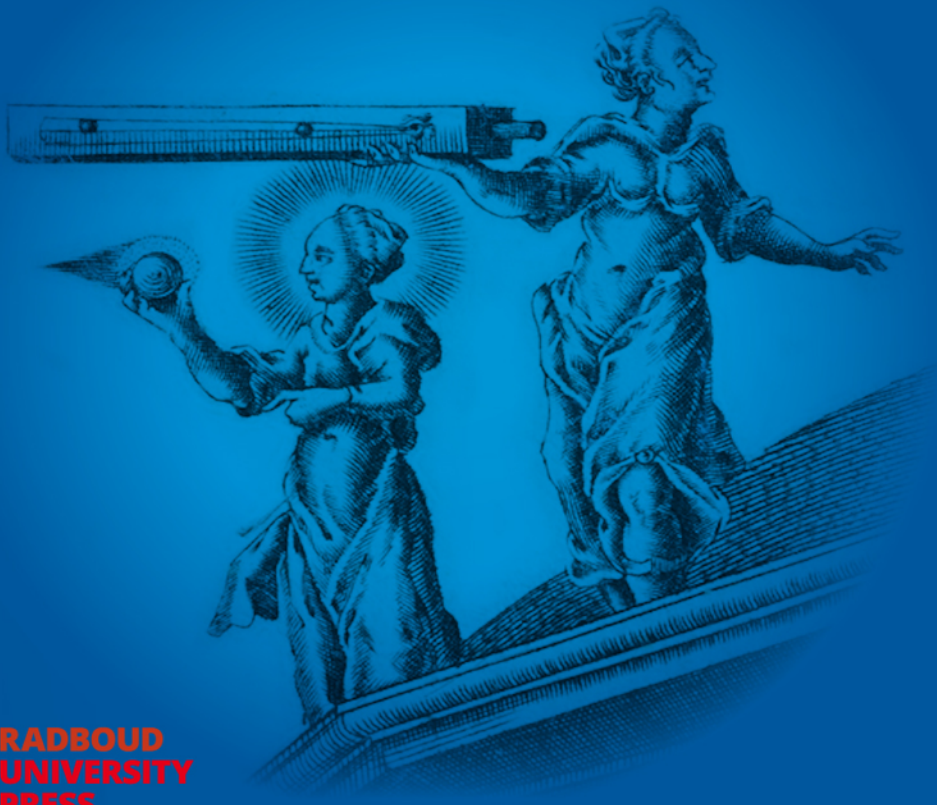
JOHANNES KEPLER

Dioptrice

TRANSLATED BY

Søren S. Larsen



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Translation and introduction

by Søren S. Larsen

The illustration on the cover of this book is a detail from the frontispiece of Kepler's *Tabulae Rudolphinae* (Ulm, 1627), showing two nymphs atop the roof of the Temple of Urania. One of them, *Optica*, holds a telescope on which an eye and two round bodies of equal size are sketched. Rays grazing the bodies illustrate the dependency of apparent size on distance from the observer. The other nymph, *Aegle*, holds a sphere representing the Earth, surrounded by its atmosphere. A cone-shaped shadow extends into space behind the Earth, which is illuminated by light emanating from Aegle's head, symbolising the Sun. The scene is described in Johann Baptist Hebenstreit's poem *Idyllion*.

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The invention of the telescope

For more than four hundred years, the telescope has been an indispensable tool for astronomers. It will probably never be known exactly when, and to whom, the idea of combining two lenses in order to magnify distant objects first occurred, but on October 2nd 1608, the States General in The Hague reviewed a patent application for a device "with which one can see all things very far away as if they were nearby".[1] This application had been submitted by a spectacle maker by the name of Hans Lipperhey from the city of Middelburg. Within weeks, however, at least two other claims to the invention of similar devices were made, and no patents were granted. The principle was, after all, also simple enough that others couldn't be prevented from copying it for very long. Indeed, news of the invention spread rapidly, and by the spring of 1609, telescopes were available across Europe. An account of the events in The Hague, which had also involved a demonstration of the instrument by Lipperhey to Count Maurice of Nassau, had already mentioned that "even the stars which ordinarily are invisible to our sight [...] can be seen by means of this instrument".[2] The Moon was observed by the English polymath Thomas Harriot on July 26th 1609,[3] and the rumour about a "spyglass […] made by a certain Dutchman"[4] reached Galileo Galilei in Venice around May 1609. Galileo put much effort and energy into improving the instrument, and in March

1 H. Zuidervaart, in *The Origins of the Telescope* (2010), p. 9.
2 A. van Helden, *The Invention of the Telescope* (1977), p. 42.
3 T. F. Bloom, *Borrowed Perceptions: Harriot's Maps of the Moon* (1978).
4 *Sidereus Nuncius*, transl. A. van Helden, p. 39.

1610, he published his famous *Sidereus Nuncius* ("Starry Messenger"), in which he announced his discovery of the four moons of Jupiter, along with observations of the Moon, the Milky Way, and various other objects, carried out between November 1609 and March 1610.

Neither the Dutch spectacle makers nor Galileo provided any detailed theoretical account of how exactly the combination of a convex and a concave lens, as was used in the first devices, makes distant objects appear larger. In the *Sidereus Nuncius*, and later in *Il Saggiatore* ("The Assayer"), Galileo described only in general terms how he had gone about constructing a telescope after he had heard about the Dutch instrument:

> Therefore, a single glass is not sufficient to produce the effect. Passing next to two, and knowing as before that a glass with parallel faces alters nothing, I concluded that the effect would still not be achieved by combining such a one with either of the other two. Hence I was restricted to trying to discover what would be done by a combination of the convex and the concave, and you see how this gave me what I sought.[5]

Johannes Kepler is today best known for the three laws of planetary motion that bear his name. However, Kepler also made important contributions to optics, and his book *Dioptrice* (a word that Kepler himself introduced) was, in fact, the first to provide a detailed optical theory for the new instrument. It also contained the first description of the astronomical or Keplerian telescope, in which the eyepiece is a convex instead of a concave lens. Two significant advantages of this design are its much larger field-of-view and the fact that it allows the placement of cross-hairs and micrometres in the focal plane (as first shown by William Gascoigne in 1641),[6] thereby turning the telescope into a far more accurate measuring device. While it also produces an inverted image, this is of no real inconvenience for astronomical observations, and the Dutch/Galilean telescope was eventually abandoned by astronomers in favour of the Keplerian variant.

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5 *The Assayer* (1623); transl. by S. Drake in *The Controversy on the Comets of 1618* (1960), p. 213.

6 D. Sellers, *In Search of William Gascoigne* (2012), p. 54; L. J. Lacour & D. Sellers, *William Gascoigne, Richard Towneley, and the micrometer* (2016).

The *Dioptrice* itself, of which the main parts were written during a few weeks in the late summer of 1610, remained an important reference for decades. It was reprinted several times between 1653 and 1683 together with Pierre Gassendi's popular textbook *Institutio Astronomica* and the *Sidereus Nuncius*, and was commonly cited by other writers:[7] Geronimo Sirtori (in his *Telescopium*, 1618) acknowledged Kepler as "foremost among those who have written about this matter"; other examples include Johannes Hevelius[8] (*Selenographia*, 1647) and Carlo A. Manzini (*Occhiale all'occhio*, 1660). René Descartes called Kepler his "first master in optics"[9] and chose the title *La Dioptrique* for his own *discourses* on the subject, published in 1637. The *Dioptrice* has thus earned its place as an important milestone in the history of the astronomical telescope and of optics in general. It is, furthermore, an integral part of Kepler's scholarship, as I will outline in the following.

Kepler's path to the *Dioptrice*

The circumstances by which Kepler, in the spring of 1610, came to learn about Galileo's discoveries are well documented. The same is indeed true for much of Kepler's life, in large part due to the about 400 letters written by him (and an even larger number addressed to him), in addition to his published works, horoscopes, and other documents, that have been compiled and published in the *Opera Omnia* (OO)[10] and in the *Kepler Gesammelte Werke* (KGW).[11] Kepler's remarkable life and career have been extensively discussed in a number of biographical works,[12] and it

7 See also A. Malet, in *The Origins of the Telescope*, p. 281.

8 Hevelius was evidently appreciative of Kepler in general. He acquired the complete collection of Kepler's writings after the death of Kepler's son Ludwig, in 1663. The manuscripts narrowly escaped destruction in a fire at Hevelius' observatory in 1679, and he thus became the first link, after Kepler's own heirs, in a long chain of custodians. In 1773, Euler successfully lobbied for Catherine II to purchase the collection for the Russian Academy of Science, in whose possession it remains today. (A. A. Mikhailov 1975).

9 Letter to Marin Mersenne, in *Oeuvres de Descartes*, vol. II, p. 86.

10 Compiled and published 1858-1871 by C. Frisch.

11 This massive effort by the Kepler Commission of the Bavarian Academy of Sciences stretched over nearly 80 years, 1937-2017.

12 Arguably, the most definitive biography of Kepler is that by M. Caspar (who also edited the first volumes of the KGW), available in English translation by C. Doris Hellman. Translations of selections of Kepler's letters have been published in *Johannes Kepler in seinen Briefen* (W. von

would be impossible, and also quite unnecessary, to cover the vast amount of material here. Nevertheless, to provide some context for the present translation of the *Dioptrice*, it may be useful to review some of the main circumstances and events related to its publication.

To start with the beginning, Kepler was born on December 27th, 1571 in Weil der Stadt, then a small but free imperial city in the Duchy of Württemberg, not far from Stuttgart. According to his self-made horoscope, he had "investigated [his] conception"[13] and determined that it took place at 4:37, on May 16th, 1571, early in the morning after his parents' wedding. Despite the very premature delivery implied by these dates, he further relates that the "weakness of the foetus ruled out the suspicion of premature impregnation". His mother, Katharina Guldenmann, was the daughter of an innkeeper (later the mayor) in the nearby town of Eltingen (today part of Leonberg), and Johannes inherited "a bodily constitution more suited to study than to other ways of life"[14] from her. Many years later, she was accused of witchcraft but was eventually acquitted thanks, in no small part, to the very significant efforts Kepler personally put into her defence.[15] The father, Heinrich Kepler, made a living as a mercenary and was characterised by Kepler as a "wicked, harsh, and contentious person", who "faced a great danger of hanging" when Kepler was five years old. Heinrich spent much of his time away from the family on campaigns in the Netherlands,[16] where he fought on the Spanish side in the Eighty Years' War for Dutch independence. On at least one occasion, Katharina also travelled to the Netherlands.[17] Kepler spent the first four years of his life in the crowded household of his paternal grandparents, with grandfather Sebald, who was the mayor of Weil der Stadt, his grandmother (also called Katharina), and several uncles and aunts. The house, located adjacent to the market square of Weil der Stadt, was destroyed in a fire in 1648 but was rebuilt in the same style and today houses the Kepler Museum.

Dyck & M. Caspar) and in *Johannes Kepler – Life and Letters* (C. Baumgardt). A more popular exposition may be found in A. Koestler's *Sleepwalkers*.

13 Quotes in this and the following paragraph: OO vol. VIII, p. 671-672; see also KGW vol. XXI.2.2.

14 *The Harmony of the World*, Book IV (transl. E. Aiton et al.), p. 376.

15 See U. Rublack's *The Astronomer and the Witch* (2015).

16 The Latin "Belgium" in Kepler's writings has been translated as "The Netherlands" (P. Geyl, "Note on the word Belgium" in *The Netherlands in the 17th century*, p. 260).

17 OO vol. VIII, p. 671: "In the year 1574, my father was already in the Netherlands. In 1575, my mother went to the Netherlands and returned with my father […] In 1576, my father was again in the Netherlands."

Kepler recalled that "I heard many things about the comet of 1577[18] and was even taken to a high place by my mother to see it" and that "I was called outside by my father to observe [the eclipse on January 31st 1580]; the Moon appeared completely red". However, apart from these two episodes, there was little in the external circumstances of his early life that might have predestined him to become one of the giants on whose shoulders Isaac Newton would later stand. Perhaps the most important catalyst to set him on that path was the fact that he could benefit from the excellent public school system and scholarships that the Dukes of Württemberg had established in order to support their need for an educated class of clergy and public servants. In 1577, he was sent to the German school in Leonberg, where his parents had bought a house in the meantime. His teachers soon noticed his academic talents, and in 1578, he was transferred to the Latin school. The situation of the family remained volatile, and it took him five years to complete the first three grades,[19] but in 1584, he eventually proceeded to the seminary in Adelberg and then, in 1586, to the Maulbronn monastery (today a UNESCO World Heritage site), with the aim of becoming a Lutheran clergyman.

In 1589, his father disappeared for good, and Kepler was admitted to the University of Tübingen on a ducal scholarship that same year. In addition to theology, the curriculum there included a thorough education in philosophy, mathematics, and astronomy. One of Kepler's professors was Michael Mästlin, who introduced him to the Copernican heliocentric system alongside the Ptolemaic geocentric system that was still officially taught. Kepler enthusiastically embraced the Copernican ideas and would later frequently acknowledge the influence of "his teacher Mästlin". In 1594, before he had even completed his studies, he was offered a position as district mathematician (with the responsibility of publishing annual astrological calendars) and teacher of mathematics at the Protestant seminary in Graz, the capital of Styria in present-day Austria. While somewhat reluctant to give up the idea of a career at the pulpit, he accepted the position.

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18 The "Great Comet of 1577" was observed around the world. Tycho Brahe used the absence of a measurable parallax to show that the comet must be more distant than the Moon.

19 OO vol. VIII, p. 671: "When my parents relocated to Ellmendingen in 1579, I was unable to attend the school in Leonberg for some time [...] From 1580 to 1582, I was heavily engaged in farm work."

Figure 1: Illustration of Kepler's model for the five regular polyhedrons nested between the six planetary spheres. From the Kepler Museum in Weil der Stadt; model design by Hermann Bühler, Stuttgart (1930). Photo by the author.

Already for some time, Kepler had been contemplating the problem of the planetary orbits, specifically "why they were such and not otherwise: the number, the size, and the motions of the circles".[20] Then, "on the 9/19th of July in the year 1595",[21] while giving a lecture, he noticed that the relative sizes of two circles circumscribed around, and inscribed within, an equilateral triangle (that is, with radii differing by a factor of two), were approximately the same as those of the orbits of Saturn and Jupiter. Developing this idea further, he eventually came to the realisation that the five regular polyhedrons (also known as the Pythagorean or Platonic solids) fit approximately in between six spheres representing the orbits of the planets known then. This scheme clearly required the Earth to be orbiting the Sun along with Mercury, Venus, Mars, Jupiter, and Saturn, and, furthermore, only the Copernican system provided the necessary constraints on the relative sizes of the planetary orbits. It seemed evident to Kepler that he had achieved a major breakthrough and that he might yet

20 *Mysterium Cosmographicum*, transl. A. M. Duncan, p. 63.

