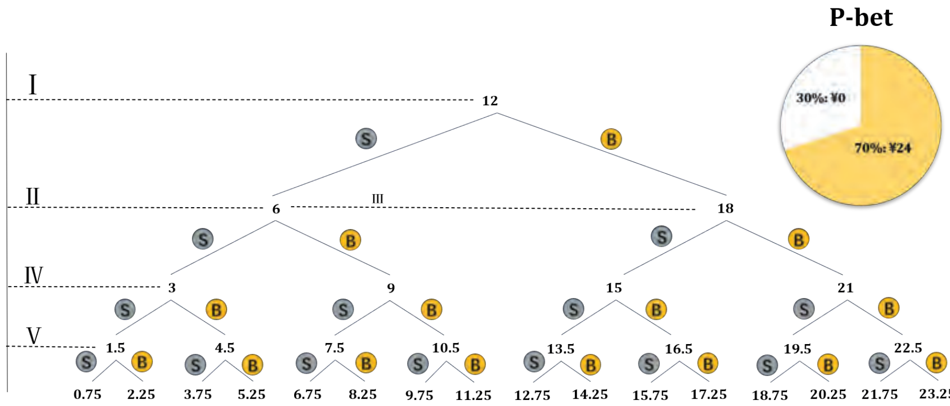


Figure B.3: The bisection process for CE(BC) of the P-bet. S stands for choosing the sure payment, B stands for choosing the bet. The numbers at the top four rows (I, II, III, IV) are the possible sure payments subjects faced in four binary choices. The numbers at the bottom are the elicited CE(BC) for the corresponding choices.

The initial possible valuation interval $int_1 = [0, y_{max}]$ is between 0 and the maximum payoff of a bet, and the sure payment in the first choice is the midpoint of the interval $\frac{y_{max}}{2}$. If the bet is chosen in the first choice, the possible valuation interval in the second choice int_2 changes to the upper sub-interval $[\frac{y_{max}}{2}, y_{max}]$ and otherwise to the lower sub-interval $[0, \frac{y_{max}}{2}]$. The sure payment option in the second choice is the midpoint of the possible valuation interval in the second choice. They conducted four such choices for the CE(BC) of the P-bet and six for the $-bet as the initial possible valuation interval is larger for the $-bet. Besides the CE(BC), they also elicited the probability equivalents through binary choices (henceforth referred to PE(BC)) between the P-bet (or the $-bet) and reference lotteries with winning probabilities changed under the bisection process. To avoid subjects choosing the sure payment disregarding true preferences to make the subsequent sure payment options lay in a higher possible valuation range, especially in the first choice with the largest possible valuation range, they add some disturbances after the first choice by facing subjects with choices in both sub-intervals no matter what they chose in the first choice. [3]

[3]For the CE of the P-bet, subjects face choices with both sure payment options 6 and 18. For the CE of the $-bet, subjects face choices with both sure payment options 20 and 60. For the PE of the P-bet, subjects face choices with reference lotteries with both winning probabilities 17.5% and 52.5%. For the PE of the $-bet, subjects face choices with reference lotteries with both winning probabilities 6.25% and 18.75%.

C Appendices to Chapter 4

C.1 Appendix: Details for theoretical analysis

C.1.1 Derivation of $me_{0,t}$:

Let $V(X_t) = \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \phi(\mathrm{ME}_{0,t})\, dF_t$. Since the individual faces no valuation uncertainty over the present monetary reward x, the decision utility of choosing x is simply $V(x) = \phi(x)$. The present monetary equivalent $(me_{0,t})$ of the reward X_t is defined by

$$V(me_{0,t}) = \phi(me_{0,t}) = V(X_t) = \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \phi(\mathrm{ME}_{0,t})\, dF_t.$$

Let $\Delta = \mathrm{ME}_{0,t} - me_{0,t}$ denote the difference of present monetary equivalent of X_t and $\mathrm{ME}_{0,t}$ given $\mathrm{ME}_{0,t}$. We can expand $\phi(\mathrm{ME}_{0,t}) = \phi(me_{0,t} + \Delta)$ at $me_{0,t}$ as a Taylor series and then take an expectation with respect to F_t:

$$\begin{aligned} V(X_t) &= \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \phi(me_{0,t} + \Delta)\, dF_t \\ &= \phi(me_{0,t}) + \phi'(me_{0,t}) E_F(\mathrm{ME}_{0,t} - me_{0,t}) + \frac{1}{2}\phi''(me_{0,t})\sigma_t^2 + O(\Delta) \\ &= \phi[me_{0,t}], \end{aligned}$$

where $\sigma^2_{F,t}$ captures the variability of $\mathrm{ME}_{0,t}$ and $O(\Delta)$ is total of Δ with moments higher than 2. This gives $me_{0,t}$ as:

$$me_{0,t} \approx E_F(\mathrm{ME}_{0,t}) - \kappa VU_{0,t},$$

where $VU_{0,t} = \frac{1}{2}\sigma^2_{F,t}$ and $\kappa = -\frac{\phi''(me_{0,t})}{\phi'(me_{0,t})}$. Similar to the mean-variance framework in the financial literature (Markowitz, 1952), this approximation is exact when $\phi(\cdot)$ is quadratic or F_t is normal.

C.1.2 Derivation of the R-range

Following Klibanoff et al. (2005), when the individual faces a lottery $(\lambda, X_t; 1-\lambda, x)$, she first performs the expected calculation using each $\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}$ as $ME(\lambda, X_t; 1-\lambda, x) = \lambda \mathrm{ME}_{0,t} + (1-\lambda)x$. Then, the individual integrates across the set of present monetary equivalents using the function $\phi(\cdot)$, and the decision utility over such a lottery is:[4]

$$V(\lambda, X_t; 1-\lambda, x) = \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \phi\left[\lambda \mathrm{ME}_{0,t} + (1-\lambda)x\right] dF_t.$$

Since the function ϕ is concave, there exists a range of x such that the individual randomizes strictly. To obtain the randomization range, we can rewrite the above equation as $V(\lambda, X_t; 1-\lambda, x) = \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \phi\left[x + \lambda(\mathrm{ME}_{0,t} - x)\right] dF_t$. Similarly, using Taylor expansion at x, we can calculate the optimal randomization probability λ^* as

$$\lambda^* = \frac{E_F\left(\mathrm{ME}_{0,t}\right) - x}{2\kappa VU_{0,t}}.$$

Solving for $\bar{x}$ such that $\lambda^* = 0$ and $\underline{x}$ such that $\lambda^* = 1$, we obtain the lower bound, the upper bound, and the R-range of x that the individual randomizes:

$$\underline{x}_{0,t} = E_F(\mathrm{ME}_{0,t}) - 2\kappa VU_{0,t}, \quad \bar{x}_{0,t} = E_F(\mathrm{ME}_{0,t}), \quad \text{R-range}_{0,t} = \bar{x}_{0,t} - \underline{x}_{0,t} = 2\kappa VU_{0,t}.$$

Recall that the present equivalent of a reward is $me_{0,t} = E_F(\mathrm{ME}_{0,t}) - \kappa VU_{0,t}$. The randomization probability at the present payment x close to the present equivalent of X_t is $\lambda^*_x \approx \frac{E_F(\mathrm{ME}_{0,t}) - me_{0,t}}{2\kappa VU_{0,t}} = \frac{1}{2}$. This implies that the individual chooses randomization probabilities close to 0.5 when the two rewards yield similar utilities.

As a concrete numerical example, let $\phi(\mathrm{ME}_{0,t}) = -e^{-\kappa \mathrm{ME}_{0,t}}$, and F_t be a normal distribution with the mean μ_t and the standard deviation of σ_t. With these parametric assumptions we have
$\underline{x}_{0,t} = \mu_t - \kappa\sigma_t^2, \quad \bar{x}_{0,t} = \mu_t, \quad \text{R-range}_{0,t} = \bar{x}_{0,t} - \underline{x}_{0,t} = \kappa\sigma_t^2, \quad me_{0,t} = \mu_t - \kappa\sigma_t^2/2.$

C.1.3 Structural analysis of valuation uncertainty attitude κ and the subjective belief F

If we make parametric assumptions about $\phi(\cdot)$ and F, we can estimate κ and F from the individual's randomization probabilities. For example, we can assume $\phi(ME) = -e^{-\kappa ME}$ and a binomial belief distribution $B(10, p)$. The binomial belief distribution offers enough flexibility to capture different shapes of subjective beliefs and an intuitive interpretation: the parameter p can be interpreted as the proba-

[4] An alternative approach is to first apply ϕ and then take the expectation:

$$V(\lambda, X_t; 1-\lambda, x) = \int_{\mathrm{ME}_{0,t} \in \mathbb{ME}_{0,t}} \lambda\phi\left(\mathrm{ME}_{0,t}\right) + (1-\lambda)\phi\left(x\right) dF_t.$$

The individual has no incentive to randomize strictly because then this formulation is basically EUT with the utility function $u(x) = \phi(x)$.

bility of receiving the reward. When an individual chooses the probability λ for the timed reward and $1-\lambda$ for x, the decision utility is:

$$V(\lambda, X_t; 1-\lambda, x) = \int_{ME=0}^{10} -e^{-\kappa[\lambda ME+(1-\lambda)x]} \times C_{10}^{ME} p^{ME}(1-p)^{10-ME} dME,$$

which allows us to compute the optimal randomization probability λ^* as a function of x. Varying x, we obtain a sequence of λ^*. We can then estimate p and κ for the individual by minimizing the sum of squared deviations of optimal randomization probabilities from the individual's actual randomization probabilities. The variance of the estimated binomial distribution $10p(1-p)$ can then be used as a measure of valuation uncertainty. We can also use $10\kappa p(1-p)$ instead of R-range in our analysis. However, this approach requires parametric assumptions, and choosing the wrong specifications of the parametric forms may bias the results. Moreover, structural estimation may be unreliable or fail when randomization probabilities behave erratically in relation to x, potentially impacting both the sample size and the results. Due to these considerations, we opt to use R-range in the main text.

C.1.4 The front-end delay analysis

Since we normalize $X = 1$, the discount between time 0 and t in the front-end delay experiment according to Equation 4.1 is simply:

$$\delta_{0,t} = \frac{me_{0,t}}{X} = me_{0,t} = E_F(\mathrm{ME}_{0,t}) - \kappa V\mathrm{ME}_{0,t} = \delta^t - \kappa VU_{0,t}.$$

It is less straightforward to derive the discount factor between time 1 and $t+1$. There are two possible approaches. In the first approach, we follow the classical view that intertemporal comparisons are based on discounted present monetary equivalents. Note that the present monetary equivalent of the timed reward X_{t+1} is defined as

$$me_{0,t+1} = E_F(\mathrm{ME}_{0,t+1}) - \kappa VU_{0,t+1} = \delta^{t+1} - \kappa VU_{0,t+1}.$$

For a monetary payment x receiving at $t=1$ (denoted by x_1), the present monetary equivalent is defined as

$$me_{0,1}^x = E_F(ME_{x_1}) - \kappa VU_{0,1}^x = x\delta - \kappa VU_{0,1}^x,$$

where $E_F(ME_{x_1}) = x\delta$ because, as explained in the main text, the individual discounts exponentially if there is no valuation uncertainty, and $VU_{0,1}^x$ is the valuation uncertainty of x_1. The equivalent of X_{t+1} in terms of the monetary payment at $t = 1$ is the value of x_1 such that $me^x_{0,1} = x_{1,t+1}\delta - \kappa VU^x_{0,1} = me_{0,t+1} = \delta^{t+1} - \kappa VU_{0,t+1}$. Denoted this value by $me_{1,t+1}$, we have

$$me_{1,t+1} = \delta^t - \kappa\Delta_{1,t+1},$$

where $\Delta_{1,t+1} = \left[VU_{0,t+1} - VU^{me}_{0,1}\right]/\delta$, capturing the change of valuation uncertainty from 1 to $t+1$. Recall $X = 1$, the discount factor from 1 to $t+1$ is then:

$$\delta_{1,t+1} = \frac{ME_{1,t+1}}{X} = me_{1,t+1} = \delta^t - \kappa\Delta_{1,t+1}.$$

The front-end delay effect is captured by:

$$\beta_{FE} \quad = \quad \frac{\delta_{0,t}}{\delta_{1,t+1}} = \frac{\delta^t - \kappa VU_{0,t}}{\delta^t - \kappa\Delta_{1,t+1}} = 1 - \frac{\kappa VU_{0,t} - \kappa\Delta_{1,t+1}}{\delta^t - \kappa\Delta_{1,t+1}},$$

which is less than 1 when the change of valuation uncertainty from 0 to t ($VU_{0,t}$) is substantially larger than the change of valuation uncertainty from 1 to $t+1$ ($\Delta_{1,t+1}$). When $\Delta_{1,t+1}$ is small and $VU_{0,t}$ is substantially larger than $\Delta_{1,t+1}$, we can write

$$\beta_{FE} \approx 1 - \kappa VU_{0,t}/\delta^t.$$

The alternative approach to derive the discount factor between 1 and $t+1$ is to assume that the individual takes x_1 as the *valuation currency* when reporting the equivalent of X_{t+1} in terms of the monetary payment at $t = 1$, just as she takes x as the valuation currency when reporting the present monetary equivalent of X_t. The discount factor between 1 and $t+1$ can then be written similarly as that between 0 and t:

$$\delta_{1,t+1} = me_{1,t+1} = E_F(\mathrm{ME}_{1,t+1}) - \kappa' VU_{1,t+1} = \delta^t - \kappa' VU_{1,t+1},$$

where $me_{1,t+1}$ is the equivalent of X_{t+1} in terms of the monetary payment at 1, and $VU_{1,t+1}$ is the valuation uncertainty about $me_{1,t+1}$. The parameter κ', denoting the cautious attitude toward the valuation uncertainty about $me_{1,t+1}$, could differ from κ. This is because x_1 also has valuation uncertainty, and it is not obvious that the individual should be equally cautious when comparing X_{t+1} with x_1 versus when comparing X_t with x (the immediate monetary payment). The front-end delay effect is captured by:

$$\beta_{FE} \quad = \quad \frac{\delta_{0,t}}{\delta_{1,t+1}} = \frac{\delta^t - \kappa VU_{0,t}}{\delta^t - \kappa VU_{1,t+1}} = 1 - \frac{\kappa VU_{0,t} - \kappa' VU_{1,t+1}}{\delta^t - \kappa' VU_{1,t+1}},$$

which is smaller than 1 when $\kappa VU_{0,t} > \kappa' VU_{1,t+1}$. Thus, the front-end delay effect arises only when the individual is less cautious when valuating X_{t+1} in terms of x_1, or she perceives lower valuation uncertainty about X_{t+1} in terms of the payment at 1 than about the present monetary equivalent of X_t.