21 Kepler often gives dates according to both the Julian and Gregorian calendars.

serve God by sharing these insights about the rationale behind the design of the Universe that had been revealed to him.[22] He did so in his first book, the *Mysterium Cosmographicum* (1596), about which he solicited the opinions of Tycho Brahe and Galileo Galilei, among others. Galileo wrote, having read only the introduction, that he was happy to have found a "companion in the search for truth",[23] and Tycho replied that he liked the book very much, although he wasn't sure "whether everything in it should be accepted".[24] Despite the somewhat non-committal responses, Kepler had earned the recognition of both men, and Tycho even extended an invitation for Kepler to visit him at the Wandesburg Castle near Hamburg, where he was residing at the time, having recently left Denmark after falling out of favour with the young King Christian IV. A visit did not materialise on that occasion, but Tycho's and Kepler's paths would soon cross again.

Kepler might have spent the rest of his life in Graz had it not been for the unrest following the Protestant reformation and subsequent Catholic counter-reformation that soon caught up with him there. During 1598-1599, his situation in Graz, as a Protestant, became increasingly precarious, and he turned to Mästlin for advice as to whether any opportunities might be available for him in Tübingen or elsewhere. The reply was not encouraging. Meanwhile, Tycho had moved to the Benatky Castle near Prague in June 1599, and at Tycho's invitation[25] Kepler visited him there between January and June of 1600. Kepler then returned to Graz for a while but was soon after definitively banished, along with anyone else who refused to convert to Catholicism.[26] In October 1600 he finally moved, "at great expense", with his wife Barbara and stepdaughter Regina Lorenz to Prague, where he arrived "destitute, abandoned, and exiled",[27] and in poor health, to work with Tycho. The brief collaboration between Tycho and

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22 In a letter to Mästlin, dated Oct 3rd, 1595 (KGW vol. XIII, Letter #23), Kepler writes: "I, however, strive for these things to be published as soon as possible for the glory of God, who wishes to be recognised through the book of Nature [...] I wanted to be a theologian and was troubled for a long time, but behold: God is now also glorified through my work in astronomy."

23 KGW vol. XIII, Letter #73.

24 KGW vol. XIII, Letter #92.

25 KGW vol. XIV, Letter #145. However, Kepler had already left for Prague before Tycho's letter reached him.

26 M. Caspar, *Kepler* (p. 111-113 in the transl. by C. Doris Hellman).

27 Letter to Tycho, Oct 17th 1600, KGW vol. XIV, Letter #177.

Kepler was a clash between two very different personalities and not always an easy one,[28] but one that would turn out to be immensely consequential for astronomy. After Tycho's death on October 24th 1601, Kepler succeeded him as imperial mathematician in the service of the Holy Roman Emperor, Rudolph II.

As Kepler had noted in the *Mysterium Cosmographicum*, the fit of the five polyhedrons between the planetary spheres was far from perfect, but he already had ideas about how to develop and refine the theory further. His new "harmonic" theory was based on relations between the planetary motions at aphelion and perihelion and the correspondence of these relations to musical intervals. A crucial ingredient in this theory was accurate knowledge of the eccentricities of the orbits, which he knew could be obtained from Tycho's unprecedentedly accurate measurements of the planetary positions, spanning a period of more than 25 years. However, the analysis of Tycho's observations turned out to be far more challenging than Kepler had initially imagined, as he could not make the observations fit the scheme based on strictly circular orbits and epicycles employed not only in the Ptolemaic but also in the Copernican system. Discrepancies up to 8 minutes of arc (about a quarter of the apparent diameter of the Moon)[29] between the calculated and observed positions of Mars remained. He spent the next several years fighting his "battle with Mars", culminating in the publication of the *Astronomia Nova* (1609), in which

> By most laborious proofs and by computations on a very large number of observations, I discovered that the course of a planet in the heavens is not a circle, but an oval path, perfectly elliptical.[30]

The *Astronomia Nova*, or *Commentaries on the Motions of Mars*, as Kepler usually refers to it, is now generally recognised as one of the most important books ever written in astronomy. This is where we find what is now known as Kepler's first two laws: that the orbits are ellipses with the Sun in one of the foci, and that a line drawn from the Sun to the planet sweeps up equal

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28 K. Ferguson, *Tycho and Kepler* (2002).

29 1° [degree] = $60'$ [minutes] = $3600''$ [seconds].

30 *Astronomia Nova*, transl. W. Donahue (2015), p. 35.

areas in equal amounts of time.[31] For Kepler himself, however, these were mere details in the grander scheme he was contemplating.

In parallel with his work on Tycho's observations of Mars, Kepler wrote another important book, the *Ad Vitellionem Paralipomena quibus Astronomiae Pars Optica Traditur*, published in 1604. Its impact, together with the *Dioptrice,* on the field of optics would rival that of the *Astronomia Nova* on astronomy. For brevity, I shall refer to the book by the English title of Donahue's translation, *Optics*. In the preface, Kepler explains that astronomy can be divided into five parts: in addition to the *arithmetical* part, there is the *mechanical* part, dealing with the instruments used for the observations (discussed by Tycho[32]); the *historical* part, comprising the observations themselves; the *optical* part (hence *astronomiae pars optica,* "the optical part of astronomy", in the title), concerned with light and the media through which it propagates; and finally the *physical* part, dealing with the motions and their causes (to be treated in the *Astronomia Nova,* then still in preparation).

Kepler's *Optics* was arguably the first major work on optical theory since Witelo's *Perspectiva* from around 1274, itself mostly based on older work by Alhacen (Ibn al-Haytam; ca. 965-1040) and others. Its conception was triggered by a solar eclipse that Kepler had observed on July 10th 1600, from Graz. Like Tycho before him, he had noticed that the silhouette of the Moon appears smaller than expected when images of the eclipsed Sun are projected through a small opening onto a screen. He set out to explain this phenomenon, having found the existing, centuries-old literature to be of little help,[33] and within a few weeks, he had fully solved the problem of imaging through a small aperture by employing basic rectilinear propagation of light rays (chapter 2 of the *Optics*). He then decided that he also needed to include an account of the means of vision, but again found work by previous writers such as Alhacen and Witelo to be entirely inadequate. A real understanding

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31 The third, or "harmonic" law, was published in Book V of the *Harmonice Mundi*, where Kepler writes: "…the genuine proportion of the periodic times to the proportion of the spheres […] was conceived mentally on the 8th of March in this year 1618 but submitted to calculation in an unlucky way and therefore rejected as false, and finally returning on the 15th of May […] it is absolutely certain and exact that the proportion between the periodic times of any two planets is precisely the 3/2 proportion of their mean distances." (from the translation by Aiton et al., p. 411).

32 Kepler refers to Tycho's *Astronomiae instauratae mechanica* (p. 230 in Donahue's transl.).

33 Straker, *Kepler's Optics* (1971).

of the anatomy and optics of the eye was still lacking,[34] and most medieval writers had considered the primary perceptive component of the eye to be the *crystalline humour* (i.e., the lens). Two decades earlier, however, the Swiss physician Felix Platter had argued that the power of perception instead resides in the retina,[35] and Kepler found himself agreeing with this view, which was by no means widely shared at the time. He proceeded to demonstrate, for the first time, how a real inverted picture is formed in the eye, with the lens acting as the opening in a *camera obscura* and the retina serving as a projection screen onto which the light is focussed (chapter 5). This enabled him to explain, furthermore, why concave and convex lenses make it possible to correct near- and far-sightedness – a "matter of great wonder […] whose cause is nonetheless hitherto unknown."[36] Another topic discussed in the *Optics* is the problem of refraction in the atmosphere (chapter 4), which is obviously relevant to accurate measurements of the positions of stars and planets. The book is also notable for introducing the inverse-square law for the attenuation of light as it propagates from a central point source (chapter 1, prop. 9), and for its discussion of conic sections to which we owe the modern use of the term *focus* in this context (chapter 4, section 4). The Prague period was indeed an extremely productive one for Kepler; *De Stella Nova* on the supernova of 1604 ("Kepler's supernova") also belongs to this period.

The telescope and Galileo's discoveries

This brings us, at last, to the *Dioptrice*. Around the 15th of March 1610, Kepler received a visit at his home in Prague from his friend Johann Matthäus Wackher von Wackenfels, a councillor to the imperial court with a keen interest in scientific matters. Von Wackenfels reported with great excitement the news from Italy that Galileo had discovered four new planets using a "double *perspicillum*", i.e., a telescope with two lenses. According to Kepler's own account,[37] von Wackenfels was in such a hurry

34 The history of the science of optics and vision before Kepler is discussed by D. C. Lindberg in *Theories of Vision from Al-Kindi to Kepler*.

35 In *De Corporis Humani Structura et Usu*, published in 1583.

36 Chapter 5, prop. 28 of the *Optics*, p. 216 in Donahue's translation.

37 *Kepler's Conversation with Galileo's Sidereal Messenger*, transl. E. Rosen.

to share the news that he told the story "from his carriage in front of my house", and

> our emotions were strongly aroused (because a small difference of opinion of long standing between us had unexpectedly been settled). He was so overcome with joy by the news, I with shame, both of us with laughter, that he scarcely managed to talk, and I to listen.

One can imagine the scene. The news was indeed startling to Kepler, who remained strongly committed to the idea that God's design for the Universe allowed for exactly the six known planets, as laid out in the *Mysterium Cosmographicum*. He thus surmised that the new planets discovered by Galileo could not be circling the Sun but had to be moons of the known planets Venus, Mars, Jupiter, and Saturn, in the same way that the Earth has its own moon (he reasoned that Galileo would not have been able to see any moon associated with Mercury, due to its proximity to the Sun).

Soon thereafter, Kepler was able to inspect a copy of the *Sidereus Nuncius* that the emperor had received. He was relieved to learn that the four new planets did not, in fact, upset the design of the Universe but were moons of the planet Jupiter. Then, on April 8th, the Tuscan ambassador to Prague, Julian de Medici, had a copy of the book delivered directly to Kepler, who was also informed, in a meeting on April 13th, that Galileo was eager to hear Kepler's opinion about it (and, no doubt, receive the endorsement of the Imperial Mathematician). The emperor himself had viewed the Moon through a telescope three months earlier and had discussed his observations with Kepler, so the suitability of the instrument for astronomical observations could not have come as a complete surprise. While he did not yet have access to an adequate instrument himself and thus could not directly verify Galileo's discoveries, Kepler promptly composed a letter which was generally supportive of Galileo, his discoveries, and the telescope, although he did express some reservations about the actual novelty of the latter.[38] The letter was completed by April 19th and sent back to Galileo via the ambassador, and Kepler also had it printed and

38 This sentiment was also expressed in a letter dated Dec 18th, 1610, addressed to an anonymous person in Dresden (KGW XVI, Letter #600): "I now say that nothing particularly new has been invented in double-lensed tubes, since simple lenses were [already] in use."

published under the title *Dissertatio cum Nuncio Sidereo* ("Conversation with the Starry Messenger"), from which much of the preceding account is adopted.

However, if Kepler had hoped to find a detailed description of the principles behind the telescope in the *Sidereus Nuncius*, he would have been disappointed. After a brief account of how he had heard about the new instrument and then set out to build one for himself, Galileo simply announced that "on another occasion we shall publish a complete theory of this instrument." But, as already mentioned, Galileo never published such a theory. This, instead, was the task that Kepler would take upon himself with the *Dioptrice*. Indeed, in the *Dissertatio* he already discussed in some detail how the combination of convex and concave lenses can produce a beam of rays suitable for being focussed on the retina:

> With eagerness, then, I await your instrument, Galileo. Yet, if fate smiles on me so that I can overcome the obstacles and attempt the mechanical construction, I shall exert myself energetically in that endeavour […] Now the rays from a single point of an object shining so far away come down to the [convex lens] practically parallel. Thereafter they converge and enter the crystalline lens of the eye. Being refracted by the crystalline lens, they meet at a point close behind it […] Therefore I shall adapt a concave lens to the individual eye of each observer in accordance with the variations in their vision. In this way the convergence of the rays from any one point will be stopped by a refraction in the opposite direction accomplished by the concave lens. The rays will diverge instead and will enter the crystalline lens as though they were coming from some nearby point. After being refracted by the crystalline lens, they will find their points of convergence on the retina itself. This is the definition of clear vision. I have proved all this on page 202 of my "Optics".[39]

This clearly shows how Kepler thought about the telescope as an extension of the optical system of the eye, as investigated in the *Optics*, and how the theory to be developed in the *Dioptrice* was already forming in his mind.

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39 E. Rosen, *Kepler's Conversation with the Sidereal Messenger*, p. 19-21.

Despite Kepler's subtle hint, he did not receive a telescope from Galileo. While telescopes were widely available by early 1610, most were of poor quality, and Kepler wrote to Galileo again, in a letter dated August 9th 1610:

> You have kindled in me a great desire to see your instrument, so that I may at last join you in enjoying the same celestial spectacles. For the best eyeglasses we have here magnify the diameter tenfold, and others scarcely triple it. My only one reaches twenty times, but with weak and poor light. I see how they can be improved, but avoid the expense.[40]

This was not only a matter of Kepler satisfying his own curiosity: not everyone was as ready to trust Galileo at his word as Kepler himself had been, so his credibility was on the line, too. In his reply (letter dated August 19th),[41] Galileo first thanked Kepler "for being the first, and nearly the only one, to fully trust my assertions with your candour and great intellect, without even having examined the matter closely", but then explained that "the most excellent [telescope] I had is no longer mine" because he had given it to the Grand Duke of Tuscany, and that "I have not constructed another of equal excellence as the work is very laborious". Therefore, alas, he could not offer one for Kepler to borrow.

Later that August, Kepler was finally able to borrow an adequate telescope from Ernest of Bavaria, the Archbishop of Cologne, a title that also made him Elector of the Holy Roman Empire. Ernest had brought the instrument to Prague, where he was meeting with other electors to mediate in the ongoing dispute between Emperor Rudolph and his brother Matthias over the right to the imperial throne (the dispute eventually concluded unfavourably for Rudolph, and the ensuing turmoil led to Kepler's departure from Prague in 1612, after Rudolph's death, and move to Linz). Ernest told Kepler that he had received the instrument from Galileo but complained that it was inferior to some others he had with him and "showed the stars as quadrangular". He was, therefore, happy to lend it to Kepler, who spent the nights between August 30th and September 9th using it to observe Jupiter, the Moon, and other objects in the company

40 KGW vol. XVI, Letter #584.

41 KGW vol. XVI, Letter #587.

of various friends and colleagues. Kepler published an account of these observations in the *Narratio de Observatis a se Quatuor Iovis Satellitibus Erronibus* ("Narrative on the Four Wandering Satellites of Jupiter Observed by Himself"),[42] which provided a stronger confirmation of Galileo's observations. The *Narratio*, furthermore, marks the introduction of the word "satellite" into the vocabulary of astronomy. Having completed these observations, Kepler returned the instrument to Ernest, although he was later able to resume observations using another instrument,[43] made in Prague from lenses that von Wackenfels had obtained from the ambassador of the Republic of Lucca, Alessandro Altogradi.