The above analysis clarifies several important points in the examination of the front-end delay. First, measuring the valuation uncertainty between 1 and $t+1$ depends on the theoretical approach. In the first approach, it is $\kappa VU_{0,t+1} - \kappa VU_{0,t}$, not $\kappa' VU_{1,t+1}$. Particularly, it is problematic to use the R-range$_{1,t+1}$ for the valuation uncertainty between 1 and $t+1$. This is because the individual perceives uncertainty both about X_{t+1} and x_1. Consequently, the individual randomizes to hedge the valuation uncertainty of both options, resulting R-range$_{1,t+1}$ capturing both R-range$_{0,1}$ and R-range$_{0,t+1}$. In the second approach, it is $\kappa' VU_{1,t+1}$. Second, to examine the relationship between the front-end delay effect and valuation uncertainty, we need to look at the change of valuation uncertainty. This can be most clearly seen when we focus on the second approach, where we should focus on $\kappa VU_{0,t} - \kappa' VU_{1,t+1}$. Notably, there is no need to include an interaction term $VU\times$Present because the front-end delay effect arises from the change in valuation uncertainty, not different reactions to $VU_{0,t}$ and $VU_{1,t+1}$.

C.1.5 Predicting β_{FE} with β

Let 4 weeks be one delay interval. With R-range$_{0,4}$ and R-range$_{0,8}$ as measures of $\kappa VU_{0,4}$ and $\kappa VU_{0,8}$, we can write:

$$\begin{aligned} me_{0,4} &= \delta - \kappa VU_{0,4} = \delta - s\text{R-range}_{0,4}, \\ me_{0,8} &= \delta^2 - \kappa VU_{0,8} = \delta^2 - s\text{R-range}_{0,8}, \end{aligned}$$

where s is an adjustment factor to bring the two sides of the equation comparable. Inserting these elicited values into the above two equations, we can calculate δ and s for each subject. With the calculated δ and s, we can predict $me_{4,8}$ based on the analysis in Appendix C.1.4. This value depends on the theoretical approach. In the first theoretical approach, we can use the following equation to predict $me_{4,8}$:

$$\hat{me}_{4,8} = \delta - \kappa\Delta_{4,8} = \delta - s\frac{\text{R-range}_{0,8} - \text{R-range}_{0,4} \times \hat{me}_{4,8}}{\delta},$$

where we use R-range$_{0,4} \times \hat{me}_{4,8}$ to approximate the valuation uncertainty of $\hat{me}_{4,8}$ by proportionality. This is because $\hat{me}_{4,8}$ is the payment at 4 weeks, and we know that R-range$_{0,4}$ measures the valuation uncertainty of the reward (normalized as 1) in 4 weeks.

In the second theoretical approach, we can use the equation:

$$\hat{me}_{4,8} = \delta - s\text{R-range}_{4,8}.$$

With the predicted $\hat{me}_{4,8}$, we can calculate the front-end delay effect $\hat{\beta}_{FE} = \frac{me_{0,4}}{\hat{me}_{4,8}}$.

In addition, we can evaluate the performance of the two theoretical approaches by using the calculated $\hat{\beta}_{FE}$ and comparing its prediction error in each approach with the actual β_{FE}.

C.2 Alternative analysis of the R-range and its relationship with present bias

In this section, we perform the analysis of the R-range and its relationship with present bias based on existing models that do not consider valuation uncertainty. We start the analysis by noting that randomizing between x and X_t generates a lottery, in which both the payoff and the payoff date are risky (DeJarnette et al., 2020; Epper et al., 2020). To properly evaluate the lottery, we need to make assumptions about the sequence that the individual integrates over time and across risk. There are three approaches: 1) the individual behaves according to the expected discounted utility (EDU) model, in which the aggregation sequence of risk or time is irrelevant; 2) the individual behaves according to the rank-dependent expected utility model (RDU), and she first integrates over time and obtain the present equivalents of the consumption streams in each state of nature, and then evaluates the resulting lottery. We call this approach the time first approach; 3) the individual behaves according to RDU, and she evaluates the lotteries at each time separately and obtain the certainty equivalents, and then integrates the certainty equivalents over time. We call this approach the risk first approach. In addition, the individual may perceive the future payment risky, which Halevy (2008) refers to as the implicit risk approach. In that case, the individual faces a compound lottery when randomizing between x and X_t, and we also need to consider how the individual evaluates the compound lottery.

Below we perform a comprehensive analysis of the randomization behavior about the delayed reward. We show that it can be difficult for RDU to simultaneously accommodate the R-range, present bias, and the relationship between them. Intuitively, for the R-range to be positive, the probability weighting function needs to be sufficiently concave such that the individual has incentive to randomize strictly. However, present bias requires (locally) convex probability weighting (Halevy, 2008), such as the inverse S-shaped probability weighting function.[5]

Throughout the analysis, we use $u(\cdot)$ to denote the Bernoulli utility function over consumption and normalize $u(0) = 0$. We let δ^t to denote the discount factor for consumption at time t. When the individual faces risk, we allow for non-additivity

[5]Some studies use subproportionality of the probability weighting function to explain hyperbolic and subadditive discounting (Epper and Fehr-Duda, 2023; Diecidue et al., 2023). While subproportionality is unrelated to the concavity of probability weighting, in order to explain other anomalies like the certainty effect, those studies generally do not deviate from the inverse S-shaped probability weighting function. Indeed, the estimated probability weighting function in Diecidue et al. (2023) is inverse S-shaped.

Rand. prob.	Now	Date t
λ	0	X
$1-\lambda$	x	0

Table C.1: The lottery generated from randomizing between x and X_t.

in risk and use $g(\cdot)$ to denote the probability weighting function (Tversky and Kahneman, 1992; Quiggin, 1982).

C.2.1 No implicit risk

For the reward at $t = 0$, all existing models predict no randomization range. To see this concretely, note that randomizing between x and X_0 generates a simple lottery $L_0 = (1-\lambda, x; \lambda, X_0)$. When $u(X_0) \geq u(x)$, a general evaluation of the lottery can be written as $V(L_0) = u(x) + g(\lambda)\left[u(X_0) - u(x)\right]$. Since $g(\lambda)$ is monotonic in λ, the optimal randomization probability is $\lambda = 1$ when $u(X_0) \geq u(x)$ and $\lambda = 0$ when $u(X_0) < u(x)$.

Consider now the delayed reward X_t, and the individual perceives no risk about the delayed reward. The present monetary equivalent of the delayed reward X_t is $me_{0,t} = u^{-1}\left[\delta^t u(X)\right]$. Thus, the individual does not exhibit present bias. Randomizing between x and X_t generates a lottery where both the reward and the reward date are risky. Table C.1 provides a concrete illustration of the lottery. To properly evaluate the lottery, as stated earlier, we need to make assumptions about the sequence that the individual integrates over time and across risk: the (EDU) model, the time first approach, and the risk first approach. We discuss the three approaches below.

The expected discounted utility model (EDU): Let δ^t denote the discount factor for t, the individual evaluates the delayed reward X_t as $V(X_t) = \delta^t u(X)$. When the individual randomizes between the immediate payment and the delayed reward, the individual evaluates the resulting lottery L as:

$$V(L) = (1-\lambda)\, u(x) + \lambda \delta^t u(X) = u(x) + \lambda\left[\delta^t u(X) - u(x)\right].$$

The individual randomizes strictly $(0 < \lambda < 1)$ only when $\delta^t u(X) = u(x)$.

The time first approach: In the time first approach, the individual first evaluates

Time first				Risk first		
Rand. prob.	Present Equivalent	Now	Date t	Rand. prob.	Now	Date t
λ	$u^{-1}[\delta^t u(X)]$	0	X	λ	0	X
$1-\lambda$	x	x	0	$1-\lambda$	x	0
				Certainty equivalent	CE_0	CE_t

Table C.2: The two further approaches of evaluating L other than the EDU model. In the time first approach, the individual first evaluates the income streams over time and obtains the present equivalents in each state of nature, and then integrates across states. The horizontal lines mean first integrating over time. In the risk first approach, the individual first obtains certainty equivalents across states at each time, and then integrates over time. The vertical lines mean first integrating across states.

the income streams over time in each state separately. The present equivalent of x is x and of X_t is $u^{-1}(\delta^t u(X))$. We can write the evaluation of the lottery L as:

$$V(L) = g(\lambda)\delta^t u(X) + (1 - g(\lambda))\, u(x) = u(x) + g(\lambda) \left[\delta^t u(X) - u(x)\right].$$

It is easy to see that, similar to EDU, the individual randomizes strictly ($0 < \lambda < 1$) only when $\delta^t u(X) = u(x)$. This result remains even if we allow for a more general form of time discount, e.g., non-exponential time discount. To see this, we can simply replace the present equivalent of $u^{-1}(\delta^t u(X))$ with a more general form $PE(X_t)$ in the above derivation.

The risk first approach: In the risk first approach, the individual evaluates a delayed reward X_t as $V(X_t) = \delta^t u(X)$. To evaluate L, the individual obtains the certainty equivalent of the lotteries at now and the delayed date t separately, and then integrates the certainty equivalents over time. We evaluate the lottery L as:[6]

$$\begin{aligned}
CE_0 &= u^{-1}\left[g(1-\lambda)u(x)\right] \\
CE_t &= u^{-1}\left[g(\lambda)u(X)\right] \\
V(CE_0, CE_t) &= u(CE_0) + \delta^t u(CE_t) \\
&= g(1-\lambda)u(x) + g(\lambda)\delta^t u(X),
\end{aligned}$$

The individual is averse to randomization when $g(1-\lambda) + g(\lambda) < 1$, since

[6]Note in this approach we do not separate preferences for intertemporal substitution from risk preferences. These two preferences are both determined by $u(\cdot)$. Epstein and Zin (1989) separate risk and time preference by using one function to evaluate atemporal payoffs $u(\cdot)$ and a different function to evaluate intertemporal rewards $v(\cdot)$. Empirical evidence in general favors a less concave utility function under time than under risk (Miao and Zhong, 2015).

$$g(1-\lambda)+g(\lambda)<1 \Longrightarrow \begin{array}{l} g(1-\lambda)u(x)+g(\lambda)\delta^t u(X) < u(x)+g(\lambda)\left[\delta^t u(X)-u(x)\right], \\ g(1-\lambda)u(x)+g(\lambda)\delta^t u(X) < u(X)+g(1-\lambda)\left[u(x)-\delta^t u(X)\right]. \end{array}$$

The individual is better off by choosing $\lambda = 1$ when $u(x) \leq \delta^t u(X)$ or $\lambda = 0$ when $u(x) \geq \delta^t u(X)$.

When $g(\cdot)$ is sufficiently concave, i.e, $g(\lambda)+g(1-\lambda) > 1$, for a given X_t the individual could prefer to randomize when x is within an interval. Simple calculation shows that the first order condition is:

$$u(x) = \frac{g'(\lambda)}{g'(1-\lambda)}\delta^t u(X).$$

By setting $\lambda = 1$ and 0 we obtain the lower bound $\underline{x}$ and the upper bound $\bar{x}$ of the randomization range as:

$$\left[\underline{x} = u^{-1}\left(\frac{g'(1)}{g'(0)}\delta^t u(X)\right), \bar{x} = u^{-1}\left(\frac{g'(0)}{g'(1)}\delta^t u(X)\right)\right].$$

Note that, for the interval to exist, a necessary (but not sufficient) condition is $\frac{g'(0)}{g'(1)} > \frac{g'(1)}{g'(0)}$, i.e., overweighting of small probabilities is stronger than the underweighting of large probabilities. This is consistent with the assumption that the probability weighting function should be sufficiently concave for the randomization incentive to exist.

We close the derivation of the risk first approach by noting its three weakness. First, the inverse S-shaped probability function typically found in the literature (Tversky and Kahneman, 1992; Wu and Gonzalez, 1996; Abdellaoui, 2000) does not generate a positive R-range. Consider, for example, the class of functions used in Tversky and Kahneman (1992); Camerer and Ho (1994); Wu and Gonzalez (1996): $g(p) = \frac{\sigma p^\gamma}{\sigma p^\gamma + (1-p)^\gamma}$ with $0 < \sigma < 1$ and $0 < \gamma < 1$. Under this specification, we have:

$$\begin{aligned} g(\lambda)+g(1-\lambda) &= \frac{\sigma\lambda^\gamma}{\sigma\lambda^\gamma+(1-\lambda)^\gamma} + \frac{\sigma(1-\lambda)^\gamma}{\sigma(1-\lambda)^\gamma+\lambda^\gamma} \\ &< \frac{\lambda^\gamma}{\lambda^\gamma+(1-\lambda)^\gamma} + \frac{(1-\lambda)^\gamma}{(1-\lambda)^\gamma+\lambda^\gamma} = 1. \end{aligned}$$

Intuitively, the inequality obtains because with $0 < \sigma < 1$ and $0 < \gamma < 1$, the individual on average underweighs probabilities. When $g(\lambda) + g(1-\lambda) < 1$, as the derivation above suggests, the individual is averse to randomization.

Rand. prob.	Now	Date t
λ	0	$D = \begin{matrix} p_t & X \\ 1-p_t & 0 \end{matrix}$
$1-\lambda$	x	0

Table C.3: The compound lottery over time.

Second, combining the risk first approach with non-linear probability could violate some sort of stochastic dominance, similar to the violation of first degree stochastic dominance in the original prospect theory. For example, since $g(\cdot)$ overweighs small probabilities, it is possible to construct a lottery such as

$$(100_{t+\epsilon}, 0.1; 100_{t+2\epsilon}, 0.1; ...; 100_{t+10\epsilon}, 0.1),$$

with ϵ for a short delay, such that the individual prefers this lottery over the option 100_t. Such a preference violates any models with positive time discount.

Finally, there is strong empirical evidence against the risk first approach. For example, Miao and Zhong (2015), Cheung (2015), and Epper and Fehr-Duda (2015) find that individuals are averse to positively correlated income streams and prefer negatively correlated ones, an observation inconsistent with the above approach. Recently, Rohde and Yu (2023) find that on average people aggregate first over time and then over risk, and this result holds even if one frames the task to encourage the aggregation first over risk and then over time.

C.2.2 The implicit risk of the delayed reward

In this subsection, we consider the framework where the individual perceives the delayed reward X_t as a risky lottery $D_t = (p_t, X_t; 1-p_t, 0_t)$, where p_t is the implicit risk of receiving the delayed reward (Halevy, 2008). The present monetary equivalent of the delayed reward X_t is:

$$me_{0,t} = u^{-1}\left[g(p_t)\delta^t u(X)\right].$$

As shown by Halevy (2008), the individual exhibits present bias and the certainty effect when $g(\cdot)$ is convex at large probabilities (the average estimated p per day is 0.995 in Epper et al., 2011 and 0.987 in Diecidue et al., 2023).

Randomizing between the delayed payment of D_t and x generates a lottery $L = (\lambda, D_t; 1-\lambda, x)$ as depicted in Table C.3. This is a compound lottery over time. In

Time First and RCL

Rand. prob.		Now	Date t
λp_t	$u^{-1}[\delta^t u(X)]$	0	X
$\lambda(1-p_t)$	0	0	0
$1-\lambda$	x	x	0

Time First and CI

Rand. prob.		Now	Date t
λ	$u^{-1}[\delta^t u(CE_D)]$	0	CE_D
$1-\lambda$	x	x	0

Risk First and CI

Rand. prob.	Now	Date t
λ	0	CE_D
$1-\lambda$	x	0
	CE_0	CE_t

Risk First and RCL

Rand. prob.	Now	Date t
λp_t	0	X
$\lambda(1-p_t)$	0	0
$1-\lambda$	x	0
	CE_0	CE_t

Table C.4: The four approaches of evaluating L. In the time first approach, the individual first evaluates the income streams over time and obtains the present equivalents in each state of nature, and then integrates across states. The horizontal lines mean first integrating over time. In the risk first approach, the individual first obtains certainty equivalents across states at each time, and then integrate over time. The vertical lines mean first integrating across states. RCL additionally assumes the reduction of compound lotteries. CI additionally assumes the compound independence (CI) and time neutrality but no reduction of compound lotteries (Segal, 1990).

addition to the sequence that the individual integrates over time and across risk, we need to make a further assumption about how the individual reduces the compound lottery. Specifically, we could either assume the axiom of the reduction of compound lotteries (RCL) or compound independence (CI) (Segal, 1990). Table C.4 depicts these four approaches. Below we start with the special case of EDU and proceed to the four approaches.