Contents of the *Dioptrice*

The main text of the *Dioptrice* was written during these busy months of August and September 1610. In a series of 141 numbered definitions, axioms, propositions, and problems, Kepler developed the theory of the Galilean telescope and much more. The book also provided a blueprint of sorts for the format Kepler would later adopt for the *Harmonice Mundi,*[44] in which he finally fulfilled his ambition to complete the project started with the *Mysterium Cosmographicum*.

The *Dioptrice* builds upon the foundation laid out in the *Optics* but develops the theory of convex, concave, and mixed lenses much further and, of course, shows how magnified images of distant objects can be obtained by combining them. In neither book did Kepler establish the sine law of refraction now known as Snell's law, but in the *Dioptrice* he uses an approximation that is adequate for small angles – that the angle of refraction is proportional to the angle of incidence – derived through experiments described in Problems 4 and 5. These experiments are included not merely for pedagogical purposes: in the aforementioned letter from December 18th 1610 (footnote #38), Kepler writes:

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42 German translation by E. Kühn.

43 The note to p. 322 in KGW vol. IV contains Kepler's description of his observations of Mars, Jupiter, Saturn, and the Moon, made in the period Oct 4th – Nov 9th 1610.

44 In the introduction to Book I, Kepler writes: "I have embraced the series of definitions, propositions, and theorems in continuous numbering, as I did in the Dioptrice, for convenience of reference." (from the translation by Aiton et al., p. 14).

> I was unaware that the refraction of glass or crystal, up to the thirtieth degree of inclination, is proportional to the inclination, to within the limits of sense perception; this had to be learned from experience.

It may be noted that Thomas Harriot appears to have discovered the sine law already sometime before 1602, but he neither published it nor mentioned it in his extensive correspondence with Kepler in the years between the publication of the *Optics* and the *Dioptrice*.[45] In Proposition 13, Kepler is the first to describe the phenomenon of total internal reflection, and in Proposition 59, he discusses the inability of lenses with spherical surfaces to focus parallel rays in a single point – an effect today known as spherical aberration. Proposition 64 goes beyond the analysis of the eye given in the *Optics* by discussing the need for a mechanism to adjust the focus ("accommodation") so that both distant and nearby objects can be seen clearly. However, Kepler suggests that this is done by changing the shape of the eyeball itself and not the lens. Along the way, he also finds opportunities to comment on colours (16-17), the functioning of the *camera obscura* (43), burning-glasses and "natural magic" (50-56,106), and the causes of near- and farsightedness and on their correction (64,99).

The description of the Keplerian telescope appears in Problem 86, "To show visible objects as larger and distinct, but in an inverted position, with two convex lenses". This point is emphasised in the letter from December 18th: "my Dioptrics teaches that even with only convex lenses, the same effects can be achieved as with a concave and convex pair". However, only a few problems are actually devoted to this arrangement, and Kepler himself does not imply that the design offers any particular advantages over the Galilean one. It is simply part of a logically arranged sequence of propositions in which Kepler first considers refraction by a single convex surface (34-37) and by a biconvex (or planoconvex) lens (38-56), the optics of vision by the eye alone and in combination with convex lenses (57-85), and then the effects of two convex lenses combined (86-89). He then proceeds to discuss concave lenses, first individually (90-100) and finally in combination with convex lenses (101-124). With the theory of the Galilean telescope thus completed, the last propositions (125-141) discuss how various devices of "concealment" (κρύψις), whose optical properties

45 See R. Riekher, *Schriften zur Optik*.

differ from what "those who construct common instruments would expect", can be constructed from combinations of more than two lenses, or by utilizing meniscus lenses (another term introduced by Kepler) with both a convex and a concave surface.

Throughout the book, Kepler is helped by the lucky coincidence that the refractive index of glass is approximately 3/2 times that of air, which simplifies many of the calculations. This means, for example, that the focal point of a biconvex lens with two equally convex surfaces can be approximated as being located at the centre of curvature of one of the surfaces (Proposition 39). Nevertheless, the *Dioptrice* contains only one quantitative numerical calculation, appearing towards the very end, of the type that fills the pages of the *Astronomia Nova*, namely in Proposition 131, where the relation between the focal length and the thickness of a meniscus lens is investigated. In many cases, the effects of lenses, individually or in the various combinations considered, are treated at a more conceptual level. Proposition 113, for example, describes how the magnification of the Galilean telescope depends on the radius of curvature of the concave lens as follows: "For a given convex lens, a concave lens defined by a smaller circle will show visible objects as larger, and a concave lens defined by a larger circle will show them as smaller". There are, however, also examples of more specific wording, such as Problem 86 which states, for the case of two convex lenses, that the image is magnified "according to the proportion of the lenses with respect to each other".[46] The proportion is here again understood to refer to the radii of curvature, or, equivalently, the focal lengths. While not rigorously demonstrated, this is equivalent to the relation $m = F/f$ for the magnification m of a telescope with focal lengths F and f for the objective and ocular lens, respectively, that will be found in modern textbooks on this subject.[47] This relation is, in fact, valid for both the Keplerian and Galilean telescopes (if f is here taken as the absolute value of the focal length, which is negative for a concave lens).

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46 This specific point was quoted e.g. by C. Manzini, *L'Occhiale all'occhio* (1660), p. 165.

47 An extensive discussion of telescope optics and designs, aimed at advanced amateur astronomers, may be found in *Telescope Optics – A Comprehensive Manual for Amateur Astronomers* (H. G. J. Rutten & M. A. M. van Venrooij). For a graduate-level exposition, see *Astronomical Optics* (D. J. Schroeder).

In September 1610, Kepler handed the manuscript over to the archbishop, Ernest, to whom he had dedicated the work. While he was waiting for the printing of the book to be arranged, he received several additional letters from Galileo via the Tuscan ambassador, announcing new discoveries made with the telescope. Famously, Galileo did not initially reveal his discoveries directly, but in the form of encrypted messages. The first of these had already been reported by Kepler in the *Narratio*, in which he had attempted to decipher the message into a "semi-barbaric" verse, based on which he believed that Galileo might have discovered two satellites around Mars.[48] This would have made Mars fit neatly into a sequence between the Earth, with its single satellite, and Jupiter, with four. But what the message actually said was that Galileo had seen Saturn as a "triple planet",[49] with a large central body flanked by two smaller orbs, as revealed by Galileo in another letter dated November 13th 1610. In December, Kepler received a letter (dated December 11th 1610) with another encrypted message about an observation which "enables the settlement of great controversies in astronomy, and in particular, contains within itself a beautiful argument for the Pythagorean and Copernican arrangement of the world." A few weeks later, Galileo revealed that he had observed the phases of the planet Venus, an indisputable confirmation that Venus must be orbiting around the Sun. It also established beyond doubt that the planets shine with light "borrowed" from the Sun and are not inherently luminous bodies, a question which until then had remained subject to debate.[50]

Kepler included Galileo's letters, in their original Italian versions and in his own Latin translations, along with his commentary on them, as a preface to the *Dioptrice*. While he was at it, he also included a commentary on the *De Usu Optices Praefatio*[51] ("Preface on the Benefits of Optics") by Jean Pena (1528-1558), a preface to Pena's Latin translations of Euclid's

48 The two tiny satellites of Mars - Phobos and Deimos - are much too faint to have been visible to Galileo and were not discovered until 1877 by A. Hall. See *The Planet Mars* (Sheehan 1997), p. 58-63.

49 Saturn's rings were first recognised as such by Christiaan Huygens, as described in his *Systema Saturnium* (1659).

50 Kepler had written in the *Optics* (chapter VI, p. 272 in Donahue's translation): "Thus in my theses on the foundation of astrology, which I published in 1602, in thesis 25 I first made it plausible by four arguments that the planets have their own light. One was this very one which I have just presented. For Venus would really change its face and waste away, like the moon, if it only shone with light communicated from the sun".

51 J. Pena (1557).

Optics and the work *Catoptrics* (the attribution of the latter to Euclid is now thought to be spurious).[52] Pena had been a student of Pierre de la Ramée (also known as Peter Ramus; 1515-1572), who was critical of Aristotelian philosophy as it was taught at the time, and whose challenge to "construct an astronomy without hypotheses" Kepler had cited at the beginning of the *Astronomia Nova*. Pena's preface starts by lamenting that

> Optics is today almost neglected, banished from schools and ignored by nearly everyone, especially by those who wish to be seen as natural philosophers. It has been reduced to such a state that it must be sought not from philosophers but from architects, sculptors, and painters.

It goes on to address a number of problems that can be investigated by means of optical principles, such as the nature of the medium filling the space between the Earth and the stars, the question of the location and motion of the Earth, the nature and distances of comets, and to disprove (from the lack of parallax) the Aristotelian notion that the Milky Way belongs to the sublunary world. Other phenomena, such as rainbows, halos, and various optical illusions, are also discussed.

Some of Kepler's comments on Pena's text can be difficult to understand without context, and I have therefore provided excerpts of the relevant parts where this seemed useful.[53] One particular point of contention concerns the direction of propagation of light rays. Pena argues, citing Euclid,[54] that vision occurs through emission of rays from the eye (extramission), an idea also promoted by Plato,[55] the Greek physician Galen of Pergamon (129-216 AD), and early Islamic writers but emphatically rejected by Kepler. According to the historian David Lindberg, the extramissionist view had already been largely abandoned by the end of the 13^{th} century[56] and Pena's opinion may, therefore, not have been widely shared in his

52 J. L. Heiberg, *Literargeschichtliche studien über Euklid* (1882); A. Jones, *On Some Borrowed and Misunderstood Problems in Greek Catoptrics* (1987).

53 Pena's work and its contemporary context are discussed by P. Barker (1985) and by S. Dupré (2002).

54 Euclid's *Optics* begins (Proposition 1): "Let it be assumed that lines drawn directly from the eye pass through a space of great extent." (transl. by H. E. Burton, 1943).

55 Especially in the *Timaeus*, 45b-47c (p. 1248-1249 in J. M. Cooper: *Plato, Complete Works*).

56 E.g. *Theories of Vision from Al-Kindi to Kepler*, p. 122.

time.[57] In certain other respects, however, the influence of Galen and other ancient writers remains apparent in the *Dioptrice*, as in Proposition 61, where Kepler mentions the idea that the eye is supplied with "visual spirit" from the brain through the optic nerve, echoing Galen's notion of "pneuma" flowing through the nerves to various body parts and sensory organs.[58]

The last letter from Galileo included in the preface to the *Dioptrice* is dated March 26th 1611. At the very end of the preface, Kepler also added a letter from the German astronomer Simon Marius (Mayr) that he had received via Nikolaus Vicke in July 1611, corroborating Galileo's observations of Venus and Jupiter's moons.[59] The names of Jupiter's four Galilean satellites used today (Io, Europa, Ganymede, Callisto) were later proposed by Marius in his *Mundus Iovialis* (1614), at the suggestion of Kepler. In his reply to Vicke,[60] Kepler notes that the *Dioptrice* is finally being printed in Augsburg with Marius' letter included.

Publication and impact

While it must have been a relief for Kepler to see the book in print, the year 1611 was one of personal tragedy for him. His 6-year-old son Friedrich and his wife Barbara died in February and July, respectively, and his attempts, once again, to return to Württemberg came to nothing as he was suspected of being too sympathetic to Calvinist ideas. An incomplete letter draft from the end of 1610, addressed to Galileo, had mentioned that "I am trying to have a few copies printed; if I succeed, I will send one to you".[61] But it is understandable that he did not find the time and energy to personally dispatch copies of the book to colleagues, and there is no evidence of any significant correspondence in the wake of the publication

57 See, however, S. Dupree (2002), especially p. 25-30.

58 See e.g. May's introduction to Galen's *On the usefulness of the parts of the body*, sect. V, and *On the doctrines of Hippocrates and Plato*, Book VII, sect. 3-4.

59 The ensuing dispute between Galileo and Marius, in which Galileo accused Marius of plagiarising his discovery of Jupiter's moons, is discussed by J. M. Pasachoff (2015).

60 KGW vol. XVI, Letters #618, #619.

61 KGW vol. XVI, Letter #603.

Figure 2: The two-day-old Moon (left) and a terrestrial scene (centre) observed through a Keplerian telescope with a 50 mm diameter objective lens and a magnification of 20 times. The objective and ocular lenses have focal lengths of +1000 mm and +50 mm, respectively. Right: the terrestrial scene observed with the same objective lens, but using a concave ocular lens (that is, the Dutch/Galilean configuration) with a focal length of −50 mm, also magnifying 20 times. The latter would not show the entire Moon, even at this modest magnification. Photos by the author.

of the work. However, by late 1613, he had settled in Linz and remarried,[62] and in a brief letter from October 1613, Octavius Pisani in Antwerp asks for Kepler's advice on how to construct a binocular telescope. Kepler replied that he had tried this himself but that the device he had asked to have fabricated "resembled a mousetrap" and that he, therefore, gave up "lest I be ridiculed" – remarking, also, on the difficulty of achieving the required similarity of the two sets of lenses.[63] It may be noted here that Lipperhey had, in fact, delivered three binocular instruments to the States General of the Netherlands between December 1608 and February 1609 and received 900 guilders in return, a large enough sum that he could buy his neighbour's house.[64]

In an appendix to the *Somnium* ("The Dream"),[65] posthumously published in 1634, Kepler described observations of the Moon carried out in 1622-1623 with a telescope he had received as a gift from the Jesuit astronomer Niccolo Zucchi. However, his efforts on the subject of optics essentially ended with the *Dioptrice,* the last of his great works from the

62 In an invitation letter for the wedding that took place on Oct 30th 1613, Kepler describes how he selected his bride, Susanna Reuttinger, among 11 candidates (KGW vol. XVII, Letter #669).