The expected discounted utility model (EDU): In this approach, the valuation of X_t is simply $V(X_t) = p_t\delta^t u(X)$. The valuation of the lottery L is:

$$V(L) = \lambda p_t \delta^t u(X) + (1-\lambda)u(x) = u(x) + \lambda\left[p_t\delta^t u(X) - u(x)\right].$$

It is straightforward to see that the individual randomizes only when $u(x) = p_t\delta^t u(X)$.

The time first and RCL approach: In this approach the individual evaluates X_t as $V(X_t) = g(p_t)\delta^t u(X)$. The evaluation of the lottery L is:

$$V(L) \quad = \quad g(\lambda p_t)\left[\delta^t u(X) - u(x)\right] + g(1-\lambda+\lambda p_t)u(x).$$

When $p_t < 1$ and the function $g(\cdot)$ is sufficiently concave, for a given X_t the individual has a strict preference to randomize when the immediate sure payment x is within certain interval. Simple calculation shows that the first order condition for the optimal λ is:

$$u(x) = \frac{g'(\lambda^* p_t)}{p_t g'(\lambda^* p_t) + (1-p_t) g'(1-\lambda+\lambda^* p_t)} p_t \delta^t u(X).$$

By setting $\lambda = 1$ and 0 we obtain the lower and upper bounds of x as:

$$\left[\underline{x} = u^{-1}\left(p_t \delta^t u(X)\right), \bar{x} = u^{-1}\left(\frac{g'(0)}{p_t g'(0) + (1-p_t) g'(1)} p_t \delta^t u(X)\right)\right]$$

As we can see, for the randomization range to exist we must have $g'(0) > g'(1)$.[7] This is consistent with the assumption that the probability weighting function should be sufficiently concave.

The time first and CI approach: In this approach $V(X_t) = g(p_t)\delta^t u(X)$. The present equivalent of the lottery D at t is $u^{-1}\left(g(p_t)\delta^t u(X)\right)$. The evaluation of the lottery receiving $u^{-1}\left(g(p_t)\delta^t u(X)\right)$ with probability λ and x with probability $(1-\lambda)$ can be written as:

$$\begin{aligned} V(L) &= g(\lambda) g(p_t) \delta^t u(X) + (1 - g(\lambda))\, u(x) \\ &= u(x) + g(\lambda)\left[g(p_t)\delta^t u(X) - u(x)\right]. \end{aligned}$$

It is straightforward to see that the individual randomizes only when $g(p_t)\delta^t u(X) = u(x)$.

The risk first and CI approach: In this approach, the individual evaluates X_t as $V(X_t) = g(p_t)\delta^t u(X)$. To evaluate the lottery L, the individual first evaluates the lottery at each time separately, and we have:

$$\begin{aligned} CE_{D_t} &= u^{-1}\left[g(p_t)u(X)\right] \\ CE_t &= u^{-1}\left\{g(\lambda)u(CE_{D_t}) + [1-g(\lambda)]\, u(0)\right\} \\ &= u^{-1}\left[g(\lambda)g(p_t)u(X)\right] \\ CE_0 &= u^{-1}\left[g(1-\lambda)u(x)\right] \end{aligned}$$

[7] This is a necessary but not sufficient condition. Further, when $g(p)$ is of the form $g(p) = \frac{\sigma p^\gamma}{\sigma p^\gamma + (1-p)^\gamma}$, $g'(0) > g'(1)$ implies $\sigma > 1$, which is inconsistent with empirical results. To be more precise, since $g'(p)$ is not well defined at $p = 0$ and 1, the condition is $lim_{n\to\infty} g'(1/n) > lim_{n\to\infty} g'(1-1/n) \longrightarrow \sigma > 1$.

The evaluation of the lottery L is:

$$\begin{aligned} V(L) &= V(CE_0, CE_t) = u(CE_0) + \delta^t u(CE_t) \\ &= g(1-\lambda)u(x) + g(\lambda)g(p_t)\delta^t u(X). \end{aligned}$$

Similar calculation as in the risk first without implicit risk approach shows that the lower bound $\underline{x}$ and the upper bound $\bar{x}$ are:

$$\left[\underline{x} = u^{-1}\left(\frac{g'(1)}{g'(0)}g(p_t)\delta^t u(X)\right), \bar{x} = u^{-1}\left(\frac{g'(0)}{g'(1)}g(p_t)\delta^t u(X)\right)\right].$$

The weakness discussed in the risk first approach in subsection C.2.1 also applies here.

The risk first and RCL approach: In this approach the individual evaluates X_t as $V(X_t) = g(p_t)\delta^t u(X)$. The evaluation of the lottery L starts with the evaluation of lotteries at different times separately, and we have

$$\begin{aligned} CE_0 &= u^{-1}\left[g(1-\lambda)u(x)\right] \\ CE_t &= u^{-1}\left[g(\lambda p_t)u(X)\right] \end{aligned}$$

The evaluation of the lottery L is:

$$\begin{aligned} V(L) &= V(CE_0, CE_t) = u(CE_0) + \delta^t u(CE_1) \\ &= g(1-\lambda)u(x) + g(\lambda p_t)\delta^t u(X). \end{aligned}$$

Similar calculation as in the risk first without implicit risk approach shows that the lower bound $\underline{x}$ and the upper bound $\bar{x}$ is:

$$\left[\underline{x} = u^{-1}\left(\frac{g'(1)}{g'(0)}p_t\delta^t u(X)\right), \bar{x} = u^{-1}\left(\frac{g'(0)}{g'(1)}p_t\delta^t u(X)\right)\right].$$

We summarize the predictions derived so far in the proposition below:

Proposition 1. *In the risk first (with/out implicit risk) approach and the time first and RCL approach, the individual has a positive R-range when the probability weighting function is sufficiently concave and the immediate sure payment is within an interval. However, these approaches have difficulty to simultaneously accommodate the R-range, present bias, and the relationship between them.*

C.3 Additional tables and figures

C.3.1 Descriptive statistics

Experiment 1 (N=291)

Variable	Mean	SD	Min	p1	p5	p50	p95	p99	Max
$\delta^M_{0,4}$	0.329	0.319	< 0.001	< 0.001	< 0.001	0.175	0.719	0.719	0.719
$\delta^M_{4,24}$	13.759	157.616	< 0.001	0.0005	0.115	0.742	1.852	6.842	> 1000
$\delta^{\bar{M}}{}_{0,4}$	0.514	0.336	< 0.001	< 0.001	0.005955	0.719	1	1	1
$\delta^{\bar{M}}{}_{4,24}$	0.804	0.392	< 0.001	< 0.001	0.153	0.872	1.068	1.633	4.917
$\underline{\delta}^M_{0,4}$	0.224	0.305	0	< 0.001	< 0.001	0.036	0.719	1	1
$\underline{\delta}^M_{4,24}$	6.863	94.864	0	< 0.001	0.001	0.698	1.298	20.356	> 1000
R-range$^M_{0,4}$	2.580	2.981	0	0	0	1.500	9.250	9.750	9.750
R-range$^M_{0,24}$	3.062	3.114	0	0	0	2.250	9.50	9.750	9.750
$(1-\beta^M)$	0.151	0.221	-1.029	-0.18	-0.084	0.091	0.564	0.975	0.986
$1-\bar{\beta}^M$	0.056	0.164	-1.091	-0.375	-0.104	0.025	0.325	0.58	0.975
$1-\underline{\beta}^M$	0.276	0.382	-1.091	-1.029	-0.14	0.145	0.975	0.975	0.986
$\delta^C_{0,4}$	> 1000	> 1000	< 0.001	< 0.001	< 0.001	0.443	26.471	> 1000	> 1000
$\delta^C_{4,24}$	19.373	316.052	0.0005	0.0009	0.070	0.698	1.646	10.108	> 1000
$\delta^{\bar{C}}{}_{0,4}$	2.568	15.022	< 0.001	< 0.001	0.004	1	4.099	120.73	160.285
$\delta^{\bar{C}}{}_{4,24}$	1.898	17.989	< 0.001	0	0.199	0.84	1.372	6.52	307.501
$\underline{\delta}^C_{0,4}$	> 1000	> 1000	0	< 0.001	< 0.001	0.367	21.791	> 1000	> 1000
$\underline{\delta}^C_{4,24}$	3.664	32.092	0	< 0.001	0.006	0.770	1.849	47.807	518.871
R-range$^C_{0,0}$	2.466	3.088	0	0	0	1	9.5	9.75	9.75
R-range$^C_{0,4}$	2.979	3.034	0	0	0	2	9.5	9.75	9.75
R-range$^C_{0,24}$	3.046	3.042	0	0	0	2.5	9.75	9.75	9.75
Ambiguity insensitivity, (a)	0.213	0.157	-0.231	-0.1	0.013	0.2	0.481	0.481	0.481
Ambiguity aversion, (b)	0.01	0.097	-0.27	-0.27	-0.17	0.01	0.15	0.26	0.3
Belief about payment	0.824	0.178	0.1	0.15	0.5	0.87	1	1	1

Table C.5: Descriptive statistics for Experiment 1 variables. The variable $\delta_{t1,t2}$ denotes the discount factor between $t1$ and $t2$ and β is the present bias parameter from the quasi-hyperbolic discount model. Variables with $\bar{\cdot}$ ($\underline{\cdot}$) are calculated from the upper (lower, respectively) bound of the R-range. Variables labelled ^{M} pertain to the monetary reward, and variables with ^{C} pertain to the Amazon coupon. Ambiguity parameter a is significantly positive but not b (Wilcoxon signed rank test, $p < 0.01$ for a and $p > 0.1$ for b). Belief about receiving the payment *on time* is significantly and negatively correlated with the R-ranges (Spearman's ρ=-0.16 with R-range$^M_{0.4}$ and $\rho = -0.18$ with R-range$^M_{0,24}$).

Monetary reward treatment

	Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	$\delta^M_{0,4}$	1													
2	$\delta^M_{4,24}$	.08	1												
3	$\delta^{\bar{M}}{}_{0,4}$	.23***	.32***	1											
4	$\delta^{\bar{M}}{}_{4,24}$	.31***	.38***	.15***	1										
5	$\underline{\delta}^M_{0,4}$	.55***	.20***	.37***	.09	1									
6	$\underline{\delta}^M_{4,24}$	.25***	.31***	.11*	.45***	.04	1								
7	R-range$^M_{0,4}$	−.49***	−.13**	−.04	−.04	−.90***	−.04	1							
8	R-range$^M_{0,24}$	−.38***	−.13**	.02	.16***	−.67***	−.32***	.77***	1						
9	$(1-\beta^M)$	−.95***	.15***	−.18***	−.21***	−.48***	−.15**	.44***	.34***	1					
10	$1-\bar{\beta}^M$	−.10*	−.14**	−.87***	.21***	−.30***	.08	.00	.03	.11*	1				
11	$1-\underline{\beta}^M$	−.45***	−.12**	−.34***	.01	−.95***	.18***	.85***	.53***	.42***	.33***	1			
12	Ambiguity insensitivity, (a)	.02	−.08	.09	.05	.01	.02	.02	.00	−.05	−.08	−.02	1		
13	Ambiguity aversion, (b)	.12**	−.02	.04	−.01	.14**	.06	−.13**	−.13**	−.11**	−.01	−.12**	.24***	1	
14	Belief about payment	.09	.11*	.05	.10*	.16***	.17***	−.16***	−.18***	−.05	.00	−.10*	−.05	.08	1

Table C.6: Spearman correlation matrix. The variable $\delta_{t1,t2}$ denotes the discount factor between $t1$ and $t2$, and β is the present bias parameter based on the quasi-hyperbolic discount model. Variables with $\bar{\cdot}$ $(\underline{\cdot})$ are calculated from the upper (lower) bounds of the R-ranges. Variables with ^{M} pertain to the monetary reward, and variables with ^{C} pertain to the Amazon coupon. *p<0.1; **p<0.05; ***p<0.01.

Non-monetary reward treatment

	Variable	1	2	3	4	5	6	7	8	9	10	11	12
1	$\delta^C_{0,4}$	1											
2	$\delta^C_{4,24}$	.09	1										
3	$\bar{\delta}^C{}_{0,4}$	.28***	.19***	1									
4	$\bar{\delta}^C{}_{4,24}$	.17***	.24***	−.02	1								
5	$\underline{\delta}^C_{0,4}$	.27***	.30***	.40***	.18***	1							
6	$\underline{\delta}^C_{4,24}$	.10	.18***	.09	.34***	.06	1						
7	R-range$^C_{0,0}$	−.12**	−.06	−.04	.00	.17***	.08	1					
8	R-range$^C_{0,4}$	−.16***	−.15**	.10*	−.05	−.25***	.09	.72***	1				
9	R-range$^C_{0,24}$	−.11*	−.15***	.06	.22***	−.15**	−.16***	.63***	.82***	1			
10	Ambiguity insensitivity, (a)	−.01	−.13**	.04	−.02	.02	.06	.00	.04	.02	1		
11	Ambiguity aversion, (b)	−.05	.04	.02	.06	.06	.14**	−.06	−.09	−.12**	.24***	1	
12	Belief about payment	.07	.03	.07	−.04	.01	.03	−.14**	−.11*	−.10*	−.05	.08	1

Table C.7: Spearman correlations between variables. The variable $\delta_{t1,t2}$ denotes the discount factor between $t1$ and $t2$ and β is the present bias parameter based on the quasi-hyperbolic discount model. Variables with $\bar{\cdot}$ ($\underline{\cdot}$) are calculated from the upper (lower) bounds of the R-ranges. Variables with ^{M} pertain to the monetary reward, and variables with ^{C} pertain to the Amazon coupon. *p<0.1; **p<0.05; ***p<0.01.