63 KGW vol. XVII, Letters #665, #675.

64 H. Zuidervaart, in *The Origins of the Telescope,* p. 15.

65 Translated into English by E. Rosen. The *Somnium* is sometimes referred to as the first science-fiction novel.

Prague period. After taking up his new position as district mathematician in Linz, a position not unlike the one he had held in Graz (although he kept the association with the imperial court), he devoted himself to other matters, including the completion of the *Harmonice Mundi*, a second edition of the *Mysterium Cosmographicum*, the *Epitome Astronomiae Copernicanae,* and the *Tabulae Rudolphinae* – not to mention the witchcraft trial against his mother.

There are no records of Kepler himself constructing a telescope of the type described in Problem 86. If he had, he would have immediately noticed the difference in the field size, which is quite striking (see figure 2). In the appendix to the *Somnium,* he mentions that his instrument "took in 12 minutes of arc of the Moon's 30 minutes" in a single glance, and as late as 1629, he wrote in a letter:

> For if the tube magnifies to such a degree that these subtle details can be discerned, then surely no tube would show the whole Moon to a fixed eye. But if the Moon is shown in its entirety, the magnification must be very small.[66]

This perfectly describes the experience of looking through a Galilean telescope, while a Keplerian telescope can still show the entire Moon comfortably at magnifications of more than 50 times. In this context, the key feature of the Keplerian design is that the eye may be placed at the intersection of the "pencils" that occurs after the ocular lens, mentioned in Problem 88, and in modern terms known as the *exit pupil*. Hence, rays coming from a wide range of directions can enter the eye, and the field size is mainly limited by optical aberrations that become increasingly severe off-axis. In the Galilean design, the exit pupil is virtual, located inside the tube between the two lenses, which gives the impression of looking through a distant, narrow opening. However, one can see different parts of the object by shifting the eye, as far as the size of the ocular lens allows (Problem 123). At least implicitly, the *Dioptrice* thus already contains essential aspects of the optical principles needed to understand the difference in field size between the Galilean and Keplerian designs.

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66 Letter to Joh. Remus Quietanus, March 2nd 1629 (KGW vol. XVIII, Letter #1103).

The exact role that the *Dioptrice* played in stimulating the first practical applications of the Keplerian design remains somewhat unclear. It is often suggested that Christoph Scheiner was the first to put the design into practice, and indeed, his book about observations of the Sun, *Rosa Ursina* (1630), describes this arrangement in some detail. Scheiner had also made an instrument for archduke Maximilian III of Austria sometime around 1615, and a letter sent to Kepler on behalf of the archduke by his chamberlain Melchior Stöltzle, dated September 27th 1615, asks Kepler for help in improving the instrument.[67] Scheiner had inserted a third lens to provide an upright image, as in Problem 89 of the *Dioptrice,* but the images were not sharp. The letter contains a detailed drawing and the lenses used, asking Kepler to see if he ("alß diser sachen Magister") could do something with them. It is, however, unclear whether Scheiner actually got the idea from Kepler, although he appears to have used instruments of the Dutch/Galilean type at least until November 1611, that is, after the publication of the *Dioptrice.*[68]

In 1646 the Neapolitan astronomer Francesco Fontana published his *Novae coelestium, terrestriumque rerum observationes* ("New observations of celestial and terrestrial things"), in which he claimed to have constructed a telescope with two convex lenses as early as 1608.[69] The book includes statements from two other Jesuit astronomers, fathers Giovanni Battista Zupi and Gerolamo Sersale, attesting that Fontana had demonstrated such a telescope to them in 1614 and 1625, respectively. Fontana points out that although it "makes the object appear inverted", the combination of two convex lenses provides a larger field of view. He further states that

> Although this method seems to have been hinted at by Johannes Kepler in his booklet on Dioptrics, problem 86, on page 42, printed in 1611, in truth I had no knowledge of this booklet until now, when I was preparing this present treatise, and I received it from Father Battista Zupi.

67 KGW vol. XVII, Letter #721.

68 KGW vol. IV, p. 477.

69 P. Molaro, *Francesco Fontana and the birth of the astronomical telescope* (2017).

Fontana certainly put the instrument to good use. He and Father Zupi observed the phases not only of Venus, but also of Mercury; he mentions "certain bands" on Jupiter, clearly shown in his drawings to be the main equatorial cloud belts of the planet; and he came close to discovering the true nature of Saturn's rings, observing them to be shaped like "half-moons" and not just like the two companion globes seen by Galileo.

Also worth mentioning is the work *Oculus Enoch et Eliæ* ("The Eye of Enoch and Elijah"), published by Antonio Maria Schyrleus de Rheita in 1645, in which a description of a binocular instrument is given. Here, Kepler is cited explicitly, and the advantage of the much larger field of a telescope with two convex lenses is again emphasised: "A convex-convex tube of 15 feet in length provides a much wider field of view with one glance than a common concave-convex tube of one foot."[70] This also shows how much telescopes had already grown in size since Galileo's first *perspicillum*.

In any case, Kepler had left his mark by showing how optical systems involving lenses could be understood from basic geometrical principles through this new field of Dioptrics.[71] His demonstration that the eye could also be understood in these terms provided a crucial foundation for his working out the theory behind the telescope.[72] This, of course, did not mean that there was nothing left to do, but as he states in the dedication to Ernest, he had indeed "opened a new field for mathematicians to exercise the power of ingenuity".

Notes on the translation

Anyone who delves into the history of the telescope, or optics in general, is bound to come across references to Kepler's *Dioptrice* sooner or later. The present translation emerged as a result of my own curiosity about the contents of the book and the subsequent realisation that no English translation was available. I thus decided to make my way through Kepler's Latin text, and my notes eventually became sufficiently detailed that

70 *Oculus Enoch et Eliæ*, p. 352.

71 See also A. Malet, in *The Origins of the Telescope*, p. 281.

72 S. Straker, *Kepler's Optics* (1971).

turning them into an actual translation that others might find useful seemed within reach.

While the *Dioptrice* has not previously been translated into English in its entirety, those parts of the preface that are related to Galileo's letters were included by Edward S. Carlos in his translation of the *Sidereus Nuncius* (originally published in 1880). Roughly the same parts are translated into German in volume I of *Johannes Kepler in seinen Briefen*. The dedication to Ernest and the main text (but not the preface) are available in a German translation from 1904, *Dioptrik*, by Ferdinand Plehn, included (together with the first five chapters of the *Optics* and the *Dissertatio cum Nuncio Sidereo*) in the *Schriften zur Optik*, with extensive commentary and historical background information by Rolf Riekher. There is also a complete French translation, *Dioptrique*, from 1990 by Jean Peyroux. These existing (partial) translations have all been valuable for reference, although my starting point has always been Kepler's original text.

I have mainly worked from the 1941 KGW edition, not only because it is set in modern typeface that makes it more easily readable than the 1611 original (though arguably less aesthetically pleasing) and implements the corrections of the errata sheet, but also because it includes numerous notes and comments that have been very helpful when tracking down some of Kepler's more implicit references to other sources and clarifying the meaning of some otherwise obscure passages. I have, however, attempted to approximate the typographic style and layout of the original book as closely as possible, and page numbers indicated in the margin refer to the 1611 edition. The illustrations have been reproduced from copies of the original 1611 edition at the library of Utrecht University.

Translating Kepler's Latin into reasonably idiomatic English is not always an easy task, and it is not made any easier by Kepler's well-known fondness for unusual words and phrases. It necessarily involves a compromise between preserving the style and flavour of the original and producing a readable translation. Hence, the boundary between translating and paraphrasing will inevitably be somewhat ill-defined. I have generally tried to stay as close to the original as possible, and I have maintained the same divisions between paragraphs. However, I do not believe it would be useful to always insist on the most literal translation possible of the Latin sentences, which would, in many cases, lead to unnecessarily awkward and convoluted English prose. Thus, I have, in many cases, rearranged the structure of individual sentences and adapted the punctuation to more closely agree with the conventions of modern English. My goal has been to produce a translation that reflects the way one might imagine Kepler would have written the text had he been writing in modern English, but without the scientific vocabulary developed during the intervening centuries. While it is sometimes tempting to "improve" the text by more extensive paraphrasing, by adopting modern terminology to express certain concepts, or by trying to reinterpret what Kepler might have meant, I have generally refrained from doing so. Today, the *Dioptrice* is chiefly of historical interest, and anyone whose goal is to actually understand the optical principles behind the telescope will be much better served by modern textbooks. I have, however, added a number of explanatory notes.

Kepler introduces the work by declaring, "I present to you, dear reader, a booklet which is mathematical, and therefore not so easy to grasp." Despite this warning, the mathematics used in the *Dioptrice* will, for the most part, present no serious obstacles to a reader who is reasonably proficient in modern high-school-level mathematics. One only needs to be aware that some conventions are different from those that are in common use today. For example, trigonometric functions do not generally refer to a unit circle but to a circle with a radius corresponding to the 'total sine', i.e., the sine of a right angle, in units that are not always specified. Certain functions, common in Kepler's day, such as the versed sine [$\text{vsin}(\theta) = \sin(90°) - \cos(\theta)$] have now fallen out of regular use. The somewhat archaic term "sesquidiameter" has been translated as "one and a half diameters". Apart from such mathematical conventions, various other concepts and terminology employed by Kepler are worth briefly discussing.

First, the title itself warrants some remarks: in the dedication, Kepler explains that since his book deals with the optics of lenses, he coined the title *Dioptrice* for it, following the example of (pseudo-)Euclid's *Catoptrice* which deals with the optics of mirrors. Kepler is thus credited with introducing the term *dioptrics* for the branch of optics concerned with refraction,[73] derived from διά ("through") and ὀπτικός/ὀπτός (pertaining

73 The translation of Plutarch's *Morals* (W. Baxter, 1874, p. 173) refers to Euclid writing "his Dioptrics", but no work by, or attributed to, Euclid bearing that title is known. Perhaps Plutarch simply meant Euclid's *Optics*. In any case, Plutarch's original text uses a slightly different form, διοπτικά (L. Lesage-Gárriga 2018, fn 20), that could also be translated as "dioptics".

to sight, vision, as in "optics"). A related word is διόπτρα (dioptra), referring to various astronomical instruments or "sighting tubes" that had been in use since antiquity and were described in Hero of Alexandria's book, also titled *Dioptra*.[74] The *dioptre,* the unit by which the refractive power of a lens is specified, also has the same origin. The word *Dioptrice*, as it appears in the title of Kepler's book, is, in fact, the singular nominative Greek form (albeit written in the Latin alphabet). This practice of using Greek words in the title was followed by Kepler in other books as well, notably in the *Harmonice Mundi* (or, as is the full title, *Harmonices Mundi Libri V,* "Five books of the Harmony of the World"). Even if a 17th-century Latin speaker would normally have pronounced the "ce" roughly as in "chess" (in contrast to the "k" sound that Julius Caesar[75] or Cicero would have used), the "c" in *Dioptrice* is really a transcribed Greek "κ" (kappa), and the proper pronunciation of the word is thus "Dioptrike". In English, it is quite naturally rendered as Dioptrics, like Optics, Physics, Mathematics, etc.

The word "telescope" or *telescopium* appears nowhere in the *Dioptrice*. It was introduced at a banquet of the *Accademia dei Lincei* (Academy of the Lynx), given for Galileo on April 14th 1611, and was most likely the brainchild of John Demisiani, a Greek poet and theologian.[76] It rapidly gained acceptance and Kepler first used it in a letter to Odo Malcotius, dated July 18th 1613,[77] and then also in later works, such as the *Epitome Astronomiae Copernicanae* (1618) and *De Cometis* (1619). Given this context, it seemed misleading to use this particular word in a translation of the *Dioptrice*, even if finding a suitable English alternative turned out to be challenging. To some extent, this simply reflects the difficulty of discussing an instrument that did not yet have a well-established name. Kepler, in fact, uses a variety of terms. On the title page (and only there), we have the word *conspicillum* and in the dedication, Kepler refers to the *arundo dioptrica*, or "dioptric pipe", but for the most part, he, like Galileo in the *Sidereus Nuncius*, uses the word *perspicillum*, derived from *perspicere*, to see (through), examine, observe. In the preface, he also uses

74 See J. Evans & J. L. Berggren, *Geminos's Introduction to the Phenomena* (2006).

75 Cf. the German word for emperor, *Kaiser.*

76 E. Rosen, *The naming of the telescope* (1947).

77 KGW vol. XVII, Letter #658.

the Greek word *dioptra* a couple of times, which, as was discussed above, means essentially the same thing. The word *perspicillum* is sometimes translated as "spyglass", although that term tends to bring to mind more basic hand-held instruments, typically used for terrestrial purposes. In Kepler's Latin translations of Galileo's letters, he uses *oculare* for the Italian *occhiale*, which can also mean "eyeglass", and I have translated it that way. I have opted to simply use the Latin word *perspicillum* directly whenever it is used in the main text, even if it does not always refer exclusively to the telescope but also more generally to (systems of) lenses. In the dedication, in particular, even this approach encounters some difficulties, since Kepler also refers to the *instrumentum e simplicibus perspicillis compositum*, i.e., the instrument composed of simple *perspicilla* (i.e. lenses), and hence I have taken the liberty of using other terms, such as "optical devices" or just "lenses". In the last part of the *Dioptrice*, where the Galilean telescope is discussed, from Definition 101 onwards, Kepler suggests that the word *tubus* should be used, which I have translated simply by its anglicised equivalent, "tube".

The terms "diverge" and "converge" are introduced in Definition 21 and are then used frequently throughout the text, but only in the specific context of referring to the relative directions of light rays that may be parallel, converging, or diverging as they propagate through a medium or the air. The actual point at which converging rays are gathered by a lens is *never* referred to as the "point of convergence", any more than Kepler would have imagined a point of divergence or parallelism, but rather as *punctum concursus*. I have chosen to preserve this distinction, using "point of concurrence" for what a modern text might refer to as the focal point, or focus. The word *focus* itself does not appear in the main text, and only once in the preface, referring to "the Sun, the common hearth [focus] of both this Terrestrial and that Jovian world".

When discussing convex and concave lenses, Kepler generally does not directly refer to radii of curvature, focal lengths, or focal points. The optical surfaces are to be imagined as sections of spherical surfaces, described by circles with a specified radius. Hence, convex or concave lenses with a large radius of curvature are described as *[lens] convexa magno circulo* or *cava magno circulo*, or, as Kepler explains in Definition 28, by using the neutral gender terms *convexum* or *cavum* to refer more generally to surfaces or bodies with a convex or concave shape. I struggled to find an equivalent and equally compact way of expressing this but settled on "convex/concave lens defined by a large circle". Occasionally, Kepler also uses a genitive construction (*convexum magni circuli*), rendered more straightforwardly (if perhaps not quite idiomatically) as "convex [surface] of a large circle". The distinction between the different grammatical genders is, quite literally, lost in translation, and difficult to convey consistently.