Experiment 2 (N=150)

Variable	Mean	SD	Min	p1	p5	p50	p95	p99	Max
$1-\beta_{FE}$	0.035	0.197	-1.053	-0.632	-0.286	0	0.323	0.421	0.857
$1-\bar{\beta}_{FE}$	-0.01	0.154	-0.905	-0.857	-0.207	0	0.175	0.414	0.513
$1-\underline{\beta}_{FE}$	-0.509	3.919	-38	-22	-0.444	0	0.588	0.947	0.963
R-range$_{0,4}$	2.36	2.777	0	0	0	1.5	8.75	9.75	9.75
R-range$_{4,8}$	2.05	2.74	0.00	0.00	0.00	1.00	8.39	9.75	9.75
R-range$_{0,8}$	2.49	2.78	0.00	0.00	0.00	1.50	9.00	9.75	9.75
Belief about receiving today	0.62	0.31	0.00	0.05	0.05	0.65	1.00	1.00	1.00
Belief about receiving in 4 weeks	0.69	0.25	0.05	0.10	0.22	0.70	1.00	1.00	1.00
Belief about receiving in 8 weeks	0.69	0.25	0.05	0.10	0.22	0.70	1.00	1.00	1.0
Ambiguity insensitivity, (a)	0.225	0.152	-0.1	-0.063	0.013	0.238	0.481	0.481	0.481
Ambiguity aversion, (b)	0.007	0.088	-0.27	-0.23	-0.15	0.01	0.14	0.21	0.22

Table C.8: Descriptive statistics for Experiment 2 variables. The variable $\delta_{t1,t2}$ denotes the discount factor between $t1$ and $t2$, and β is the present bias parameter based on the quasi-hyperbolic discount model. Variables with $\bar{\cdot}$ ($\underline{\cdot}$) are calculated from the upper (lower) bounds of the R-ranges. Ambiguity parameters a and b are significantly positive (Wilcoxon signed rank test, $p < 0.01$ for a and $p < 0.10$ for b). Belief in receiving payment in 4 weeks is significantly correlated with R-range$_{0.4}$ and belief in receiving payment in 8 weeks is significantly correlated with R-range$_{0.8}$ (Spearman's $\rho = -0.18$ with R-range$_{0.4}$ and $\rho = -0.30$ with R-range$_{0,8}$).

	Front-end delay treatment											
	Variable	1	2	3	4	5	6	7	8	9	10	11
1	$1-\beta_{FE}$	1										
2	$1-\bar{\beta}_{FE}$	.26***	1									
3	$1-\underline{\beta}_{FE}$	.34***	.38***	1								
4	R-range$_{0,4}$	.20**	-.02	.28***	1							
5	R-range$_{4,8}$	.10	.05	-.08	.82***	1						
6	R-range$_{0,8}$	.19**	-.08	.17**	.82***	.74***	1					
7	Belief about receiving today	.00	.00	-.02	-.04	-.06	-.03	1				
8	Belief about receiving in 4 weeks	-.13	-.07	-.09	-.18**	-.21***	-.19**	.72***	1			
9	Belief about receiving in 8 weeks	-.16**	.00	-.16*	-.29***	-.26***	-.30***	.42***	.79***	1		
10	Ambiguity insensitivity, (a)	.06	.06	.01	.00	.05	.03	.05	-.01	-.04	1	
11	Ambiguity aversion, (b)	.10	.22***	.03	-.02	.07	.09	.10	.06	.12	.28***	1

Table C.9: Spearman correlation matrix. The variable β is the present bias parameter based on the quasi-hyperbolic discount model. Variables with $\bar{\cdot}$ ($\underline{\cdot}$) are calculated from the upper (lower) bounds of the R-ranges. *p<0.1; **p<0.05; ***p<0.01.

Overall	β^M		β^c	
	(5%,95%)	outside (5%,95%)	(5%,95%)	outside (5%,95%)
13%	11%	35%	11%	40%

Table C.10: The percentage of subjects who failed the R-range task control question in the full sample, within and outside the (5%,95%) interval of the present bias parameters. The variable β^M denotes the present bias toward monetary reward, β^C denotes the present bias toward Amazon coupon.

C.3.2 Additional results about the monetary reward

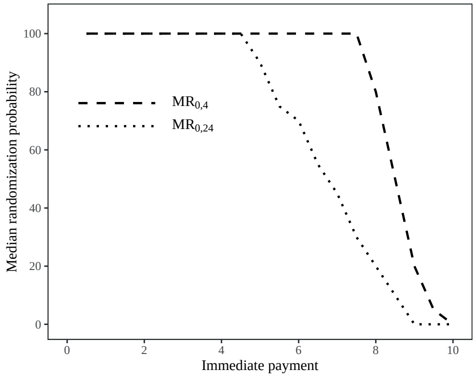


Figure C.1: The median randomization probability at each present payment. Two notable patterns are: 1) randomization probabilities decrease systematically with the immediate payment and; 2) for rewards with longer delay, median randomization probabilities exhibit a more gradual decline in randomization probability over a narrower range of immediate reward.

		Delay: 4 weeks			Delay: 24 weeks		
		Full	Test	(5%,95%)	Full	Test	(5%,95%)
Rand. frequency							
$=0$		32%	37%		29%	32%	
		(93)	(92)		(85)	(81)	
$=1$		10%	11%		4%	4%	
		(29)	(28)		(12)	(11)	
>1		58%	52%		67%	63%	
		(169)	(132)		(194)	(160)	
Rand. range							
$\bar{x}$	Median	9.75	9.75	9.75	8.75	8.75	8.75
	Mean	9.08	9.05	9.21	8.07	8.02	8.29
		(1.23)	(1.25)	(0.72)	(2.11)	(2.10)	(1.57)
$\underline{x}$	Median	7.75	8.25	6.75	4.75	5.25	4.75
	Mean	6.50	7.02	6.09	5.01	5.43	5.36
		(3.20)	(2.87)	(2.66)	(3.35)	(3.21)	(2.55)
R-range	Median	1.50	1.00	2.50	2.25	1.75	3.50
	Mean	2.58	2.04	3.25	3.06	2.59	3.82
		(2.98)	(2.48)	(2.41)	(3.11)	(2.73)	(2.45)
Present equivalent							
R-range>0	Median	8.25	8.25	7.75	6.75	7.25	6.75
	Mean	7.76	7.90	7.45	6.55	6.64	6.52
		(2.03)	(1.81)	(1.39)	(2.22)	(2.13)	(1.64)
R-range$=0$	Median	9.75	9.75	8.75	9.25	9.25	7.50
	Mean	9.16	9.15	8.22	8.19	8.17	7.29
		(1.53)	(1.54)	(1.32)	(2.25)	(2.27)	(1.84)

Table C.11: Randomization frequency, randomization range, and present equivalent for the monetary reward of €10. The Test sample includes subjects who answered correctly in the R-range control task, and the (5%,95%) sample includes subjects whose values fall within the (5%,95%) interval. The numbers in parentheses pertaining to randomization frequency represent the absolute number of subjects, while the numbers in parentheses pertaining to randomization range and present equivalent represent the standard deviations. The present monetary equivalents of subjects without R-ranges are not statistically different from the upper bound of the subjects with at least one positive R-range (medians: 9.75 vs. 9.75 for 4 weeks, and 9.25 vs. 8.75 for 24 weeks, two-sample Wilcoxon signed rank tests, $p > 0.10$ for both comparisons).

Subjects		Payoffs directly			CRRA adjusted			CARA adjusted		
		β	$\bar{\beta}$	$\underline{\beta}$	β	$\bar{\beta}$	$\underline{\beta}$	β	$\bar{\beta}$	$\underline{\beta}$
Subjects: All										
	Median	0.91	0.97	0.85	0.96	0.98	0.90	0.94	0.98	0.87
	Mean	0.86	0.95	0.73	0.91	0.96	0.80	0.89	0.96	0.76
		(0.15)	(0.07)	(0.28)	(0.12)	(0.06)	(0.25)	(0.13)	(0.07)	(0.28)
Subjects: R-range>0										
	Median	0.85	0.97	0.71	0.92	0.98	0.84	0.91	0.98	0.80
	Mean	0.83	0.94	0.65	0.89	0.96	0.75	0.87	0.96	0.70
		(0.15)	(0.08)	(0.29)	(0.13)	(0.06)	(0.26)	(0.14)	(0.07)	(0.30)
Subjects: R-range=0										
	Median	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98
	Mean	0.94	0.95	0.94	0.96	0.96	0.93	0.94	0.96	0.94
		(0.11)	(0.06)	(0.10)	(0.07)	(0.06)	(0.15)	(0.11)	(0.07)	(0.11)

Table C.12: Median, mean and SD (in parentheses) of β (present bias) from the monetary reward treatment. Mean and SD are calculated using β values within the (5%,95%) interval.

	$1-\beta$		$1-\bar{\beta}$		$1-\underline{\beta}$	
	(1)	(2)	(3)	(4)	(5)	(6)
R-range$_{0,4}$	0.025***	0.025***	−0.002	−0.002	0.106***	0.108***
	(0.004)	(0.004)	(0.003)	(0.003)	(0.004)	(0.004)
Aversion (b)		−0.098		0.047		0.155
		(0.130)		(0.102)		(0.133)
Insensitivity (a)		−0.027		−0.028		0.015
		(0.080)		(0.062)		(0.081)
Belief about		0.004		0.094*		0.154**
payment		(0.070)		(0.055)		(0.072)
Intercept	0.086***	0.090	0.063***	−0.012	0.001	−0.135**
	(0.016)	(0.065)	(0.013)	(0.051)	(0.017)	(0.066)
Observations	291	291	291	291	291	291
R^2	0.118	0.121	0.002	0.014	0.688	0.695

Table C.13: OLS regressions of present bias estimated from the present equivalents (β), the upper bound ($\bar{\beta}$), and the lower bound ($\underline{\beta}$) of the R-range based on the full sample. *p<0.1; **p<0.05; ***p<0.01.

	$1-\beta$		$1-\bar{\beta}$		$1-\underline{\beta}$	
	(1)	(2)	(3)	(4)	(5)	(6)
R-range$_{0,4}$	0.20***	0.19***	0.00	0.00	0.95***	0.95***
	(0.04)	(0.04)	(0.01)	(0.01)	(0.03)	(0.03)
R-range$_{0,24}$ -	-0.01	0.00	0.03	0.03	0.24***	0.24***
R-range$_{0,4}$	(0.05)	(0.06)	(0.02)	(0.02)	(0.07)	(0.07)
Aversion, (b)		-0.106		-0.002		-0.122
(b)		(0.097)		(0.056)		(0.077)
Insensitivity, (a)		-0.020		-0.005		-0.020
(a)		(0.064)		(0.034)		(0.053)
Belief about		-0.087		0.007		0.070
payment		(0.071)		(0.025)		(0.049)
Intercept	0.090***	0.172**	0.057***	0.053**	0.064***	0.011
	(0.012)	(0.067)	(0.006)	(0.025)	(0.010)	(0.047)
Observations	260	260	263	263	256	256
R^2	0.138	0.152	0.005	0.006	0.812	0.815

Table C.14: OLS regressions of present bias estimated from the present equivalents (β), the upper bound ($\bar{\beta}$), and the lower bound ($\underline{\beta}$) of the R-range. The sample restricts β to values within the $[5\%, 95\%]$ interval. *p<0.1; **p<0.05; ***p<0.01.

C.3.3 Additional results about the Amazon coupons

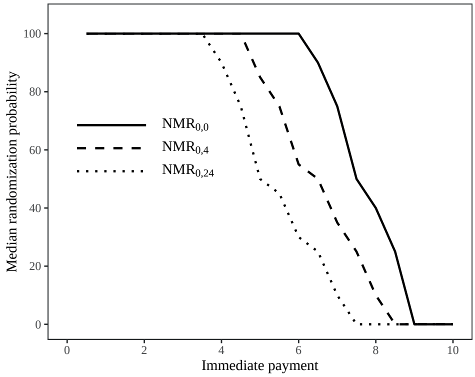


Figure C.2: The median randomization probability at each level of immediate monetary payment. Results are broadly similar to the results pertaining to the monetary rewards.

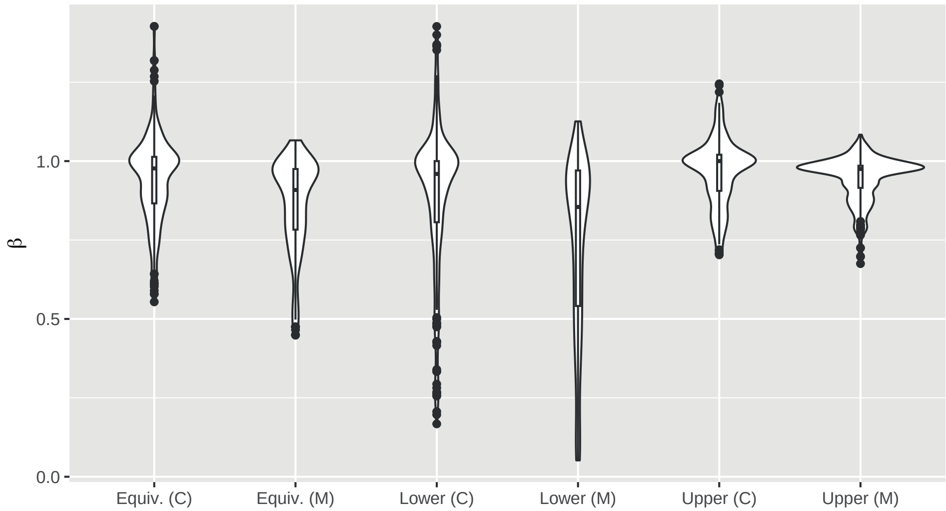


Figure C.3: Present bias (β) toward monetary and non-monetary rewards. The violin plots are based on β values within the (5%,95%) interval. More details can be found in Table C.16. The label (C) denotes the non-monetary reward treatment, and (M) denotes the monetary reward treatment. The box in the violin plot illustrates the median (horizontal line in the box), the 25% quantile (the lower line), and the 75% quantile (the upper line).

		Today			Delay: 4 weeks			Delay: 24 weeks		
		Full	Test	(5%,95%)	Full	Test	(5%,95%)	Full	Test	(5%,95%)
Rand. frequency										
$= 0$		0.40	0.44		0.27	0.30		0.29	0.33	
		(116)	(111)		(78)	(76)		(85)	(82)	
$= 1$		0.07	0.08		0.06	0.06		0.04	0.04	
		(20)	(19)		(18)	(16)		(11)	(10)	
> 1		0.53	0.48		0.67	0.63		0.67	0.63	
		(155)	(122)		(195)	(160)		(195)	(160)	
Rand. range										
$\bar{x}$	Median	8.75	8.75	8.75	8.25	7.75	8.25	7.25	7.25	7.25
	Mean	8.25	8.12	8.45	7.63	7.48	7.79	6.80	6.66	6.97
		(2.18)	(2.23)	(1.46)	(2.40)	(2.38)	(1.75)	(2.67)	(2.64)	(2.10)
$\underline{x}$	Median	6.25	6.75	6.25	4.75	4.75	5.25	3.75	4.25	4.75
	Mean	5.78	6.20	5.84	4.65	5.00	5.26	3.76	4.04	4.56
		(3.33)	(3.10)	(2.25)	(3.11)	(2.99)	(2.21)	(2.95)	(2.91)	(2.13)
R-range	Median	1.00	0.50	3.00	2.00	2.00	3.00	2.50	2.00	3.50
	Mean	2.47	1.92	3.42	2.98	2.48	3.52	3.05	2.62	3.88
		(3.09)	(2.56)	(2.45)	(3.03)	(2.60)	(2.35)	(3.04)	(2.71)	(2.39)
Present equivalent										
R-range > 0	Median	7.75	8.00	7.25	7.25	7.25	6.75	4.75	5.25	4.75
	Mean	7.47	7.65	6.87	6.61	6.71	6.35	5.40	5.46	5.42
		(2.33)	(2.03)	(1.84)	(2.20)	(1.96)	(1.74)	(2.32)	(2.20)	(1.86)
R-range $= 0$	Median	8.75	8.50	7.25	7.75	7.75	7.25	6.75	6.75	6.75
	Mean	6.99	6.95	6.79	6.59	6.54	6.76	6.09	6.03	6.21
		(3.39)	(3.41)	(1.90)	(3.35)	(3.36)	(2.08)	(3.30)	(3.30)	(2.12)

Table C.15: Randomization frequency, randomization range, and present equivalent for the monetary reward of €10. The Test sample includes subjects who answered correctly in the R-range control task and the (5%,95%) sample includes subjects whose values fall within the (5%,95%) interval. The numbers in parentheses pertaining to randomization frequency represent the absolute number of subjects, while the numbers in parentheses pertaining to randomization range and present equivalent represent the standard deviations.