When discussing the anatomy of the eye, Kepler refers to the *tunica retiformis* (the "net-like tunic") or simply *retiformis*. Following the example of Donahue in his translation of the *Optics*, I have translated this as the "retiform" rather than with the modern term "retina". Kepler frequently uses the Greek forms of the words "presbyope" (πρεσβύτης) and "myope" (μυωπός) with reference to Aristotle's usage of these terms. I have generally rendered them using the Latin alphabet, and as these terms for far- and near-sightedness are still common, they should cause no particular difficulties. The Latin term *sensus communis*, however, is not equivalent to the way in which the term "common sense" is now generally used. Rather, it should be understood as some general faculty of perception by which input from each of the specific external senses – sight, hearing, smell, taste, touch – is turned into an internal perception by the mind or "soul" of that which is sensed, as described e.g. by Aristotle in Book III of *On the Soul*:

> The senses perceive each other's special objects incidentally; not because the percipient sense is this or that special sense, but because all form a unity.[78]

To signify this, I have kept the original Latin as *sensus communis*.

The concept of an image, naturally, plays an important role in a book about telescopes. As in the *Optics*, Kepler uses the word *pictura* very specifically to refer to "the figures of objects that really exist on paper or upon another surface" (*Optics*, chapter 5, section 3), and I have translated this as "picture". In modern optical terms, this is what we would refer to as a real image, formed in the focal plane of an optical system. Kepler uses the verb *pingere* to describe the process by which such a picture is formed by a lens (e.g. on the retina), translated as "depict" or "paint". In

78 Transl. by J. A. Smith in *The Complete Works of Aristotle*, vol. 1, 424a30, p. 676.

contrast, the word *imago*, translated as "image", refers to the (inverted or upright) representation of a visible object perceived by the eye when looking through a lens or a system of lenses – a virtual image, in modern terminology. We also find the word *species*, which can have a wide range of meanings that are not easily conveyed by a single English word. This term has a long history in medieval philosophy of sensory perception,[79] dating back at least to Augustine (354-430).[80] The details evolved over time and vary somewhat from author to author, but *species* was described by Roger Bacon (ca. 1220-1292) as

> the first effect of an agent; for all judge that through species [all] other effects are produced. [...] the agent sends forth a species into the matter of the recipient, so that, through the species first produced, it can bring forth out of the potentiality of the matter [of the recipient] the complete effect that it intends.[81]

Hence, one may think of the *species* as a sort of form or representation that propagates from the object to the recipient. In the *Dioptrice*, we first encounter it in Proposition 61, where Kepler discusses the transmission of the *pictura* formed on the retina through the optic nerve to the *sensus communis*, and then also later in the book. Rather than attempting to find an English equivalent and obscuring the link to Kepler's predecessors, I follow the practice of Donahue, Lindberg, and others and keep the original Latin.

One should also be aware that the term *physicus* did not mean exactly the same to Kepler and his contemporaries as "physicist" does to us today, but had somewhat broader connotations and could also be rendered as "natural philosopher". Finally, Kepler often uses the word *artifex* when referring to Tycho Brahe, Copernicus, and others. Following Donahue, I have rendered this as "practitioner", although one might also read it as "master" or "artisan".

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79 D. Lindberg (1976) and *Roger Bacon's Philosophy of Nature, A Critical Edition* (1986).

80 T. Nawar, *Augustine on Active Perception, Awareness, and Representation* (2021).

81 In *De multiplicatione specierum*; English translation in Lindberg (1983); p. lvi.

Final remarks

At first glance, the *Optics* and the *Dioptrice* may seem like somewhat of an anomaly among Kepler's other works. On closer examination, however, they are clearly part of a whole. Kepler's aim was to understand and formulate the inner workings of Nature through geometrical and mathematical principles, and he found these to be equally applicable to optical systems and to the planetary orbits. At the same time, the *Dioptrice*, perhaps more clearly than any of Kepler's other major works, puts the distinction between "Kepler the philosopher" and "Kepler the mathematician" on full display. The main text is, for the most part, a fairly terse and concise mathematical treatise stripped of excess verbiage. It stands in sharp contrast to the preface, which is ripe with philosophical considerations and commentary on the discoveries enabled by the telescope, often expressed in richly embellished language to which a translation cannot do full justice.

Kepler's two books on optics reinvigorated the discipline and set it on a path towards the vastly more sophisticated telescopes and their instruments now in use. A practical limit to the size of telescope lenses was reached with the 40-inch refractor at Yerkes observatory, completed in 1897, and all major telescopes built since the early 20th century use concave mirrors rather than lenses to focus the light. Even an entry-level modern amateur telescope far surpasses the best instruments made by Galileo, and Kepler and his contemporaries would, no doubt, have been astonished to see the sheer scale and advanced technology of the professional observatories with which research is conducted today. Professional astronomers no longer spend long nights peering through an eyepiece. Instead, they rely on a broad range of specialised instruments to process the light (or, indeed, radiation across the electromagnetic spectrum) and record it on electronic detectors, not unlike those found in consumer electronics such as digital cameras and cell phones. Nevertheless, the basic principle by which an image is formed in the focal plane of a modern telescope is the same as that by which the *pencils* focussed by a lens *paint* a *picture* on a sheet of paper, as described in Problems 43, 88, and 105 of the *Dioptrice*.

One of the two main reasons why telescopes have grown ever larger was already anticipated in the *Dioptrice*: "For a given concave lens, visible objects are shown as brighter or stronger with a larger or wider convex lens than with a smaller one" (Proposition 119). In other words, Kepler recognised that a large objective lens (or mirror) collects more light than a smaller one, which in turn allows fainter objects to be detected and measured.

The other reason, that a larger telescope aperture makes it possible to resolve finer details, could not have been anticipated by Kepler, and in fact, appears contrary to his Proposition 122: "With a narrow part of a convex lens, other things being equal, visible objects are shown as more distinct, and with a wide part they are shown as more confused." Indeed, one of the crucial features of the early telescopes made by Lipperhey, Galileo, and others was that the aperture could be stopped down, allowing light to pass through only the central parts.[82] This was necessary because of the difficulty of achieving the correct figure of the optical surfaces across the entire lens, and it also helped suppress chromatic aberration caused by the wavelength dependency of the refractive index of glass. In the absence of such effects, the smallest angle that can theoretically be resolved by a telescope is inversely proportional to the diameter of the aperture and (for a perfect optical system) a larger aperture will give a sharper image. This follows from the principles of diffraction and interference, which can only be understood through the wave theory of light, unknown to Kepler. Somewhat ironically, this theory represents a departure from the concept of rectilinear propagation of light rays that Kepler had so successfully employed. The corresponding framework of *physical optics* is due to the efforts of Christiaan Huygens,[83] Thomas Young,[84] and Augustin-Jean Fresnel,[85] among others, and the implications for the resolution of a telescope were not clearly formulated until 1835 (by George Airy).[86]

Kepler did not get everything right, and some of his ideas may seem odd to us today. He might have been somewhat puzzled to learn that we remember him chiefly for the three laws of planetary motion named after him and not for his broader theory of regular polyhedrons and harmonic ratios.[87] He does not dwell much on the Keplerian telescope in the

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82 R. Willach, in *The Origins of the Telescope*, p. 93-114.

83 C. Huygens, *Treatise on Light* (1690), transl. by S. P. Thompson.

84 T. Young, *The Bakerian Lecture: Experiments and calculation relative to physical optics* (1804).

85 In H. Crew, *The Wave Theory of Light* (1900).

86 G. B. Airy, *On the Diffraction of an Object-glass with Circular Aperture* (1835).

87 Some insight into Kepler's thoughts on the importance of various aspects of his work (and his attitude towards astrology) may be gathered from this passage near the end of Book IV of

Dioptrice, and he probably did not consider in detail its advantages over the combination of a convex and concave lens, which was then standard. But such is the nature of scientific progress: sometimes, we stumble across ideas that turn out to stand the test of time, even if their merits may not be immediately recognised. Others, which might *a priori* have seemed promising, may instead be destined to end up as historical footnotes or curiosities. Rarely does the advancement of knowledge proceed along a well-charted course, but it is often greatly accelerated, today no less than in Kepler's time, when the power of human imagination is aided by new technological innovations.

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the *Harmonice Mundi*: "An astrologer will seek in vain from the arrangement of the stars at my nativity the reasons for my discovery in the year of 1596 of the proportions between the heavenly spheres; in the year 1604 of the way of seeing; in this year, 1618, of the reasons why to each planet has fallen an eccentricity of a particular size, neither smaller nor larger; in the intervening years, of my demonstration of the physics of the heavens, and of the ways in which the planets move and of their true motions; and lastly of the basis of the influentiality, which is metaphysical, of the heaven on this Earth below."

a scan. Finally, I am thankful to my family for putting up with the many weekends and evenings and even the occasional holiday that I have spent in Kepler's company over the past year.

Let me conclude by stressing that my main motivation and qualification for translating Kepler's book is that of an observational astronomer with an interest in the history of the instruments and methods used in my field. I am neither a historian nor an expert in classical languages, and this will no doubt be apparent in some places in this translation. Nevertheless, to paraphrase Kepler: I hope that the interested reader will forgive my shortcomings and find this effort to be of some value.

IOANNIS KEPLERI
S^æ. C^æ. M^tis. MATHEMATICI

DIOPTRICE

SEV

Demonſtratio eorum quæ viſui & viſibilibus propter Conſpicilla non ita pridem inventa accidunt.

Præmiſſa Epiſtola Galilæi de ijs, quæ poſt editionem Nuncij ſiderij ope Perſpicilli, nova & admiranda in cælo deprehenſa ſunt.

Item

Examen præfationis Ioannis Penæ Galli in Optica Euclidis, de uſu Optices in philoſophia.

AVGVSTAE VINDELICORVM,
typis Davidis Franci.

Cum priuilegio Cæſareo ad annos XV.

M. DCXI.

JOHANNES KEPLER
MATHEMATICIAN OF HIS HOLY IMPERIAL MAJESTY

DIOPTRICS

OR

Demonstration of those things that occur to vision and visible objects by means of the recently invented Conspicilla.

Preceded by Letters from Galileo concerning those new and wonderful things which have been discovered in the heavens with the help of the Perspicillum after the publication of the Starry Messenger.

And also

Examination of the preface by Jean Pena of France to Euclid's Optics, concerning the benefits of Optics in philosophy.

AUGSBURG,
by the press of David Franck.

With Imperial privilege for fifteen years.

M. DCXI

To the Most Reverend and Most Serene Prince And Lord,

Lord Ernest, Archbishop of Cologne,

Septemvir Elector[88] of the Holy Roman Empire, Archchancellor for Italy, Bishop of Liège, Administrator of Münster, Hildesheim, and Freising, Prince of Stavelot, Count of the Rhine Palatinate, Duke of Upper and Lower Bavaria, Westphalia, Angria, etc., Margrave of Franchimont.
My most merciful Lord.

Most Reverend and Most Serene Prince-Elector, Most Merciful Lord: When in recent years the dioptric pipe, certainly not to be counted among common contrivances, was added to the great heap of inventions of this last century, some would contend for the laurels of its invention, while others would devote themselves above all to perfecting the instrument, for the former is chiefly a matter of chance, whereas the latter must be governed by reasoning. Indeed, Galileo achieved a most splendid triumph in demonstrating its benefits for the investigation of astronomical secrets, as someone to whom industriousness had provided a purpose and fortune had not denied success. I, for my part, driven by some honourable emulation, have opened a new field for mathematicians to exercise the power of ingenuity, that is, to employ the principles of geometry in the demonstration of the causes that underpin such sought-after and delightfully various and numerous effects. Because I had, in fact, published the *Optical Part of Astronomy*[89] six years earlier, in which I used this new way of reasoning to demonstrate, for the first time as far as I know, a number of things concerning the means of vision and about optical devices,[90] which remain unshaken up to this day, it seemed fitting that I should demonstrate that the same foundations, upon which I had constructed my account of the means of vision and the effects of simple optical devices, also suffice for the combination of different transparent lenses

88 The Electoral College of the Holy Roman Empire consisted of seven prince-electors (German "Kurfürsten") who elected the ruler of the Empire. One of these was the Archbishop of Cologne.

89 That is, the *Optics*.

90 In fact, Kepler also uses *perspicilli* here, clearly not referring to telescopes (which are not discussed in the *Optics*).

in one pipe: and consequently, that it is not even possible (which is an argument for truthfulness) that this demonstration be accomplished by any principles other than those I have used. And since Euclid had fashioned catoptrics[91] as the kind of optics which deals with reflected light, the name being derived from the principal apparatus of this kind, mirrors, and their wonderful and delightful variety, the name Dioptrics was born for my booklet following this example, because it mainly deals with light refracted by dense transparent media, both natural in the human eye and artificial in a variety of *perspicilla*. By its subject, dioptrics is thus distinguished from catoptrics as one kind [of optics] from another; in such a way, however, that dioptrics comes first and catoptrics follows, above all because catoptrics is concerned with images, the true nature of which cannot be fully understood without knowledge about the eye that must be obtained from dioptrics.

For this reason, I have also revisited the means of vision and the principles of simple lenses, both in order that dioptrics, in a certain sense, might be completed, and also because the principles of the instrument[92] are connected to the human eye and the instrument itself is composed of simple lenses,[93] so that one cannot be explained without the other. Finally, some have judged[94] that I treated these things somewhat obscurely in the *Optics* so that, for many, it is not a lack of intellect but rather the fault of the teacher that hinders them from comprehending what has been written and demonstrated. Therefore, in order to address their concerns, I have presented some things here more briefly, others more extensively, and expressed some in different words; I have listed definitions of the

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91 Johann G. Brengger, in a letter dated 23rd Dec 1604 (KGW vol. XV, Letter #310) had expressed doubts to Kepler about Euclid's authorship of the *Catoptrice*. Kepler, however, dismissed these concerns (KGW vol. XV, Letter #317).

92 That is, the telescope.

93 Also here, Kepler uses *simplicibus perspicillis*.

94 Note to p. 332 in KGW vol. IV: among those who had written to Kepler about the difficulty of the *Optics* were David Fabricius, Michael Mästlin, and Johannes Papius.
From Mästlin's letter, Jan 28th 1605: "I must admit that you sometimes pursued subjects loftier than my intellect and learning could satisfy." (KGW vol. XV, Letter #322)
From Papius' letter, Feb 26th 1606: "If only your Paralipomena [Optics] were as clear as it is ingenious and subtle. In my whole life, nothing so difficult has been presented to me on any mathematical, or I dare say, nearly any philosophical subject matter […] If I were with you, I would be a most troublesome student, always doubting." (KGW vol. XV, Letter #375)

terms I use with geometric liberty and included them in a continuous sequence among the propositions at suitable locations; and I have added more diagrams (which are the true letters of geometers). Even if, by this effort, I have not eliminated all obscurity, I hope that those dedicated to philosophy will, to some degree, forgive my shortcomings and find this effort to be of some value.