	β			$\bar{\beta}$			$\underline{\beta}$		
	Full	Test	(5%,95%)	Full	Test	(5%,95%)	Full	Test	(5%,95%)
Subjects: All									
Median	0.98	0.97	0.98	1.00	1.00	1.00	0.96	0.95	0.96
Mean	1.14	1.05	0.94	0.97	0.96	0.97	1.22	1.07	0.89
	(2.12)	(1.58)	(0.14)	(0.20)	(0.19)	(0.11)	(2.84)	(2.15)	(0.23)
Subjects: R-range>0									
Median	0.97	0.96	0.96	1.00	1.00	1.00	0.95	0.92	0.95
Mean	1.21	1.09	0.93	0.97	0.97	0.97	1.30	1.10	0.87
	(2.45)	(1.87)	(0.15)	(0.21)	(0.19)	(0.11)	(3.27)	(2.53)	(0.25)
Subjects: R-range=0									
Median	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Mean	0.96	0.96	0.97	0.94	0.94	0.96	0.99	0.99	0.97
	(0.13)	(0.13)	(0.12)	(0.18)	(0.18)	(0.11)	(0.30)	(0.30)	(0.16)

Table C.16: Median, mean and SD (in parentheses) of β (present bias) from the non-monetary reward treatment. The Test sample includes subjects who answered correctly in the R-range control task andthe (5%,95%) sample includes subjects whose values fall within the (5%,95%) interval. The present bias β, $\bar{\beta}$, and $\underline{\beta}$ are calculated from present monetary equivalents, the upper bounds and the lower bounds of the R-ranges, respectively. The parameters $\bar{\beta}$ and $\underline{\beta}$ sometimes differ from each other among subjects with R-range=0 because the (5%,95%) samples for $\bar{\beta}$ and $\underline{\beta}$ are not always identical.

	Payoffs directly			CRRA adjusted			CARA adjusted		
	β	$\bar{\beta}$	$\underline{\beta}$	β	$\bar{\beta}$	$\underline{\beta}$	β	$\bar{\beta}$	$\underline{\beta}$
Subjects: All									
Median	0.98	1.00	0.96	0.99	1.00	0.99	0.99	1.00	0.98
Mean	0.94	0.97	0.89	0.96	0.97	0.93	0.96	0.97	0.91
	(0.14)	(0.11)	(0.23)	(0.10)	(0.07)	(0.16)	(0.12)	(0.08)	(0.21)
Subjects: R-range>0									
Median	0.96	1.00	0.94	0.99	1.00	0.98	0.97	1.00	0.95
Mean	0.94	0.97	0.87	0.95	0.98	0.91	0.95	0.98	0.89
	(0.15)	(0.11)	(0.25)	(0.10)	(0.07)	(0.17)	(0.13)	(0.09)	(0.23)
Subjects: R-range=0									
Median	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Mean	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.98
	(0.11)	(0.09)	(0.13)	(0.09)	(0.07)	(0.09)	(0.10)	(0.08)	(0.10)

Table C.17: Median, mean and SD (in parentheses) of β (present bias) from non-monetary reward treatment. Mean and SD are calculated using β values within the (5%,95%) interval.

	$\delta_{0,4}$ and $\delta_{4,24}$			
	(1)	(2)	(3)	(4)
ΔR-range$_{t,t+1}$	-4.68^{***}	-3.43^{***}	-3.18^{***}	-3.17^{***}
	(0.57)	(0.57)	(0.54)	(0.55)
D_{coupon}	0.004	-0.021	0.009	-0.051
	(0.022)	(0.023)	(0.058)	(0.130)
ΔR-range$_{t,t+1}$ $\times$ D_{coupon}	1.26	0.32	-0.32	-0.28
	(1.09)	(1.06)	(1.02)	(1.03)
Present		-0.246^{***}	-0.166^{***}	-0.165^{***}
		(0.025)	(0.028)	(0.028)
$D_{coupon}\times$Present		-0.072^{*}	-0.048	-0.047
		(0.039)	(0.042)	(0.043)
$\delta^{\bar{x}}_{t,t+1}$			0.342^{***}	0.349^{***}
			(0.047)	(0.046)
D_{coupon} $\times$ $\delta^{\bar{x}}_{t,t+1}$			-0.001	-0.003
			(0.069)	(0.069)
Aversion, (b)				-0.161
				(0.147)
$D_{coupon}\times$ Aversion				-0.081
				(0.185)
Insensitivity, (a)				0.118
				(0.075)
$D_{coupon}\times$ Insensitivity				-0.033
				(0.110)
Belief about payment				-0.052
				(0.073)
$D_{coupon}\times$ Belief about payment				0.117
				(0.121)
Intercept	0.625^{***}	0.721^{***}	0.446^{***}	0.507^{***}
	(0.016)	(0.019)	(0.043)	(0.080)
Observations	951	951	951	951
R^2 (clustered SE)	0.090	0.172	0.256	0.261

Table C.18: OLS regressions of discount factors ($\delta_{0,4}$ and $\delta_{4,24}$). The sample is restricted to $\delta_{0,4}$, $\delta_{4,24}$ and $\delta^{\bar{x}}_{t,t+1}$ values within their (5%,95%) interval. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

C.3.4 Additional results about the front-end delay experiment

		4 weeks vs. 0		8 weeks vs. 4 weeks		8 weeks vs. 0	
		Full	(5%,95%)	Full	(5%,95%)	Full	(5%,95%)
Rand. frequency							
$= 0$		0.31		0.39		0.32	
		(46)		(59)		(48)	
$= 1$		0.09		0.08		0.06	
		(13)		(12)		(9)	
> 1		0.61		0.53		0.62	
		(91)		(79)		(93)	
Rand. range							
$\bar{x}$	Median	9.75	9.75	9.75	9.75	9.75	9.75
	Mean	9.24	9.45	9.24	9.29	8.89	9.09
		(1.03)	(0.47)	(1.01)	(0.72)	(1.49)	(0.90)
$\underline{x}$	Median	7.75	7.25	8.25	7.25	7.25	6.75
	Mean	6.88	6.53	7.19	6.66	6.41	6.27
		(2.89)	(2.29)	(2.80)	(2.18)	(2.97)	(2.16)
R-range	Median	1.50	2.00	1.00	2.00	1.50	3.00
	Mean	2.36	2.89	2.05	2.78	2.48	3.10
		(2.78)	(2.18)	(2.74)	(2.12)	(2.78)	(2.05)
Present equivalent							
R-range> 0	Median	7.75	7.75	8.75	7.75	7.75	7.50
	Mean	7.71	7.71	8.11	7.78	7.15	7.18
		(1.89)	(1.11)	(1.74)	(1.17)	(2.16)	(1.43)
R-range$= 0$	Median	9.75	8.75	9.75	9.00	9.75	8.50
	Mean	9.36	8.54	9.64	8.75	9.24	8.17
		(0.79)	(0.99)	(0.38)	(0.71)	(1.04)	(1.29)

Table C.19: Randomization frequency, randomization range, and present equivalent in the front-end delay experiment. The (5%,95%) sample includes subjects whose values fall within the (5%,95%) interval. The numbers in parentheses pertaining to randomization frequency represent the absolute number of subjects, while the numbers in parentheses pertaining to randomization range and present equivalent represent the standard deviations.

	$\delta_{0,4}$ and $\delta'_{4,8}$			$\delta_{0,4}$ and $\delta_{4,8}$		
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	−0.11	−0.29	−0.28	−0.02	0.13	0.13
	(0.31)	(0.23)	(0.23)	(0.13)	(0.13)	(0.14)
$\bar{x}_{t+1}/\bar{x}_t$	0.78**	0.98***	0.95***	0.39***	0.35***	0.36***
	(0.32)	(0.24)	(0.23)	(0.13)	(0.13)	(0.13)
Present	−0.32***	−0.19***	−0.19***	−0.05**	−0.03	−0.03
	(0.04)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)
R-range$_{0,4}$		−0.51***	−0.51***		−0.49***	−0.52***
		(0.08)	(0.08)		(0.06)	(0.06)
Aversion, (b)			−0.21			−0.47**
			(0.26)			(0.24)
Insensitivity, (a)			−0.02			−0.19
			(0.15)			(0.14)
Gender			0.04			0.07*
			(0.04)			(0.04)
Observations	289	289	283	300	300	294
Adjusted R^2	0.23	0.33	0.33	0.04	0.25	0.29

Table C.20: OLS regressions of discount factors ($\delta_{0,4}$ and $\delta'_{4,8}$) with clustered robust standard errors (in parentheses). We remove 11 outliers with $\delta > 1$. We did not include the interaction term R-range×Present because in our theoretical analysis discount factors react to valuation uncertainty R-range$_{0,4}$ and R-range$_{4,8}$ with the same sensitivity, and the front-end delay effect arises from the difference in valuation uncertainty from 0 to 4 weeks and 4 weeks to 8 weeks (R-range$_{0,4}$ >R-range$_{4,8}$). Significance levels: *p<0.1; **p<0.05; ***p<0.01.

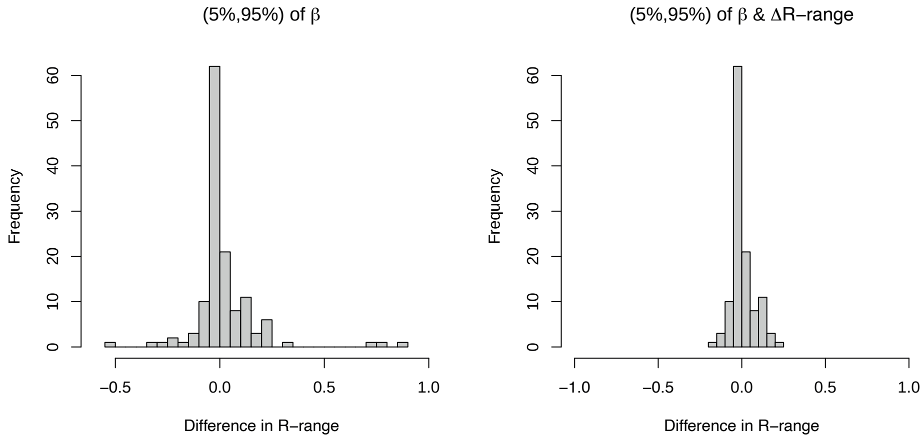


Figure C.4: Histograms of ΔR-range including and excluding subjects with ΔR-range values outside its (5%, 95%) interval.

	Approach 1			Approach 2		
	$1-\beta_{FE}$	$1-\bar{\beta}_{FE}$	$1-\underline{\beta}_{FE}$	$1-\beta_{FE}$	$1-\bar{\beta}_{FE}$	$1-\underline{\beta}_{FE}$
	(1)	(2)	(3)	(4)	(5)	(6)
R-range$_{0,4}$	0.009	-0.007	0.024			
	(0.006)	(0.005)	(0.118)			
ΔR-range				0.022**	-0.022***	1.144***
				(0.01)	(0.007)	(0.167)
Aversion (b)	-0.103	0.248*	-0.6	-0.051	0.196	2.386
	(0.19)	(0.145)	(3.783)	(0.19)	(0.143)	(3.314)
Insensitivity (a)	0.046	0.107	1.804	0.05	0.103	2.025
	(0.109)	(0.083)	(2.177)	(0.108)	(0.082)	(1.891)
Belief about	0.053	0.021	1.781	0.04	0.03	1.966
payment	(0.072)	(0.055)	(1.422)	(0.07)	(0.053)	(1.219)
Intercept	-0.031	-0.033	-2.155*	-0.009	-0.048	-2.648***
	(0.006)	(0.005)	(0.118)	(0.01)	(0.007)	(0.167)
Observations	150	150	150	150	150	150
Adjusted R^2	-0.007	0.033	-0.012	0.013	0.075	0.237

Table C.21: OLS regressions of the front-end delay effect (β_{FE}) based on the full sample. *p<0.1; **p<0.05; ***p<0.01.

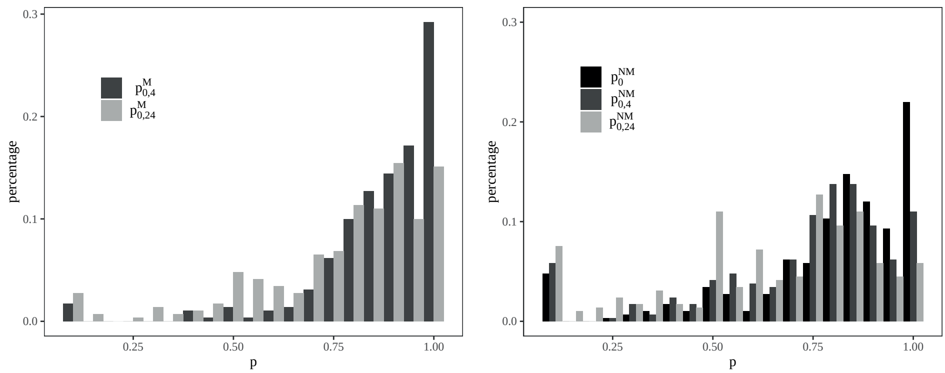
	R-range$_{4,8}$	
	(1)	(2)
Intercept	0.03	0.31
	(0.17)	(0.35)
R-range$_{0,8}$	0.23***	0.43***
	(0.08)	(0.13)
R-range$_{0,4}$	0.61***	0.39***
	(0.08)	(0.12)
Observations	150	83
Adjusted R^2	0.67	0.58

Table C.22: OLS regressions of R-range$_{4,8}$ by R-range$_{0,4}$ and R-range$_{0,8}$. Regression (1) is based on the full sample while regression (2) includes only subjects with positive R-range$_{0,4}$, R-range$_{4,8}$ and R-range$_{0,8}$. *p<0.1; **p<0.05; ***p<0.01.

C.4 Structural analysis of the attitude toward valuation uncertainty κ and the subjective belief F

p	Monetary reward		Amazon coupon		p	Front end delay	
	Mean	Median	Mean	Median		Mean	Median
p_0	-	-	0.79 (0.22)	0.87	$p_{0,4}$	0.90 (0.10)	0.93
$p_{0,4}$	0.87 (0.16)	0.92	0.73 (0.23)	0.79	$p_{4,8}$	0.90 (0.11)	0.96
$p_{0,24}$	0.78 (0.21)	0.83	0.64 (0.25)	0.72	$p_{0,8}$	0.88 (0.14)	0.92

Table C.23: The fitted p of the binomial distribution $B(10, p)$ across timed rewards and treatments. We restrict $\kappa \in [0.1, 3]$ and $p \in [0.1, 1]$.