Furthermore, I devoted myself to this task chiefly at a time when my mind, numbed by a mournful coldness, was warmed by the most munificent Sun of the presence of Your Most Reverend and Serene Highness, and was awakened from sleep by its gentle urging and repeated encouragements, as if by some Mercury. Finally, the various delightful and ingenious contrivances of your mathematician and noble chamberlain, the esteemed Mr. Johannes Zuckmesser, as well as his most skilful glass polishing, which I saw were bringing remarkable delight to Your R. S. H., have prompted me to emulate his same diligence. But if these reasons alone had not impelled me to dedicate this Dioptrics of mine to Your R. & S. H., then even just this would suffice: that mathematical booklets, as they are far removed from the understanding of the common people and therefore regarded with contempt by them, are offered to no-one more appropriately than to those who are able to judge them; those endowed by nature with a sharp intellect, whose contemplation and love of philosophy have led them to a perfect understanding of these matters. It is unknown to me whether, at this time, you have an equal amongst princes in this understanding. Certainly, among the professors of academies, those who are equal in this judgement are fewer than would be beneficial.

If, in the very numerous dedications of books, the praises of patrons were no more embellished than these, I believe the trust in the virtues of patrons, which dedications have cooked up almost to the point of evaporation, would soon be restored. And to this very end, I refrain from further commemorating the virtues of Your R. & S. H. (as is customary in dedications), lest I would appear to be a cobbler wanting to judge beyond the shoe.[95]

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95 Proverb from the *Natural History*, Book 35, of Pliny the Elder (vol. VI, p. 258 in the transl. by J. Bostock & H. T. Riley, 1857). Cf. "Let not the cobbler go beyond his last".

Furthermore, I hang no other ivy[96] for the reader than to indicate that the book has been approved by so eminent a prince and ordered to see the light of day. And now, I most humbly commend myself to Your R. & S. Highness. Farewell. January 1st of the eleventh year of the seventeenth century: which I pray will be most fortunate for Your R. & S. H. in governance, in the pursuit of wisdom, and in the preservation of health.

Your Most Reverend and Most Serene Highness'

Most Devoted

Mathematician to His Holy Imperial Majesty

Johannes Kepler

96 Roman metaphor - that is, "I shall offer no further assurance".

JOHANNES KEPLER'S PREFACE TO THE DIOPTRICS,

on the benefits and excellence of the recently invented *perspicillum*, and on the new celestial discoveries made through it.

I present to you, dear reader, a booklet which is mathematical, and therefore not so easy to grasp. Not only does it require intellect on the part of the reader, but also an extraordinary attention of the mind and an incredible desire to understand the causes of things. 1

While pondering this, it seemed fitting to discuss some things pertaining to the excellence of the dioptras[97] *or perspicilla, and to their remarkable effect on the advancement of the frontiers of philosophy so that ingenious youths and other cultivators of knowledge, encouraged by this stimulus of utility, may be incited to acquire the principles of the instrument from this booklet.*

Many great examples of the benefits of all of optics were given by Jean Pena of France, the former Royal Mathematician, in the preface to his translations of Euclid's Optics and Catoptrics. However, as important as these may be, they can be considered quite childish compared to what has been revealed during the past two years through the benefit of the dioptras.

And because I have recommended that preface to the reader by mentioning it here, let us now briefly examine its main points, lest I may appear to have knowingly and deliberately promoted also the doubtful and false things that, I cannot deny, are interspersed among the true and splendid ones contained therein. Once I have resolved this, I will then finally add the things that have been revealed at the present time by the new perspicillary science.

In agreement with Pena, I hold the first teaching concerning the heavens as firmly demonstrated from optics: without a doubt, those physicists[98] *are completely mistaken, and indeed also some theologians, who believe that*

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97 The term *dioptra* can refer to astronomical instruments used in antiquity (of various degrees of sophistication), but the literal meaning of the Greek word is similar to that of the Latin word *Perspicillum* (see the introduction) and is used in that sense by Kepler here.

98 Or "natural philosophers".

there are nine or ten transparent spheres surrounding this elemental world, like the white of an egg usually surrounds the yolk, or like the layers of an onion enclose one another. For since the paths of the planets are established
2 *as eccentric by indisputable reasoning, the optician rightly concludes that rays descending obliquely from the stars through these vast revolving orbs (towards the Earth, indeed, which is situated away from the centres of some of the orbs) are going to be refracted according to the laws of optics. And with this being granted, all certainty of observations would be removed, to which experience nevertheless bears testimony. The same also follows from the quite perceptible proportion of the Earth's body to the orb of the Moon. For even if we were to ignore that the orbs are eccentric and placed the Earth at the centre of all of the orbs, the surface of the Earth would nevertheless be at a considerable distance from the centre of the lunar sphere, which it occupies with its own centre. And again, the rays from the stars would intersect the orb of the Moon obliquely as they descend to the surface of the Earth that we inhabit, and it follows that those refracted rays would disturb the certainty of observation.*

Having not yet left the vestibule of this most beautiful demonstration, Pena improvidently pushes the argument too far, removing the division not only between the orbs but also between the air and the ether. By making the substance of the ether the same as the substance of the air we breathe, he teaches with his own misstep how important it is for someone walking in the temple of philosophy to keep the eyes of optics wide open. For with the same argument by which the divisions between the spheres are removed, in turn, the distinction is firmly established between the air and what follows it just above the summits of the mountains – that is, the ether.

For although astronomical observations are not disturbed by some complex manner of mutually intertwined refractions, as the divisions and solidity of the orbs would require if they existed, they are nevertheless disturbed by a certain uniform manner of refractions when the stars approach the horizon. These refractions cannot come from anywhere else except the surface of the air we breathe, to such a degree of certainty, indeed, that in the Optical Part of Astronomy, I was even able to investigate the altitude of this surface above the surface of the Earth. Pena appeals to experience, bringing in an eyewitness, Gemma Frisius, with his

astronomical staff,[99] *who denied having detected any such refractions.*[100] *Of course, Pena did not then know about the remarkable industriousness of the greatest practitioner*[101] *of all, Tycho Brahe, who, partly through extensive efforts and partly through the magnitude and exactness of his instruments, was able to measure that minute refraction which had escaped* 3
notice by the crude instrument of Gemma and the attention of a single and solitary man. In the Optical Part of Astronomy, I also produced witnesses for refraction held in reserve by Brahe, summoned from antiquity and therefore impartial and uncorrupted.[102]

I hear that the esteemed Dr. Helisäus Röslin has suggested that I should solve the problem of the Sun being seen by the Dutch in the northern lands 14 days earlier than it should have been. I have not seen his book[103] *during these tumults.*[104] *However, I point out that I have dealt with this question through refraction by the air in the Optical Part of Astronomy, chapter IV, section 9, page 138.*

Pena devoted the second parts to the teaching concerning the truly eccentric paths of the planets, and he did so rightly. Optics provides very strong arguments for these. We must only be careful to avoid that the same happens to us as what befell the ancients: that, relying much too confidently on one eye of optics for fully perceiving the planetary orbits, we close the other eye of physics and thus, by attributing to optics alone, what had to be attributed equally to both optical and physical reasoning, we stray from the

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99 An instrument related to the dioptra used by Hipparchus. Also known as the "Radius Astronomicus", or Jacob's staff, it is described by Frisius in his *De Radio Astronomico & Geometrico Liber* (1545). Commentary and transl. by B. Goldstein (1987).

100 From Goldstein's translation (p. 173): "Though it is true that images of things which appear in air that is denser seem larger, in fact, they do not become larger as one can see from ordinary experience. For, though the distances between stars near the horizon appear to be greater than when they are high in the sky, nevertheless, when they are measured with the Radius, they do not differ at all."

101 *artifex.*

102 Chapter 4, sect. 10 of the *Optics*. Kepler here discusses possible evidence of atmospheric refraction in the works of Pliny, Ptolemy, Hipparchus, Proclus, and others.

103 From chapter 9, p. 79 of Röslin's *Mitternächtige Schiffarth…* (1610): "I leave it for others to think about that and to provide calculations about it; especially for Mr. Kepler, Mathematician of His Imperial Majesty, who, with his sharp intellect, will know how to adequately handle these matters and provide explanations for them." (my translation)

104 The "tumults" refer to the unrest resulting from the feud between Emperor Rudolph II and his brother Matthias.

goal again. Regarding this matter, see my Optical Part of Astronomy and the Commentaries on the Motions of Mars.

Thirdly, Pena examines the question concerning the order of the planets[105] *by means of optics. He does not reason badly, following Aristotle, that if indeed the Earth remains immobile in its place, it appears improbable that the Sun, Venus, and Mercury would traverse three distinct orbs of unequal size in an equal period of time. Rather, as held by Martianus Capella,*[106] *Campanus,*[107] *and Brahe, and as Galileo most manifestly proves below, if indeed the Sun moves, it is more fitting that they are all carried together in one orb, with the Sun encircled by the epicycles of Venus and Mercury, as if by segments of two wheels, with the Sun like an axle of the wheels. But on the contrary, it is indeed far more likely, as Copernicus held, and as the oldest Samian philosophy held so many centuries ago, that the Sun remains immobile in the middle, and that not only Mercury and Venus travel around it with their respective periods, but also the Earth itself with its companion, the Moon, in the course of one year, and the other three planets similarly with their own periods.*

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105 Pena writes: "Optics teaches that of objects moving at equal speeds, the one that is more distant appears to move more slowly. And since among the three planets - namely, the Sun, Venus, and Mercury - none is slower than the others, what will optics conclude from this? (Even if I remain silent, the matter speaks for itself). Without a doubt, it will assert that the Sun, Venus, and Mercury move in the same orb. For why should it hesitate to pronounce what is not only true but also in agreement with the very wise teaching of Aristotle? Aristotle says that the farther each planet is from the highest part of the heavens, the shorter the time it takes to traverse its orb. This view could have suggested to keen interpreters the position Aristotle would assign to each planet. For if none of these three planets is farther from the highest part of the heavens than the other two, then whether or not the eternal globes of the stars revolve around the Earth, balanced and stationary at the centre of the Universe, as we seem to see, with Mercury and Venus riding on epicycles, then surely these epicycles will revolve in the same orb as the Sun, with the Sun as their centre. Or (as many great minds have conceived, and which is possible according to optics), if the Earth is a star traversing the zodiac in the space of a year, around the Sun, which is stationary at the centre of the Universe, the same epicycles of Mercury and Venus will still have the Sun as their centre. Thus, it is established from optics that the centres of the epicycles of Mercury and Venus are in the same orb as the Sun." (my translation)

106 In: *Libri Novem de Nuptiis Philologiae et Mercurii et de Septem Artibus Liberalibus* ("Nine Books on The Marriage of Philology and Mercury and the Seven Liberal Arts"), Book VIII, §856. Martianus Capella lived in the 5th Century in Carthage, then a province of the Roman Empire.

107 Campanus of Novara (1220-1296) in his *Theorica Planetarum*, actually adopted the Ptolemaic order of the planets (p. 333, Benjamin & Toomer): "From this it will be clear […] that Venus and Mercury are below the sun, as he [Ptolemy] assumed." That is, Campanus believed the epicycles of Mercury and Venus to be located between the Earth and the Sun, not centred on the Sun.

Here again, however, Pena frees himself from the thorny shrubs of intricate reasoning with some damage to the truth. And this argument 4
was indeed constrained by no particularly evident necessity but supported by probability alone. Therefore, lacking confidence in the uncertain argument, Pena timidly abandons the mobility of the Earth, as taught by Copernicus, and instead, with a light blink of the eye of optics, places his trust in some other, very slow motion of the Earth that he has investigated in detail.[108] *Having made this assumption, he thinks it follows that the fixed stars will seem to have been allotted an unequal motion, and that the lack of consistency throughout the ages testifies to the existence of this motion. But, oh Pena, this is not the way to commend the excellence of optics, applying its powers to impossible matters. Bucephalus*[109] *was truly noble, even though he could not imitate the wings of Pegasus. And if someone who has testified that Bucephalus was seen flying were accused of falsehood, this would not diminish the glory of Bucephalus. Too much, oh Pena, does this reasoning of yours depart from the principles of optics; too many things intervene between your assumed optical principle and what you conclude from it. First, concern for the accuracy of those observations that we today bring forward from that deep antiquity did not touch you. Then, you adduce the motion of the fixed stars as if it were something seen with the eyes. But it is very far removed from the perception of the eyes: the astronomer hardly dares, by a combination, which is not even such a very tight one, of three very subtle reasonings into one, to eventually declare in which position of the zodiac any fixed star may be located in any century.*

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108 Pena writes: "I proceed to explain something that cannot be denied in any way. The optical law is as true as it is brief: of objects moving with equal speed, the one that appears to move more slowly is farther away. The fixed stars, however, move with equal speed (for the astronomical hypotheses teach that celestial motions are uniform, even if they appear unequal). Yet, they seem to progress unevenly, as the observations of different times show. At the beginning of the Calippic periods, that is, during the time of Alexander the Great, the fixed stars traversed one degree of the sky in seventy-two years. In Ptolemy's era, they took one hundred years. In the time of Al-Battani, they took sixty-six years. In this century, they progress at almost the same rate as they did in the early times of Calippus. From this, it is evident that the Earth was farthest from the heavens during Ptolemy's time, came closest during the era of Al-Battani, and in our age is at a moderate distance from both extremes […] I assert solely from Optics that the Earth progresses from place to place by some motion, and that this progression over time is very slow, as scarcely any inequality of such motions can be perceived in less than four hundred years." (my translation)

109 The horse of Alexander the Great.

Finally, what you call the motion of a fixed star from the equinoctial point is, instead, truly the retrocession of the equinoctial point from the fixed star, where the equinoctial point deviates greatly from Pena's conception. For what else is the equinoctial point but the imaginary intersection of two imaginary circles, one of which is understood to extend from the Sun through the Earth's orbit up to the highest ether, and the other is similarly understood to extend from the centre of the Earth through the terrestrial equator up to beneath the fixed stars, and not for every position of the Earth, but only when the Earth is at the equinoctial points. But this matter is taught in astronomy and in my Commentaries on Mars. Thus, Pena, poorly informed on account of such uncertainties, needlessly attributes some new motion to the Earth, and a very slow one at that, by which
5 *motion the Earth is banished from the centre of the world. It would have been better if he had kept the motion of the Earth that the most excellent practitioners introduced: a motion that, in fact, carries the Earth around the centre of the planetary world at a distance as large as the semidiameter of the sphere of the Sun is imagined to be.*

I cannot neglect to also expose this misstep of Pena's in his preface, where he accuses Copernicus' refutation regarding the Ptolemaic hypothesis of the Moon[110] *of falsehood. For this allegation greatly harms the reputation of such a great practitioner among those who are inexperienced in such matters. Copernicus refuted Ptolemy, whose suppositions place the Moon in its bisected state nearly twice as close to the Earth as when it is full. Copernicus used an optical argument, and a most excellent one indeed, to show that this is false: the Moon should then also appear nearly twice as wide*

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110 Pena writes: "Ptolemy asserted that the greatest distance of the Moon from the Earth is sixty-four Earth radii, and the smallest is thirty-three, and he did so for good reasons. However, Copernicus, certainly a remarkably shrewd man but less perceptive in this matter, criticised these distances when he examined them, arguing that since the ratio of the greatest to the smallest distance is nearly double, the diameter of the Moon at perigee should appear twice as large as at apogee. This argument of Copernicus is flawed: for if there are two equal magnitudes, one at a hundred and the other at two hundred paces from you, it is not necessary for the nearer magnitude to appear twice as large as the more distant one. No demonstration in optics or geometry teaches this. On the contrary, the art of optics dictates that equal magnitudes, unequally distant from the eye, have a smaller ratio of the angles which they appear to subtend than of their distances. Hence, it can also be concluded that Ptolemy's distances may indeed be true, and yet the apparent diameters of the Moon may not differ significantly." (my translation)

when bisected, compared to when it is full,[111] *whereas experience testifies to a constant or only slightly variable diameter. But now Pena, having abused the subtlety of an optical axiom brought forward by Copernicus, which is recalled as Number 67 in the present propositions, improperly makes light of the argument. For even granting that the apparent diameters of the Moon are not precisely in inverse proportion to the distances, then what? If, nevertheless, they are nearly so, then the argument of Copernicus surely remains valid, does it not? The commander of an army may deny that he is able to storm a city in which there are ten thousand garrison soldiers unless he has fifty thousand. But if he is just one individual short of this number, then surely he will not hesitate about the assault, will he?*

But I return to the category of teachings which Pena demonstrates very truthfully from optics, of which the fourth is this: that it is very correctly argued from optics that there is no sphere of fire above us; and with this foundation undermined, none of the philosophers of this time can be unaware of the great collapse that follows in Aristotelian meteorology. For if there were fire beneath the heavens, of either the visible or invisible kind, there would undoubtedly be great refraction of rays. For fire seeks higher regions because it is of a less dense substance than air. Just as an inflated bladder emerges from the depths of the water, lifted upwards by the weight of the water; so also, the fiery substance obtains the cause of its ascent from its tenuousness, being pushed upwards by the denser body of the surrounding air.

Therefore, when physicists state that a transparent substance, more **6**
tenuous than our air, is surrounding us above our heads, they will not be able to deny that the rays from visible objects, when passing through the adjacent surfaces of the denser air and the more tenuous fire, are refracted wherever they pass obliquely. However, all except one pass obliquely to the location of the observer. Thus, there would be great refractions of the rays from all directions.

The power of the argument can be illustrated by an experiment as clearly as if the effect were seen directly. Let the Sun shine against a wall and let a censer with live coals be placed in between: if the air is calm,

111 *De Revolutionibus*, Book IV, sect. 2. Copernicus indeed makes this argument, but also notes that the large difference between the distances in the Ptolemaic system should produce corresponding differences in the lunar parallax. Pena does not address this point.

something like a stream of fiery substance will ascend straight from the censer, unmixed with smoke; but if a slight breeze blows, that stream will deflect slightly to the side, yielding to the wind, yet still gushing upwards with its undulation. You will not perceive this stream of fire with your eyes, as it is transparent and lacks all colour. But if you look at the opposite wall, you will see the shadows of objects placed behind the coal towards the Sun tremble as these shadows are transported through the stream of fire. Trembling, indeed, is a semblance of motion. Therefore, the rays of the Sun that surround the shadow tremble because they are broken as they pass through that fiery ebullition, and this happens variously depending on the various transformations of the surface of that fiery stream. This varying deflection of the rays at the surface of the ebullition causes an inconstant incidence of the deflected or refracted rays on the wall and an inconstant, that is, a trembling, projection of the shadow. Therefore, from this experiment, it is evident that the rays of light are perceptibly refracted at the surface of a fiery substance, however invisible it may be. Therefore, no such fiery substance is spread out under the heavens and overhanging our heads, neither fluctuating nor calm, for observers of the stars perceive neither any trembling refraction of the stars and a change of their place nor any constant one commensurate with the nature of a fiery sphere. In short, there is no refraction other than the one that belongs to the surface of the air.

Again, Pena handles this very strong argument carelessly, and while he
7 *shakes the walls of the fiery sphere, he injures himself with the excessive force of this battering ram. He thinks it will be conducive to the strength of the argument if he admits no refractions*[112] *of the stars at all. Accordingly, he does not hesitate to also diminish faith in the observations of astronomers adduced by Witelo. Witelo had stated that the rays of light are refracted, and that this is discernible in the Moon, whose latitude is often seen to differ from what the tables of motions allow. Pena objects*[113] *that the cause is not refraction but rather parallax, an effect known to astronomers. By Hercules,*

112 While "refraction" is typically treated as uncountable in modern texts, I adopt Kepler's usage here.

113 Pena: "It is not necessary that, if the Moon appears closer to the pole when it touches the horizon than when it reaches the meridian, any refraction should intervene, nor that a double medium should be involved at all. For who, with even moderate knowledge of Astronomy, does not know that the Moon, due to the perceptible relation of its orb to the size of the Earth,

what a perplexing affair! For they both employ a valid principle, and both demonstrate it impertinently, and while attempting to demonstrate it, they go astray regarding related matters. Witelo correctly states that a bending of rays from the stars occurs due to the density of the air. And he also states correctly, but fortuitously, that this is detected in the Moon. But he is greatly mistaken when he presupposes that the unrefracted locations of the Moon could be determined with very great certainty from the calculations of his time, and thereby established a [sufficiently accurate] reference for comparison with the observations and for detection of refractions through them. Therefore, I would not readily say that refractions of the Moon were detected by anyone before Tycho Brahe, not only because of the uncertainty of the ancient calculations but also because of the negligence of earlier observers. But Brahe detected refractions not only for the Moon, which is more difficult due to its varied and rapid motion, but especially for the fixed stars. Yet even for the Moon alone, although its calculation is not very reliable, the refractions could be easily detected [by him]. So much for Witelo's delusion. Let us now also examine Pena's criticism. He correctly defends the point that no refractions occur due to a fiery sphere; however, he falsely adds that they do not occur at all, not even due to the air. Finally, he ineptly confronts Witelo's argument, although, as mentioned, it is useless and in ruins, by attributing to parallaxes what Witelo attributed to refractions. Yet, astronomers have learned that the effects of these two things are opposite. Refraction raises the Moon, while parallax lowers it. Pena does not consider this. But as I mentioned, there is no harm even if Pena does not refute Witelo's refractions, for they are the effects of air, **8**
not fire. Hence, they firmly establish the denser surface of the air, as mentioned above, whereas they indeed thoroughly shatter and eliminate the more tenuous region of fire, as Pena wishes. Thus, the pre-eminence of optical demonstrations shines forth on both sides, both in establishing the distinction between the air and the ether and in removing the fictitious sphere of fire.

In the fifth place, Pena exposes the great ignorance that will hold physicists back regarding the matter, location, and effects of comets, unless they have entered the schools of optics, and he shows what this discipline

is almost always seen in different locations in the sky than where it actually is? Therefore, this cause should be attributed to parallax, not to refraction."

teaches about such portents of nature to those who do not disdain it.

Accordingly, I again point this fact out to the reader: that it is taught from optics that the cometary, or hairy, bearded, and tailed stellar bodies, are entirely translucent, by the argument that they turn their tails away from the Sun.

Secondly, it is also true that those translucent bodies are denser than the ether in which they roam. Thirdly, it is also true that we learn many things about the location of comets from the proportion[114] *of their motions, and most hold it for certain that they travel in the highest ether above the Moon. But the fourth point that Pena adds is dubious, namely whether the laws of optics imply that there is a force of heating in comets, such that when the rays of the Sun are refracted as they enter and leave the cometary body and are forced towards the apex of a cone, the power of ignition is conceived through this compression. For, even if I grant that the rays are gathered into a cone in this way, the violence of inflammation will follow nowhere except at the very apex of the cone, in the depths of the ether. But what does this have to do with that heat which is induced here on Earth?*[115] *Next, that conspicuous tail of the comets is not the actual cone of the rays, which has the body of the comet as its base, but if we assign much to this speculation, the tail is a new cone, beginning where the other cone, whose base is in the body of the comet, ends in an apex. The laws of optics teach that this apex occurs just after the body of the comet. Then the rays of the*

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114 Kepler here, as elsewhere (e.g. in the *Bericht von dem im Jahre 1607 Erschienenen Kometen*, in *De Cometis Libelli Tres*, and in the *Astronomia Nova*) uses the term *analogia motus*, in the sense "ratio" or "proportion" of the motions. In *Harmonice Mundi*, Book III (KGW vol. VI, p.193), Kepler makes it clear that he uses the Greek word "Αναλογία" in the sense "Proportion". Pena does not use this specific term, but his argument is again based on the assumption that all celestial bodies move with a constant, uniform motion so that differences in their apparent motions can be translated to differences in their distances. This is what Pena writes: "Nor can I easily persuade myself of what the schools assert, namely, that a comet travels in the elementary region below the Moon. Nor does optics always recognise this as true. But from the motion of comets, which is sometimes slower than the motion of the Moon, sometimes faster, it follows that some comets travel far above the Moon, for among things moving with equal speed, those which appear to move more slowly are further away."

115 Pena: "Indeed, the cause of heat induced by comets is also derived from this. For optics teaches that fire can be generated by the concurrence of refracted rays upon the encounter with a transparent body denser than air, as demonstrated by glass spheres and water-filled flasks, through all of which the refracted rays of the Sun ignite a fire on the other side. What wonder is it, then, that the rays of the Sun, passing through the transparent body of the comet, denser than air, induce great heat through the air?"

Sun, insofar as they make up that conspicuous tract which we call the tail, once again diverge. However, ignition arises not from separation but from the intersection of rays. Therefore, there is no power of ignition in the tail; but if there is any, it is at the intersection of the rays closest to the body, 9
whence such a tail begins.

Although, as I said, this reasoning of Pena's about the effect of a comet is dubious, I am still far from declaring that it should be completely disregarded; rather, I commend it as most noble to everyone, and I believe that this most profound mystery of Nature concerning the tails of comets may be unravelled from it. I wrote something about this matter in a German description of the comet that shone brightly in the year 1607,[116] *which I had also adorned with a most beautiful demonstration in Latin of the straight trajectory of the comet through the depths of the ether. But my hopes were thwarted by the printer, and the booklet remains in the chest, awaiting another opportunity.*[117]

A sixth utility of optics recalled by Pena is that of shattering the opinion of the Aristotelians[118] *about the Milky Way: Pena teaches, from optics, that the Milky Way is poured around us in the ether itself, far above the Moon, since it reveals no change to the eyes in its position with respect to the fixed stars when observed from different locations on Earth or at different positions relative to the horizon. Without a doubt, even those who have until now admired and valued Aristotle's Meteorology will acknowledge this great benefit of optics. However, the discoveries about the Milky Way made by Galileo with the help of the perspicillum now render this reasoning of Pena's obsolete.*

Next in Pena's preface follow some unseemly remarks, by which I urge the student of optics not to be swayed. Witelo is absolutely correct in holding that vision occurs through the reception of rays, as I have confirmed through very clear experiments. Great glory was claimed by opticians because of

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116 Later to be known as Halley's comet; see KGW vol. IV, p. 428.

117 The Latin text was eventually published in 1619, *De Cometis libelli tres.*

118 From the beginning of Aristotle's *Meteorology* (translation by E. W. Webster in *The Complete Works of Aristotle*): "There remains for consideration a part of this inquiry which all our predecessors called meteorology. It is concerned with events that are natural, though their order is less perfect than that of the first of the elements of bodies. They take place in the region nearest to the motion of the stars. Such are the Milky Way, and comets, and the movements of meteors."

their mutual agreement in opposing the Aristotelians, who defended the emission of rays [from the eye].[119] *It is, therefore, regrettable that Pena tarnishes that glory of the opticians with contrary testimony, especially since Pena himself is an optician and also praises optics in this very preface. However, let the cultivator of philosophy ponder that the investigation of abstruse matters is not a task of a single century; it often happens that the truth appears almost stealthily and, offended by the negligence of philosophers, suddenly withdraws again, not deeming humans worthy of an unadulterated look at itself unless they are diligent and industrious. Add to this that Pena, because of esteem for the ancients, follows a misguided approach to philosophy and attributes undue importance to the fact that Euclid, whose work Pena translated anew and presented in that booklet, employed the emission of rays. I therefore respond to Pena with words adopted from his own mouth:*

"I want a physicist to be anything but credulous, and therefore to be experienced in optical demonstrations; to carefully examine Euclid (Pena himself had spoken about Witelo) and other opticians and to believe only as much as he sees demonstrated by them. Euclid was a man who was second to no one in learning and erudition,[120] as shown by his writings. However, as is a common lot at the emergence of new sciences, he held preconceived opinions, which he presented as axioms for his demonstrations, such as the notion that sight occurs through rays hastening from the eyes to the object seen. Yet, this is no more necessary than if you were to say that vision occurs through the reception of rays."

These words, I say, should, in my opinion, be thrown back at Pena at

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119 It is not entirely clear which Aristotelians Kepler has in mind here: Aristotle himself rejected the extramission theories of Plato and Empedocles, e.g. in *Sense and Sensibilia* 438a25: "It is, to state the matter generally, an irrational notion that the eye should see in virtue of something issuing from it." (p. 696 in *The Complete Works of Aristotle*, vol. I). Nevertheless, some peripatetics may have been more inclined towards the extramissionist view (B. Bakhouche, 2018; S. Berryman, 1998).