(a) Belief distribution of monetary rewards (b) Belief distribution of non-monetary rewards

Figure C.5: The density distribution of the estimated p in the binomial distribution $B(10, p)$. Notations $p^M_{0,4}$, $p^M_{0,24}$ denote the belief of monetary timed rewards in 4 weeks and in 24 weeks, respectively. Notations p_0^{NM},$p^{NM}_{0,4}$, $p^{NM}_{0,24}$ denote the belief of non-monetary timed rewards at present, in 4 weeks and in 24 weeks respectively.

VU	Monetary reward		Amazon coupon		VU	Front-end delay	
	Mean	Median	Mean	Median		Mean	Median
VU_0	-	-	1.45 (1.68)	0.83	$VU_{0,4}$	1.06 (1.22)	0.58
$VU_{0,4}$	1.31 (1.60)	0.49	2.02 (1.92)	1.62	$VU_{4,8}$	0.89 (1.27)	0.37
$VU_{0,24}$	1.95 (1.99)	1.31	2.38 (2.11)	2.27	$VU_{0,8}$	1.19 (1.36)	0.59

Table C.24: The parametric valuation uncertainty $10\kappa p(1-p)$.

VU	Monetary reward	Amazon coupon	VU	Front-end delay
VU_0	-	0.79***	$VU_{0,4}$	0.88***
$VU_{0,4}$	0.88***	0.78***	$VU_{4,8}$	0.87***
$VU_{0,24}$	0.80***	0.75***	$VU_{0,8}$	0.82***

Table C.25: Spearman correlation between the parametric valuation uncertainty measure $10\kappa p(1-p)$ and the corresponding R-range. Significance levels: *p<0.1; **p<0.05; ***p<0.01.

VU	Spearman correlation between VU and present bias		
	$\rho(VU_{0,4}, \beta)$	$\rho(VU_{0,4}, \beta_{FE})$	$\rho(VU_{0,4} - VU_{4,8}, \beta_{FE})$
$10\kappa p(1-p)$	-0.45***	-0.29***	-0.21***
R-range	-0.41***	-0.198**	-0.261***

Table C.26: Spearman correlation between different measures of valuation uncertainty and present bias. Significance levels: *p<0.1; **p<0.05; ***p<0.01.

C.5 Experiment Instructions

C.5.1 Instructions about experiment payouts

Welcome

You are invited to participate in an experiment in which we examine how individuals make decisions. Your decisions in the experiment are about choices between different options. You will learn more about these options in the experiment. The whole experiment will take approximately 15 minutes. Please complete the experiment within 1 hour. Please do not open any other applications on the computer during the experiment.

At the end of the experiment, one of the questions that you face in the experiment will be randomly selected. You will receive either a monetary payment of up to €15 or an Amazon.nl coupon of up to €10, depending on the decisions you make in the selected question. The average payment of this experiment is higher than most online experiments with similar length. Please read the experimental instruction carefully. You will face two control questions. Answering each control question incorrectly will reduce your final experimental payment by €0.50.

You will receive your payment via a bank transfer (monetary payment) or through an email (Amazon.nl coupon) before 23:59 today or on the date specified in the chosen option. For this, we will ask for your IBAN number, email address, and living address. This information will only be used for the payment and will be permanently deleted afterward.

If you do not receive the experimental payment on time, you can contact us at liu.shi@ru.nl, and you will receive your experimental payment **plus an additional compensation of €5.**

Thank you for your participation!

Sincerely yours,

Figure C.6: Instructions about experiment payouts

C.5.2 Binary choices

Instruction about task 1

In this task (and some future tasks) you will face a few questions. In each question you will see a table with some rows. Each row has two options: A and B. The exact options will be explained to you in each question.

Across rows one option stays the same, while the other option becomes more attractive or less attractive. **Your choice is to decide in which row you want to switch from A to B.** You can do this by clicking "Switch here" at the row where you prefer B over A for the first time. Your preferred option will then be highlighted in **bold**. If one of the questions in this task is selected to decide your payoff, the computer will randomly select one row from the table in that question. You will receive your preferred option in the selected row.

Please pay attention to the payment option and payment date. They may vary across questions.

Before this task starts, you face one control question to check whether you understand the experimental instruction (with only 10 rows, not 20 rows like in real questions). *Answering the control question incorrectly will reduce your final experimental payment by €0.50.* So, please read the instruction carefully.

Figure C.7: Instruction about the binary choice task

Task 1 Question 1:
In this question the two options in each row are:

- Option A: an Amazon coupon worth €10 in four weeks (December 13, 2022)
- Option B: a certain amount of euro today (November 15, 2022)

#	Option A	Option B	Your choice
1	**€10.00 coupon in four weeks**	€0.50 today	Switch here
2	**€10.00 coupon in four weeks**	€1.00 today	Switch here
3	**€10.00 coupon in four weeks**	€1.50 today	Switch here
4	**€10.00 coupon in four weeks**	€2.00 today	Switch here
5	**€10.00 coupon in four weeks**	€2.50 today	Switch here
6	**€10.00 coupon in four weeks**	€3.00 today	Switch here
7	**€10.00 coupon in four weeks**	€3.50 today	Switch here
8	**€10.00 coupon in four weeks**	€4.00 today	Switch here
9	**€10.00 coupon in four weeks**	€4.50 today	Switch here
10	**€10.00 coupon in four weeks**	€5.00 today	Switch here
11	**€10.00 coupon in four weeks**	€5.50 today	Switch here
12	**€10.00 coupon in four weeks**	€6.00 today	Switch here
13	**€10.00 coupon in four weeks**	€6.50 today	Switch here
14	**€10.00 coupon in four weeks**	€7.00 today	Switch here
15	**€10.00 coupon in four weeks**	€7.50 today	Switch here
16	**€10.00 coupon in four weeks**	€8.00 today	Switch here
17	€10.00 coupon in four weeks	**€8.50 today**	**Switch here**
18	€10.00 coupon in four weeks	**€9.00 today**	Switch here
19	€10.00 coupon in four weeks	**€9.50 today**	Switch here
20	€10.00 coupon in four weeks	**€10.00 today**	Switch here

If this decision is chosen to be paid out for real, the computer will randomly pick one row of the list. Your payoff is determined according to your decision for that row.

To confirm your decision please click on the arrow button below.

Figure C.8: Binary choice between coupon in the future and money today

In this question the two options in each row are:

- Option A: €10 in eight weeks (August 22, 2023)
- Option B: a certain amount of euro in four weeks (July 25, 2023)

#	Option A	Option B	Your choice
1	**€10.00 in eight weeks**	€0.50 in four weeks	Switch here
2	**€10.00 in eight weeks**	€1.00 in four weeks	Switch here
3	**€10.00 in eight weeks**	€1.50 in four weeks	Switch here
4	**€10.00 in eight weeks**	€2.00 in four weeks	Switch here
5	**€10.00 in eight weeks**	€2.50 in four weeks	Switch here
6	**€10.00 in eight weeks**	€3.00 in four weeks	Switch here
7	**€10.00 in eight weeks**	€3.50 in four weeks	Switch here
8	**€10.00 in eight weeks**	€4.00 in four weeks	Switch here
9	**€10.00 in eight weeks**	€4.50 in four weeks	Switch here
10	**€10.00 in eight weeks**	€5.00 in four weeks	Switch here
11	**€10.00 in eight weeks**	€5.50 in four weeks	Switch here
12	**€10.00 in eight weeks**	€6.00 in four weeks	Switch here
13	**€10.00 in eight weeks**	€6.50 in four weeks	Switch here
14	**€10.00 in eight weeks**	€7.00 in four weeks	Switch here
15	**€10.00 in eight weeks**	€7.50 in four weeks	Switch here
16	€10.00 in eight weeks	**€8.00 in four weeks**	**Switch here**
17	€10.00 in eight weeks	**€8.50 in four weeks**	Switch here
18	€10.00 in eight weeks	**€9.00 in four weeks**	Switch here
19	€10.00 in eight weeks	**€9.50 in four weeks**	Switch here
20	€10.00 in eight weeks	**€10.00 in four weeks**	Switch here

If this decision is chosen to be paid out for real, the computer will randomly pick one row of the list. Your payoff is determined according to your decision for that row.

To confirm your decision please click on the arrow button below.

Figure C.9: Binary choice between money in two future periods

C.5.3 Ambiguity task

Instruction about task 2

In this task you will face 3 questions. In each question, the options you will face are some computerized boxes: box K and box U. Each computerized box contains 100 balls of different colors. Some colors are specified as the winning colors. Importantly:

- the proportion of balls of the winning color(s) in box K will be provided to you in each question.
- the proportion of balls of the winning color(s) in box U is unknown in each question. The box U is constructed by an 11-year-old child using Scratch (**https://scratch.mit.edu/about**) without any intervention, apart from requiring 100 balls of specified colors in the box. You will see a link to the computer program for box U at the end of the experiment.

More information regarding these boxes will be provided in the questions.

You will choose between box U and box K. The computer draws one ball from your chosen box. **You receive TODAY €15 if the drawn ball is the winning color(s) (specified in each question) and €5 otherwise.**

→

Figure C.10: Instruction about the ambiguity task

Task 2 Question 1:

In this question you face a table with 11 rows. In each row, you need to choose between Box K and Box U. **They both contain 100 balls of either purple or yellow**, but the difference is:

- Box K (known): You know the proportion of the winning color balls. This proportion is clearly stated in each row of the table.
- Box U (unknown): You do not know the proportion of the winning color balls. The box U is constructed by an 11-year-old child using Scratch without any intervention (**https://scratch.mit.edu/about**), apart from requiring 100 balls of of either purple or yellow.

For the payment, the computer draws one ball from your chosen box. **You receive TODAY €15 if the drawn ball is the winning color purple, and €5 if the drawn ball is yellow.**

#	Box U	Box K	Your choice
1	**U(?%, purple; ?%, yellow)**	K(35%, purple; 65%, yellow)	Switch here
2	**U(?%, purple; ?%, yellow)**	K(38%, purple; 62%, yellow)	Switch here
3	**U(?%, purple; ?%, yellow)**	K(41%, purple; 59%, yellow)	Switch here
4	**U(?%, purple; ?%, yellow)**	K(44%, purple; 56%, yellow)	Switch here
5	U(?%, purple; ?%, yellow)	**K(47%, purple; 53%, yellow)**	**Switch here**
6	U(?%, purple; ?%, yellow)	**K(50%, purple; 50%, yellow)**	Switch here
7	U(?%, purple; ?%, yellow)	**K(53%, purple; 47%, yellow)**	Switch here
8	U(?%, purple; ?%, yellow)	**K(56%, purple; 44%, yellow)**	Switch here
9	U(?%, purple; ?%, yellow)	**K(59%, purple; 41%, yellow)**	Switch here
10	U(?%, purple; ?%, yellow)	**K(62%, purple; 38%, yellow)**	Switch here
11	U(?%, purple; ?%, yellow)	**K(65%, purple; 35%, yellow)**	Switch here

If this decision is chosen to be paid out for real, the computer will randomly pick one row of the list and draw one ball from your chosen box in that row.

To confirm your decision please click on the arrow button below.

Figure C.11: Ambiguity task (one winning color out of two possible colors)

Task 2 Question 2:

In this question you face a table with 11 rows. In each row, you need to choose between Box K and Box U. **They both contain 100 balls of 10 possible colors, including the winning color purple**, but the difference is:

- Box K (known): You know the proportion of the winning color balls. This proportion is clearly stated in each row of the table.
- Box U (unknown): You do not know the proportion of the winning color balls. The box U is constructed by an 11-year-old child using Scratch without any intervention (**https://scratch.mit.edu/about**), apart from requiring 100 balls of 10 possible colors (including purple).

For the payment, the computer draws one ball from your chosen box. **You receive TODAY €15 if the drawn ball is the winning color purple, and €5 if the drawn ball is of other colors.**

#	Box U	Box K	Your choice
1	**U(?% purple; ?% other colors)**	K(0% purples; 100% other colors)	Switch here
2	**U(?% purple; ?% other colors)**	K(3% purples; 97% other colors)	Switch here
3	**U(?% purple; ?% other colors)**	K(6% purples; 94% other colors)	Switch here
4	U(?% purple; ?% other colors)	**K(9% purples; 91% other colors)**	**Switch here**
5	U(?% purple; ?% other colors)	**K(12% purples; 88% other colors)**	Switch here
6	U(?% purple; ?% other colors)	**K(15% purples; 85% other colors)**	Switch here
7	U(?% purple; ?% other colors)	**K(18% purples; 82% other colors)**	Switch here
8	U(?% purple; ?% other colors)	**K(21% purples; 79% other colors)**	Switch here
9	U(?% purple; ?% other colors)	**K(24% purples; 76% other colors)**	Switch here
10	U(?% purple; ?% other colors)	**K(27% purples; 73% other colors)**	Switch here
11	U(?% purple; ?% other colors)	**K(30% purples; 70% other colors)**	Switch here

If this decision is chosen to be paid out for real, the computer will randomly pick one row of the list and draw one ball from your chosen box in that row.

To confirm your decision please click on the arrow button below.

Figure C.12: Ambiguity task (one winning color out of ten possible colors)

Task 2 Question 3:

In this question you face a table with 11 rows. In each row, you need to choose between Box K and Box U. **They both contain 100 balls of 10 possible colors, including purple**, but the difference is:

- Box K (known): You know the proportion of the winning color balls. This proportion is clearly stated in each row of the table.
- Box U (unknown): You do not know the proportion of the winning color balls. The box U is constructed by an 11-year-old child using Scratch without any intervention (**https://scratch.mit.edu/about**), apart from requiring 100 balls of 10 possible colors (including purple).

For the payment, the computer draws one ball from your chosen box. **You receive €15 TODAY if the drawn ball is any OTHER color than purple, and €5 if the drawn ball is purple**.

#	Box U	Box K	Your choice
1	**U(?%, other colors; ?%, purple)**	K(70%, other colors; 30%, purple)	Switch here
2	**U(?%, other colors; ?%, purple)**	K(73%, other colors; 27%, purple)	Switch here
3	**U(?%, other colors; ?%, purple)**	K(76%, other colors; 24%, purple)	Switch here
4	**U(?%, other colors; ?%, purple)**	K(79%, other colors; 21%, purple)	Switch here
5	**U(?%, other colors; ?%, purple)**	K(82%, other colors; 18%, purple)	Switch here
6	**U(?%, other colors; ?%, purple)**	K(85%, other colors; 15%, purple)	Switch here
7	U(?%, other colors; ?%, purple)	**K(88%, other colors; 12%, purple)**	**Switch here**
8	U(?%, other colors; ?%, purple)	**K(91%, other colors; 9%, purple)**	Switch here
9	U(?%, other colors; ?%, purple)	**K(94%, other colors; 6%, purple)**	Switch here
10	U(?%, other colors; ?%, purple)	**K(97%, other colors; 3%, purple)**	Switch here
11	U(?%, other colors; ?%, purple)	**K(100%, other colors; 0%, purple)**	Switch here

If this decision is chosen to be paid out for real, the computer will randomly pick one row of the list and draw one ball from your chosen box in that row.

To confirm your decision please click on the arrow button below.

Figure C.13: Ambiguity task (nine winning colors out of ten possible colors)

C.5.4 Randomization choices (R-range)

Introduction about task 3

In this task you will face a few questions. In each question you will see a table with 20 rows. Each row has two options: A and B.

Across rows A stays the same, while B becomes more attractive. **Your choice is to decide the chances you want to receive A and B.** You can do this by clicking the slider and moving the bar. Below are two examples:

Example 1: You want to receive A with a 100% chance and B with a 0 % chance

100% A **0% B**

Example 2: You want to receive A with a 55% chance and B with a 45 % chance

55% A **45% B**

If one of the questions in this task is selected to decide your payment, the computer will randomly select one row and generate a number between 1 and 100. You will receive A in that row if the randomly generated number is between 1 and the chance (in %) you specify for receiving A, and you will receive B otherwise. In Example 1, you will receive A if the randomly generated number is between 1 and 100, which is always true and thus you will receive A for sure (100% chance). In Example 2, you will receive A if the randomly generated number is between 1 and 55, and recdeive B if the randomly generated number is between 56 and 100.

Before this task starts, you face one control question to check whether you understand the experimental instruction (with only 10 rows, not 20 rows like in real questions). *Answering the control question incorrectly will reduce your final experimental payment by €0.50.* So, please read the instruction carefully.

Figure C.14: Instruction about the R-range task

Task 3 Question 1:

In this question you face a table with 20 rows and two options in each row:

- Option A: €10 in four weeks (December 13, 2022)
- Option B: a certain amount of euro today (November 15, 2022)

Your choice is to decide **the chances you want to receive Option A and Option B**. You can do this by clicking the slider and moving the bar to set the chances.

#	Option A	Your choice		Option B
1	€10 in four weeks	100% A	0% B	€0.50 today
2	€10 in four weeks	100% A	0% B	€1.00 today
3	€10 in four weeks	100% A	0% B	€1.50 today
4	€10 in four weeks	100% A	0% B	€2.00 today
5	€10 in four weeks	100% A	0% B	€2.50 today
6	€10 in four weeks	100% A	0% B	€3.00 today
7	€10 in four weeks	100% A	0% B	€3.50 today
8	€10 in four weeks	100% A	0% B	€4.00 today
9	€10 in four weeks	100% A	0% B	€4.50 today
10	€10 in four weeks	100% A	0% B	€5.00 today
11	€10 in four weeks	100% A	0% B	€5.50 today
12	€10 in four weeks	100% A	0% B	€6.00 today
13	€10 in four weeks	100% A	0% B	€6.50 today
14	€10 in four weeks	100% A	0% B	€7.00 today
15	€10 in four weeks	100% A	0% B	€7.50 today
16	€10 in four weeks	85% A	15% B	€8.00 today
17	€10 in four weeks	50% A	50% B	€8.50 today
18	€10 in four weeks	0% A	100% B	€9.00 today
19	€10 in four weeks	0% A	100% B	€9.50 today
20	€10 in four weeks	0% A	100% B	€10.00 today

For the payment: the computer will randomly select one row and randomly generate a number between 1 and 100 to decide your payment. If the generated number is between 1 and the chance (in %) you specify for receiving A in that row, you will receive A, and you will receive B otherwise.

Before you submit, please make sure you have moved the bars in all sliders. Otherwise, you cannot proceed.

Figure C.15: Randomization choice between money in the future and money today

In this question you face a table with 5 rows (not 20 rows like before) and two options in each row:

- Option A: €10 today (November 15, 2022)
- Option B: a certain amount of euro today (November 15, 2022)

Your choice is to decide **the chances you want to receive Option A and Option B**. You can do this by clicking the slider and moving the bar to set the chances.

#	Option A	Your choice			Option B
1	€10 today	A		B	€9.90 today
2	€10 today	A		B	€9.75 today
3	€10 today	A		B	€9.50 today
4	€10 today	A		B	€9.25 today
5	€10 today	A		B	€9.00 today

For the payment: the computer will randomly select one row and randomly generate a number between 1 and 100 to decide your payment. If the generated number is between 1 and the chance (in %) you specify for receiving A in that row, you will receive A, and you will receive B otherwise.

Before you submit, please make sure you have moved the bars in all sliders. Otherwise, you cannot proceed.

Figure C.16: The R-range control (in Experiment 1 only)

C.5.5 Risk task

Task 4 Question 1:

In this question the two options in each row are:

- Option A: €15 or €5 today with an equal chance.
- Option B: a certain amount of euros today.

Your choice is to decide in which row you want to switch from A to B. You can do this by clicking "Switch here" at the row where you prefer B over A for the first time. Your preferred option will then be highlighted in bold.

#	Option A	Option B	Your choice
1	€15 or €5 today with an equal chance	€5.00 today	Switch here
2	€15 or €5 today with an equal chance	€5.50 today	Switch here
3	€15 or €5 today with an equal chance	€6.00 today	Switch here
4	€15 or €5 today with an equal chance	€6.50 today	Switch here
5	€15 or €5 today with an equal chance	€7.00 today	Switch here
6	€15 or €5 today with an equal chance	€7.50 today	Switch here
7	€15 or €5 today with an equal chance	€8.00 today	Switch here
8	€15 or €5 today with an equal chance	€8.50 today	Switch here
9	€15 or €5 today with an equal chance	€9.00 today	Switch here
10	€15 or €5 today with an equal chance	€9.50 today	Switch here
11	€15 or €5 today with an equal chance	€10.00 today	Switch here
12	€15 or €5 today with an equal chance	€10.50 today	Switch here
13	€15 or €5 today with an equal chance	€11.00 today	Switch here
14	€15 or €5 today with an equal chance	€11.50 today	Switch here
15	€15 or €5 today with an equal chance	€12.00 today	Switch here
16	€15 or €5 today with an equal chance	€12.50 today	Switch here
17	€15 or €5 today with an equal chance	€13.00 today	Switch here
18	€15 or €5 today with an equal chance	€13.50 today	Switch here
19	€15 or €5 today with an equal chance	€14.00 today	Switch here
20	€15 or €5 today with an equal chance	€14.50 today	Switch here
21	€15 or €5 today with an equal chance	€15.00 today	Switch here

If this decision is chosen to be paid out for real, the computer will randomly select one row to determine your payment.

To confirm your decision please click on the arrow button below.

Figure C.17: The risk task

C.5.6 Control questions about binary and randomization choices

Control question 1:

Suppose you face a table with 10 rows and two options in each row:

- Option A: Receive a bar of chocolate today
- Option B: Receive a certain amount of euro today

Suppose you choose to switch from A to B at the #7 row (see the picture below for illustration). Which of the following statement about the payment in this question is correct?

#	Option A	Option B	Your choice
1	**a bar of chocolate today**	€1 today	Switch here
2	**a bar of chocolate today**	€2 today	Switch here
3	**a bar of chocolate today**	€3 today	Switch here
4	**a bar of chocolate today**	€4 today	Switch here
5	**a bar of chocolate today**	€5 today	Switch here
6	**a bar of chocolate today**	€6 today	Switch here
7	a bar of chocolate today	**€7 today**	**Switch here**
8	a bar of chocolate today	**€8 today**	Switch here
9	a bar of chocolate today	**€9 today**	Switch here
10	a bar of chocolate today	**€10 today**	Switch here

- If the computer selects row #3 to decide your payment, you will receive €3 today.
- If the computer selects row #8 to decide your payment, you will receive a bar of chocolate today.
- If the computer selects row #9 to decide your payment, you will receive €9 today.

Figure C.18: Control question about binary choices

Control question 2:

Suppose you face the options as the following (options in the real task are different).

- Option A: €4 today.
- Option B: a package of chocolate in one month

Suppose after some thinking you decide that you want to receive A with 95% chance and receive B with 5% chance, as illustrated in the picture below,

Option A	Your choice	Option B
€4 today	**95% A** **5% B**	a package of chocolate in one month

After your choice, the computer will randomly generate a number between 1 and 100. You will receive A if the randomly generated number is between 1 and 95, and you will receive B if the randomly generated number is between 96 and 100.

Which of the following statements about the lottery is correct?

- ◯ I will receive 0.95*4=3.8 euro today and 0.05 package of chocolate in one month.
- ◯ There is a 95% chance I will receive 4 euro today and nothing in one month, and a 5% chance I will receive nothing today and a package of chocolate in one month.
- ◯ There is a 95% chance I will receive 4 euro today and a package of chocolate in one month, and a 5% chance I will receive nothing today and nothing in one month.

→

Figure C.19: Control question about randomization choices

C.5.7 Beliefs about payments

We would like to ask you some final, general questions.

Q1:

As we explained in the invitation, we will make sure that you will receive your experimental payment on time. Specifically,

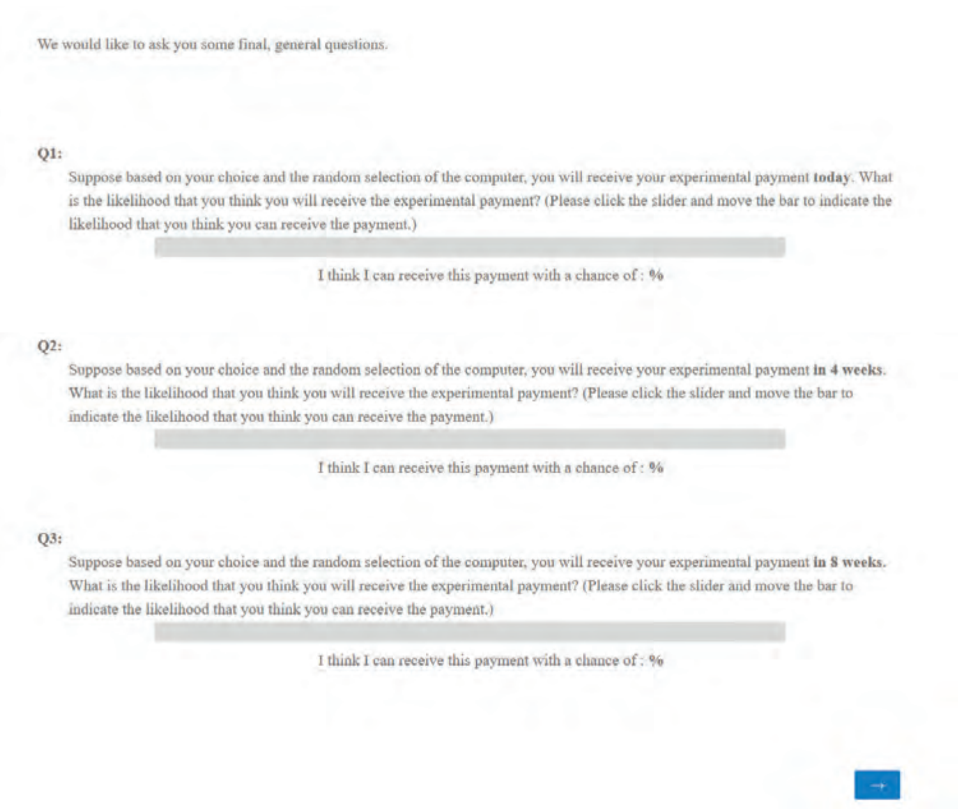
- if an Amazon.nl coupon is selected by the computer as your payment, you will receive it through an email before 23:59pm on the specified date;
- if a monetary payment is selected by the computer as your payment, you will receive it via a bank transfer before 23:59pm on the specified date.

If you do not receive the experimental payment on time, you can contact us at lin.shi@ru.nl, and you will receive your experimental payment plus the additional compensation of €5.

Nevertheless, some of you may still worry that you may not receive your experimental payment on time. How likely do you think you can receive the experimental payment on time? (Please click the slider and move the bar to indicate the likelihood that you think you can receive the payment.)

I think I can receive this payment with a chance of : **%**

Figure C.20: Belief about payment: Experiment 1



We would like to ask you some final, general questions.

Q1:

Suppose based on your choice and the random selection of the computer, you will receive your experimental payment **today**. What is the likelihood that you think you will receive the experimental payment? (Please click the slider and move the bar to indicate the likelihood that you think you can receive the payment.)

I think I can receive this payment with a chance of : %

Q2:

Suppose based on your choice and the random selection of the computer, you will receive your experimental payment **in 4 weeks**. What is the likelihood that you think you will receive the experimental payment? (Please click the slider and move the bar to indicate the likelihood that you think you can receive the payment.)

I think I can receive this payment with a chance of : %

Q3:

Suppose based on your choice and the random selection of the computer, you will receive your experimental payment **in 8 weeks**. What is the likelihood that you think you will receive the experimental payment? (Please click the slider and move the bar to indicate the likelihood that you think you can receive the payment.)

I think I can receive this payment with a chance of : %

Figure C.21: Beliefs about payments: Experiment 2

Research Data Management

The data management in this dissertation complies with the regulations and guidelines described by Radboud University, in accordance with the General Data Protection Regulation (GDPR).

Data were collected through experiments involving students from the Lab of Experimental Economics at Dongbei University of Finance and Economics, as well as students from the Individual Decision-Making Lab at Radboud University. Participants were informed of the data privacy guidelines via consent forms provided at the beginning of each experiment.

All privacy-sensitive data collected for payment purposes were securely deleted once administrative tasks were completed. Anonymous research data were used by scientists as part of data sets, articles and presentations and were stored securely on Radboud University's servers, in compliance with university protocol. The anonymized research data will be accessible to other scientists for a period of a minimum of 10 years.

List of Notations

List of symbols

CE	Certainty equivalent
PE	Probability equivalent
$U(\cdot)$	Von Neumann–Morgenstern expected utility
$c(\cdot)$	A convex and continuously differentiable cost function
A, B	Two options in a binary choice
$p(A, B)$	The probability of choosing A over B
$A \wedge B$	The greatest lottery dominated by A and B
η	Stochastic choice function parameter
γ	Utility curvature parameter
$\overline{y}$	Upper bound of a monetary randomization range
$\underline{y}$	Lower bound of a monetary randomization range
X_t	A timed reward at t
$F_t(\cdot)$	Belief distribution
κ	Cautious attitude toward valuation uncertainty
$ME_{t1,t2}$	Monetary equivalent at $t2$ of a timed reward received at $t1$
$VU_{0,t}$	Valuation uncertainty of present monetary equivalent discounted from t
δ	Discount factor
β	Present bias parameter
$\overline{\beta}$	Present bias parameter calculated from the upper bound of R-range
$\underline{\beta}$	Present bias parameter calculated from the lower bound of R-range
$\delta_{t1,t2}$	Discount factor from $t2$ to $t1$
β_{FE}	Front-end delay effect parameter
$\bar{\beta}_{FE}$	Front-end delay effect parameter calculated from the upper bound of R-range
$\bar{\beta}_{FE}$	Front-end delay effect parameter calculated from the lower bound of R-range

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Summary

This dissertation contains three studies to improve the understanding of decision-making under preference uncertainty. Many daily decisions involve trading off between conflicting objectives, where individuals may be uncertain about their preferences. Under preference uncertainty, individuals may exhibit stochastic choice by changing their decisions across repeated choices or by reporting a range of possible values when evaluating an asset. This dissertation contributes to the empirical investigation of preference uncertainty by proposing novel methods to reveal and measure it with incentives. These methods are applied in economic experiments to examine whether preference uncertainty can explain two economic anomalies: preference reversal and present bias.

Chapter 2 examines the prevalence and implications of conscious stochastic choice arising from preference uncertainty in preference reversal experiments. This chapter proposes a novel method that allows individuals to either pay a small cost to select a specific option or opt for a free randomization option. By combining this method with repeated choices, this chapter separately estimates stochastic choice functions from choice tasks and valuation tasks. The analysis is based on three major conscious stochastic choice models: preference incompleteness, preference imprecision, and hedging of preference uncertainty. The results suggest prevalent stochastic choice and substantial preference reversals. However, the estimated stochastic functions depend on the elicitation procedures. This chapter concludes that choices are not only stochastic but also procedure-dependent, and that both are important in explaining preference reversals.

Chapter 3 focuses on stochastic choice arising from random shocks in preferences and employs it to explain the preference reversal. Using two independent data sets, this chapter bases the analysis on a wide range of random utility models and estimates a consistent stochastic choice function for each subject. Using the estimated stochastic function, this chapter calculates the distribution of preference reversal patterns and compares it with the observed proportions from the experiments. The results suggest that stochastic choice alone is insufficient to explain the preference reversal

phenomenon, as a consistent stochastic function cannot simultaneously account for decisions in direct choice tasks and valuation tasks.

Chapter 4 investigates whether uncertainty in valuing delayed rewards, coupled with caution toward this uncertainty, can result in present bias. This chapter formalizes a behavioral model of intertemporal valuation and shows that present bias can arise from the discontinuous jump in valuation uncertainty as soon as a delay is introduced. Following the theoretical framework, this chapter estimates subjects' present bias and measures their valuation uncertainty toward timed rewards in two pre-registered experiments. In addition to a baseline treatment that employs monetary rewards, the experiments also include a coupon treatment using Amazon coupons as non-monetary rewards and a front-end delay treatment to study the role of valuation uncertainty in the front-end delay effect. The results align with the theoretical analysis. For monetary timed rewards, subjects with higher valuation uncertainty exhibited significantly stronger present bias. For non-monetary rewards, subjects revealed valuation uncertainty for both the immediate and delayed rewards and exhibited substantially weaker present bias. Finally, higher valuation uncertainty was significantly related to stronger front-end delay effects.

In general, this dissertation contributes to the measurement of preference uncertainty and its application in explaining economic anomalies. It has theoretical implications by differentiating prominent non-deterministic decision-making models and demonstrating their potential to explain economic anomalies. Additionally, the proposed incentivized measurement method has methodological implications for future research on preference uncertainty. Furthermore, by deepening the understanding of uncertainty in individuals' preferences, the dissertation has policy implications, including improving the effectiveness and flexibility of policies and reducing social discrimination.

Samenvatting

Deze dissertatie bevat drie studies die bijdragen aan een beter begrip van besluitvorming onder preferentieonzekerheid. Veel dagelijkse beslissingen vereisen een afweging tussen conflicterende doelen, waarbij individuen onzeker kunnen zijn over hun voorkeuren. Wanneer er sprake is van preferentieonzekerheid, kunnen individuen stochastisch gedrag vertonen. Zij wisselen van keuze bij herhaalde beslissingen of rapporteren een reeks van mogelijke waarden voor een object. Deze dissertatie draagt bij aan het empirisch onderzoek naar preferentieonzekerheid door het gebruik van nieuwe methoden om deze onzekerheid met behulp van prikkels te identificeren en te meten. Deze methoden worden toegepast in economische experimenten om te onderzoeken of preferentieonzekerheid twee economische anomalieën kan verklaren: preferentiereversal en present bias.

Hoofdstuk 2 bestudeert de prevalentie en implicaties van bewuste stochastische keuze als gevolg van preferentieonzekerheid in experimenten met preferentiereversal. Dit hoofdstuk introduceert een nieuwe methode waarbij individuen ofwel een klein bedrag betalen om een specifieke optie te kiezen, of kiezen voor een gratis randomisatieoptie. Door deze methode te combineren met herhaalde keuzes, worden stochastische keuzefuncties afzonderlijk geschat op basis van keuzetaken en waarderingstaken. De analyse is gebaseerd op drie belangrijke modellen van bewuste stochastische keuze: preferentie-incompleetheid, preferentie-imprecisie en hedging van preferentieonzekerheid. De resultaten suggereren wijdverspreid stochastisch keuzegedrag en substantiële preferentiereversals. De geschatte stochastische functies blijken echter afhankelijk te zijn van de elicitatieprocedures. Dit hoofdstuk concludeert dat de keuzes van proefpersonen zowel stochastisch als procedureafhankelijk zijn, en dat beide aspecten een belangrijke rol spelen in het verklaren van preferentiereversals.

Hoofdstuk 3 richt zich op stochastische keuze die voortkomt uit willekeurige schokken in voorkeuren en past deze toe om preferentiereversal te verklaren. De analyse in dit hoofdstuk is op basis van twee onafhankelijke datasets en maakt gebruik van een breed scala aan willekeurige-utiliteitsmodellen. Daarbij wordt voor elke

proefpersoon een consistente stochastische keuzefunctie geschat. Op basis van de geschatte stochastische functie berekent dit hoofdstuk de verdeling van preferentiereversalpatronen. Vervolgens vergelijkt het deze met de geobserveerde proporties uit de experimenten. De resultaten suggereren dat stochastische keuze alleen onvoldoende is om het fenomeen van preferentiereversal te verklaren. Dit komt doordat een consistente stochastische functie niet tegelijkertijd de beslissingen in directe keuzetaken en in waarderingstaken kan verklaren.

Hoofdstuk 4 onderzoekt of onzekerheid bij het waarderen van uitgestelde beloningen, in combinatie met voorzichtigheid ten opzichte van deze onzekerheid, kan leiden tot present bias. Dit hoofdstuk formaliseert een gedragsmodel van intertemporele waardering. Het laat zien dat present bias kan ontstaan door de discontinue sprong in waarderingsonzekerheid zodra er een vertraging wordt geïntroduceerd. In lijn met het theoretische raamwerk schat dit hoofdstuk de present bias van proefpersonen en meet het hun waarderingsonzekerheid voor getimede beloningen in twee gepreregistreerde experimenten. De experimenten omvatten naast een baselinebehandeling met monetaire beloningen ook een couponbehandeling en een front-end delaybehandeling. In de couponbehandeling worden Amazon-coupons als niet-monetaire beloningen gebruikt. De front-end delay-behandeling onderzoekt de rol van waarderingsonzekerheid in het front-end delay-effect. De resultaten komen overeen met de theoretische analyse. Voor monetaire getimede beloningen vertoonden proefpersonen met een hogere waarderingsonzekerheid significant sterkere present bias. Voor niet-monetaire beloningen rapporteerden proefpersonen onzekerheid voor zowel de onmiddellijke als de uitgestelde beloningen en vertoonden zij aanzienlijk zwakkere present bias. Ten slotte bleek een hogere waarderingsonzekerheid significant samen te hangen met sterkere front-end delay-effecten.

Over het algemeen draagt deze dissertatie bij aan de meting van preferentieonzekerheid en de toepassing ervan bij het verklaren van economische anomalieën. De studie heeft theoretische implicaties door onderscheid te maken tussen prominente nietdeterministische besluitvormingsmodellen en hun potentieel aan te tonen om economische anomalieën te verklaren. Daarnaast heeft de voorgestelde meetmethode met behulp van prikkels methodologische implicaties voor toekomstig onderzoek naar preferentieonzekerheid. Bovendien heeft deze dissertatie beleidsimplicaties, doordat een dieper begrip van onzekerheid in individuele voorkeuren kan bijdragen aan effectievere en flexibelere beleidsmaatregelen en het verminderen van sociale discriminatie.

Acknowledgments

The story begins in the golden autumn of 2019. I traveled across 9,000 kilometers to the Netherlands. The radiant sunlight, the smell of bread and butter, and the perfect temperature at Schiphol Airport set the stage for an incredible adventure. Along the way, I received a lot of support, and I want to express my sincere thanks to everyone who helped me during this journey.

First and foremost, I would like to thank my daily supervisor, Dr. Jianying Qiu, for your unwavering support, invaluable guidance, and insightful feedback throughout this journey. Your expertise and dedication have shaped my research, and your commitment to academia has been remarkable. I still remember our first project, when you reviewed my data analysis with great attention and provided quick, insightful feedback, even in the evenings. Beyond academics, you also supported me in my daily life. During the pandemic, when I had to move, you generously helped me with the relocation. Whenever a holiday came around, you always warmly invited me to your home to share dinner with your family, offering a sense of belonging and care in a foreign country. I feel truly fortunate to have had the opportunity to learn and grow under your guidance, which has enriched both my professional and personal life immensely.

I would like to thank my promotor, Prof. dr. Stefan Zeisberger, for your consistent encouragement and supportive supervision throughout my PhD journey. I greatly appreciated the freedom you gave me to explore my ideas while always providing guidance when needed. Whenever I faced difficulties, you were always willing to offer me help, giving me the confidence to move forward. Your support made this journey both enriching and enjoyable, and I feel privileged to have worked under your supervision.

I would like to thank my Manuscript Committee: Prof. dr. André van de Hoorn, Prof. dr. Georgios Gerasimou and Dr. Kim Fairley. It's my pleasure to have you as my manuscript committee members. Thanks for your time and effort to read and review my thesis. Thank Prof. dr. Songfa Zhong, Dr. Fadong Chen and Dr.

Jana Vyrastekova, I feel extremely honored to invite you as the members of my Examination Board. Your favor is so much appreciated!

I would like to thank my colleagues. Thank you, Anita, Jan, Jorgo, Sascha, and Sven, for your valuable suggestions during the IdeaLab. Your feedback whenever I presented my project helped me refine my ideas and improve my work. Thank you, Sara, Markus, and Ben, for many discussions we had together about academia and life. I still cherish our trip to Utrecht, it was a pleasure to create those memories together. A special thanks to Sara for helping me with many Dutch translations during my research, including the summary. Your kindness and support mean so much to me. Thank you, Constantina, Daniel, Emiliano, Galina, Janine, and Rein, for sharing the office with me. The time we spent working together was enjoyable. Thank you, Hatice, Daan, Aäron, Rinze, Deborah, and Irma, for your support during my PhD journey. Your help has been truly heartwarming.

I would like to thank my outstanding coauthors, Jiangyan, Frank, Qiyan, Pavlo, and Elias. I have learned so much from you, and it has been a great pleasure working with you. Your insights and collaboration have been invaluable. I truly appreciate your expertise and the constructive discussions that make our work better. Special thanks to Jiangyan for providing valuable suggestions and sharing your experiences with me on the job market.

I would like to thank every subject who participated in my experiment. After each experiment, I received many emails of discussions and feedback, which provided me with invaluable insights. You were not just participants but also contributors and advocates of my research. Your involvement has made my work more grounded in reality and filled with vitality. Your enthusiasm will inspire my future research, and make me focus on improving the benefits of individual decision makers.

I would like to especially thank my two paranymphs, Zixin and Zhenyuan. Your support and help on my PhD defense means a lot to me. Beyond the paranymphs, you are dear friends, sharing the memorable moments of this journey. I deeply appreciate your kindness, patience, and the many ways you have been there for me.

I would like to thank all of my friends. Without you, I could not have made it through this journey. Thank you, Xue, Zhuoran, and your lovely daughter Rourou, for welcoming me into your home with such warmth. I truly cherished the time playing and reading books with Rourou, it was both relaxing and healing. Thank you, Hongling and Stefan, for your generous hospitality and the wonderful meals. I treasure the moments we shared during our visits to Kleve and Roermond, especially the serene afternoon at Kleve's Schwanenburg Castle. Thank you, Fugang and Yong, for all the moments we shared. From making mooncakes and dumplings to celebrating the New Year and walking along the Waal River at sunset, these moments will always stay with me. Thank you, Ting and Niels for all the cozy dinners. The Chinese pancake impressed me the most. You even gifted me a pancake pot, though I must admit I lack Ting's talent to make it successfully. I will keep practicing and hopefully make one for you someday. I will also miss your adorable cats, Lulu and Jimbob. Thank you, Jinhan. Your care has been like that of a sister to me. You invited me to your home, prepared delicious food, and hiked with me. I got to try many new dishes at your place that I had never eaten before. Thank you, Yueyue, for the warm Christmas lunch and for all our shared adventures exploring new cities. I especially enjoyed the game we played together during the Christmas lunch. Thank you, Xiaolei (Amy). Ever since we became neighbors, I have often turned to you for advice. Your patience and encouragement have helped me overcome many difficulties. Thank you, Hongtao and Nan, Zhengyu and Wei, and Chao, for helping me move and inviting me to your homes. Your support means a lot to me, and your kindness is truly unforgettable. Thank you, Jinbiao and Sizhu, for having interesting conversations and exploring creative recipes with me. And heartfelt thanks to Chen, Tiemei, Jinshuo, Hao, Yidong, Keyang, Mingliang, Shuang, Ruipeng, Tianfu, Xuemei, Sailiu, Yiran, Zhen, and Meilin. I feel so lucky to have met you in the Netherlands.

I would like to thank my parents. You have always supported my decisions. Rather than mapping out my future for me, you encouraged me to follow my own path and pursue my dreams. When many of my peers were entering the job market and starting their careers, you stood by my choice to pursue a PhD abroad. During the five years I've been away, you made video calls to me every day and shared stories from home, which greatly reduced my stress. During the Covid pandemic, I was unable to return home for over three years. In that time, my father, who had always been healthy, had heart surgery. I deeply regretted not being able to support you during such a critical moment. I hope there will be more opportunities to spend time by your side in the years to come. I am grateful to my cousins, aunts, and uncles, for taking care of my parents in my absence. Your kindness and support have made me realize how lucky I am to be part of such a loving family. I also want to thank our family's little companions, my cat Yanquan and my dog Xiongmao, whose presence brings joy and warmth to our home.

I would like to thank my husband Xuepeng. Maintaining a relationship across 9,000 kilometers and a 7-hour time difference has not been easy, but we persevered. Leaving you to move to another country was incredibly challenging at the beginning. However, by sharing stories from our daily lives, we found a way to stay connected and feel involved in each other's worlds despite the distance. In 2022, you came to Europe for a one-year scholar visit, and it was only then that we were finally able to see each other again. During that time, we created countless beautiful memories together. We visited each other often and explored many countries. We strolled together in the heavy snow in France in April, watched the sunset over the Seine, admired the scenery on the Philosopher's Path in Heidelberg, cheered for Bayern Munich at the Allianz Arena, tasted delicious food in Prague together, and enjoyed the night view in Brussels. These moments have become treasures in my life, reminding me of how fortunate I am to have met you.

Finally, I would like to thank myself. Throughout my PhD studies, I faced many unexpected challenges and difficulties, but not once did I think about giving up. This mindset sustained me, enabling me to complete my doctoral thesis and helping me develop not only research skills but also the courage to overcome difficulties. I will carry this strength with me as I move forward in life.

The story continues in the peaceful winter of 2024, with more beautiful memories than when it first began.

Curriculum Vitae

Liu Shi was born on March 16th, 1995 in Xi'an, Shannxi Province, China. She had studied Economics at Xian Jiaotong University (Shannxi, China) since 2013 and obtained her bachelor's degree in 2017. In 2017 she started her master's study supervised by Prof. Lirong Yang in Finance at Xian Jiaotong University (Xian, China) and received her Master's degree in 2019.

In 2019, Liu received the scholarship for PhD study from China Scholarship Council and started her doctoral research in Department of Economics at Radboud University (Nijmegen, the Netherlands). She studied the behavior economics under the supervision of Prof. dr. Stefan Zeisberger and Dr. Jianying Qiu. One of her research lines has been published on *Journal of Risk and Uncertainty*.

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