120 Pena: "I want a physicist to be anything but credulous, and therefore to be experienced in optical demonstrations, to carefully examine Witelo and other opticians, and to believe only as much as he sees demonstrated by them. Witelo was a man not inferior to Euclid in learning and erudition, as shown by his writings, but he suffered from the weakness, common to all times, that he held preconceived opinions which he often presented as demonstrations, such as the notion that vision occurs through the reception of rays. Yet, this is no more necessary than if you were to say that it happens by emission. And Euclid openly teaches in this booklet that vision occurs by rays hastening from the eyes to the objects seen." (my translation)

this place. For in certain demonstrations, it makes no difference which of the two is true, and you will see that both are used indiscriminately by me in propositions 3 and 19. However, the following distinction should be noted: if we are concerned with the nature of a luminous object, it is expedient for us to speak clearly and to insist on teaching nothing other than emission of rays from luminous points. But if we are talking about the perception of luminous objects and the deceptions of sight, those deceptions themselves often invite us to speak deceitfully and assume that the rays are emitted from the eye when, in reality, the rays are received into the eye.

Regarding the reasons why a single thing may be seen as one with two eyes, Pena correctly refutes Witelo's false reasoning, but he incorrectly praises the equally false reasoning of Galen.[121] *Galen used optical terminology but hardly in accordance with optical laws: he reasoned as if pyramids of vision, formed by the very act of seeing and extending continuously to the eyes from the object seen, as if from a common base, became something real and material which could be twisted away from the object seen by turning the eyes. However, you will find the true cause below in proposition 62.*

The explanations of halos, rainbows, mock suns, and mock moons must be sought from the science of optics. Aristotle himself already saw this long ago, and those things which are still lacking in Aristotle's Meteorology **11**
cannot be supplied from elsewhere.

I had also considered appending a booklet about the rainbow here, which would serve as a supplement to Aristotle's inquiries on the rainbow.[122] *However, the true causes of mock suns were still lacking, and these are intertwined with the causes of the marvellous rainbows. Therefore, for the time being, I abandoned this business.*

Since the science of optics benefits natural philosophy so greatly, Pena quite rightly expects still more from optics, in Porphyrian[123] *theology and*

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121 *On the usefulness of the parts of the body,* chapter X (p. 493-503 in May's transl.), and *On the doctrines of Hippocrates and Plato*, Book VII, sect. 5.

122 The rainbow was among the topics discussed by Kepler and Harriot in their correspondence.

123 Porphyry of Tyre, c. 234 - 305 AD. Pena references the text *On the Cave of the Nymphs in the Odyssey* (Lamberton 1983): "Porphyry, in the book he wrote about Homer's cave, says that spirits and geniuses have bodies, but very tenuous, airy, and humid ones […] For just as we do not see our breath in the summer because of its tenuousness but do see it in the winter, Porphyry thinks that these spirits, like a kind of breath, are not seen during the day because of the heat but are seen at night due to the opposite reason. Thus, Porphyry attributed the presence

magic, and in uncovering sleights of hand[124]. *And the Natural Magic*[125] *of Giovanni Battista della Porta promises more than a few things for the reader to undertake, and they will see that the science of optics extends wonderful benefits to all of human life.*

Thus far, let then Pena have been heard by us, speaking about the excellence of optics and concluding most wisely about its astounding power to uncover the phenomena of the natural world.[126]

Now it is time for me to make good on my promises and discuss those marvellous new wonders of the natural world we have learned about from this part of optics, which we call dioptrics, with the help of its subject, the perspicillum, in a short period of time. Whatever we have so far presented, revealed through the benefit of optics and brought forth from Pena, may indeed seem quite childish in comparison.

Galileo's Starry Messenger is now widely accessible to everyone, along with my Conversation, such as it is, with that Messenger and my little Narrative confirming the Starry Messenger. Let, therefore, the reader briefly ponder the main points of that Messenger and the nature and significance of the things laid bare through the benefit of that perspicillum of which I demonstrate the principles in this booklet. Visual experience bore witness to the fact that there is a luminous body in the heavens, which we call the Moon, and it was demonstrated through optical reasoning that this body is round. With some further reasoning built upon optical foundations, Astronomy established its altitude above the Earth to be

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of spirits to material and physical causes, which optics shows to be neither true nor natural, but mere hallucinations." (my translation).

124 Pena here embarks on a long discussion of various kinds of deceptions through the use of mirrors that might be employed by tricksters, pretending to be summoning demons and spirits. Pena writes: "…acknowledging that spirits and ghosts can be summoned and displayed by witches, I say that many things are accomplished through incredible human cunning and deceit, which the unlearned attribute to sorcery but could deceive anyone except those skilled in optics. For who wouldn't be amazed at the promises of witches, who claim they can summon the spirits of any deceased person? For these secret rites, they employ a mirror cleansed with sacred prayers, which they say can attract spirits. The entire matter seems suspicious to me, and it is possible that some great cunning is at work."

125 The *Magia Naturalis* was first published (in four books) in 1558 and extended to 20 books in 1589. An English translation, *Natural Magick*, was published in 1658. Optics is treated in Book 17, "Wherein are propounded Burning-glasses, and the wonderful sights to be seen by them."

126 *rerum natura* - also often translated (literally) as "the nature of things", as in the famous poem by Lucretius.

about sixty semidiameters of the Earth. Various spots were apparent on that body, and this led to the obscure opinion of a few philosophers, as it was recounted by Hecataeus in the fables about the island of the Hyperboreans,[127] *that alternating likenesses of mountains and valleys, water and continents were seen there. But now the perspicillum places* 12
all of these things so clearly before our eyes that anyone who enjoys such a view must be quite timid in order to still believe that their existence must be doubted. Nothing is more certain than that the southern parts of the Moon abound with numerous and immense mountains, while the northern parts, which are lower, receive water flowing from the south into vast lakes. The teachings, which Pena had previously presented as obtained through the benefits of optics, were demonstrated with only tenuous support of visual experience, deriving their origin from long interconnected arguments, such that they might rather be ascribed to human reasoning than to the eyes. But now our very eyes are being led into view of hidden things as if a new door to the heavens had been opened. And to those who may now find it pleasing to extend the power of reasoning beyond these new observations: who does not realise how far the boundaries of the contemplation of nature will be extended, as we ask for whose benefit there are regions of mountains and valleys and vast expanses of sea on the Moon, and whether some creature, more ignoble than man, might not be imagined to inhabit those regions?[128]

No less does this also allow that question to be settled, which is nearly as old as philosophy itself, and which is entertained today by the noblest minds: namely, whether the Earth can move (as the theoretical science of the planets greatly desires) without overthrow of everything heavy or without disturbing the motion of the elements. For if the Earth were banished from the centre of the world, some fear that the waters would flow back towards

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127 Hecataeus of Abdera, c. 360 BC - c. 290 BC. The Hyberboreans were a mythical people inhabiting the far North. Hecataeus' account of them is discussed by Diodorus of Sicily (English translation by C. H. Oldfather), Book 2, 47 1-5: "They also say that the moon, as viewed from this island, appears to be but a little distance from the earth and to have upon it prominences, like those of the earth, which are visible to the eye."

128 The passage could also be translated as: "…and whether some creature, no more ignoble than man, might be imagined to inhabit those regions." However, the translation adopted here seems more consistent with the *Dissertatio*, in which Kepler argues (p. 43 in Rosen's translation) that "we humans live on the globe which by right belongs to the primary rational creature, the noblest of the (corporeal) creatures".

the centre of the world and leave the globe of the Earth desolate. However, we see that there is also an abundance of water on the Moon, filling the depressed cavities of that globe. And although that globe is carried around in the very ether, not only outside the centre of the world but also away from our Earth, nothing prevents the abundance of lunar waters from tending towards the centre of its body and continuing to adhere to the globe of the Moon. Accordingly, optics also reforms the doctrine of heavy and light[129] *by this example of the lunar globe, and in this regard also confirms my introduction to the Commentaries on the Motion of Mars.*[130]

The supporters of the Samian philosophy (for this epithet may be used to refer to its founders, Pythagoras and Aristarchus of Samos) also have in
13 *the Moon a ready defence against the apparent immobility of the Earth. For we are taught by optics that if any of us were on the Moon, he would perceive the Moon, his dwelling, as entirely immobile, while our Earth, the*

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129 That is, the Aristotelean idea that heavy things (earth, water) have a natural tendency towards their natural place at the centre of the Universe, which is also the centre of the Earth, or downwards, and the opposite for light things (air, fire). See *Physics*, Books V and VIII, and *On the Heavens*, Book IV. In sect. 3 of the latter (310b2), Aristotle writes: "If one were to move the earth to where the moon now is, the various fragments of the earth would each move not towards it but to the place in which it now is." (translation by J. L. Stocks in *The Complete Works of Aristotle*). This is exactly the opposite view of what Kepler states in the *Astronomia Nova*.

130 In the *Astronomia Nova*, Kepler writes (p. 24 in Donahue's translation):
"The true theory of gravity rests upon the following axioms:
Every corporeal substance, to the extent that it is corporeal, has been so made as to be suited to rest in every place in which it is put by itself, outside the orb of power of a kindred body. Gravity is a mutual corporeal disposition among kindred bodies to unite or join together […] Heavy bodies (most of all if we establish the earth in the centre of the world) are not drawn towards the centre of the world because it is the centre of the world, but because it is the centre of a kindred spherical body, namely, the earth. Consequently, wherever the earth be established, or whithersoever it be carried by its animate faculty, heavy bodies are drawn towards it […] The orb of the attractive power in the Moon is extended all the way to the Earth and calls the waters forth beneath the torrid zone, in that it calls them forth into its path wherever the path is directly above a place."
To be sure, Copernicus had already hinted at similar ideas, as he writes in chapter I of *De Revolutionibus* (from the translation by C. G. Wallis, p. 19): "I myself think that gravity or heaviness is nothing except a certain natural appetency implanted in the parts by the divine providence of the universal Artisan, in order that they should unite with one another in their oneness and wholeness and come together in the form of a globe. It is believable that this affect is present in the sun, moon, and the other bright planets and through its efficacy they remain in the spherical figure in which they are visible, though they nevertheless accomplish their circular movements in many different ways."

Sun, and all the other bodies would appear to move.[131] *This, indeed, is how the principles of visual experience are constituted.*

Pena previously recalled how astronomers, using optical principles, elevated the Milky Way, by great effort of reasoning, from the elementary world where Aristotle had placed it to the highest ether. But now, through the benefit of the recently invented perspicillum, the very eyes of astronomers are led straight to a direct perception of the substance of the Milky Way so that anyone who delights in this spectacle will be compelled to acknowledge that the Milky Way is nothing but a heap of the tiniest stars.[132]

What a nebulous star might be was entirely unknown until now. But when the perspicillum is directed towards any such nebulous convolution[133] *(as Ptolemy calls it), it again reveals, as in the Milky Way, two, three, or even four very distinct stars assembled within the narrowest of spaces.*

Who would have believed that the number of fixed stars is, in fact, ten or perhaps twenty times greater than what is included in the Ptolemaic catalogue[134] *of the fixed stars, were it not for this instrument? And whence, I ask, can we seek evidence that the boundary or limit of this visible world is the very sphere of the fixed stars, if not from this multitude of fixed stars detected through the perspicillum, which is like a certain vaulting of the mobile world?*

Furthermore, how much the astronomers may err when determining the sizes of the fixed stars, unless they inspect the stars anew through the use of the perspicillum, can likewise be seen in the writings of Galileo. Below, we will also produce a letter from a certain German as evidence. [135]

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131 This idea was explored in the *Somnium*, in which Kepler describes a journey to the Moon and gives a detailed account of the motions of the celestial bodies as they would appear to its inhabitants.

132 Compare with the phrase in the *Sidereus Nuncius*: "For the Galaxy is nothing else than a congeries of innumerable stars distributed in clusters" (translation by A. van Helden).

133 In his translation of the Almagest, G. J. Toomer uses the term "nebulous mass" for the Greek νεφελοειδής συστροφή. E. S. Carlos uses "nebulous ball". I have chosen to adopt Kepler's Latin word "convolutio" directly here.

134 Ptolemy's catalogue (as reproduced in Toomer's translation of the Almagest) contains 1025 unique entries (and three duplications), of which 5 are noted as "nebulous"; two of these are stellar clusters (Praesepe and h Per) while the other three are probably stars (F. Verbunt & R. H. van Gent, 2012).

135 See the remark in Galileo's last letter (March 26th 1611) below: "…the disc of the Dog Star appears no larger than a fiftieth part of Jupiter's disc". (The letter of Simon Marius only mentions that "the opinion of those who believe the celestial bodies to be of such monstrous bulk will be refuted").

But all wonder is surpassed by that part of the Starry Messenger in which another world, as it were, the Jovian world, detected through the benefit of a perspicillum of the greatest perfection,[136] *is described to us. And the mind of the philosopher considers, not without astonishment, that there exists a certain huge globe, the bulk of which equals fourteen terrestrial globes*[137] *(unless this perspicillum of Galileo's will shortly reveal something more accurate to us than the Brahean measurements), and*
14 *that four moons, not dissimilar to our Moon, travel around this globe. The slowest does this in a period of fourteen of our days, as Galileo has reported, and the one next to it, the most prominent of them all, in a period of eight days, as I discovered last April and May,*[138] *while the other two take a much shorter period of time.*[139] *Here, reasoning summoned from my commentaries on Mars to a similar cause induces us to establish that also the globe of Jupiter revolves very rapidly, undoubtedly faster than in the period of one of our days, so that the perpetual revolutions of those four moons follow in the same direction as the rotation of this largest globe around its axis. And in those regions, our Sun, the common hearth* [focus] *of both this Terrestrial and that Jovian world, which we reckon*

Kepler wrote to Vicke (Letter #619): "I suppose [Galileo], as he usually does, is comparing the discs, of which the diameters are in a ratio of seven to one." The apparent diameter of Jupiter is between 30" and 50" (depending on the distance), so this would then imply a diameter of about 4"-7" for Sirius. Tycho had estimated stellar diameters between 20" and 2' (*Progymnasmata*, p. 481-482). In reality, these measurements were all spurious: the disc of any fixed star is far too small to be resolved even with telescopes much better than those available to Galileo and Marius (Graney 2010). The first direct measurements of stellar diameters were made in the early 20th century (0.047" for Betelgeuse; A. A. Michelson & F. G. Pease, 1921).

136 Galileo explains that he discovered three (and eventually four) moons of Jupiter with a new "superlative instrument" (in van Helden's translation, from *admodum excellens*), of superior quality compared to those he had previously been using.

137 The value is from Tycho's *Progymnasmata*, p. 476. The actual volume of Jupiter is about 1300 times that of the Earth. Tycho's pre-telescopic estimate of Jupiter's apparent diameter (2'45") was about three times too large, but his assumed mean Earth-Jupiter distance (3990 Earth radii) was about 30 times too small. In the *Epitome Astronomiae Copernicae*, Book IV (p. 39 in the translation by C. G. Wallis), Kepler says that he had observed Jupiter "to occupy approximately 50 seconds of arc" at opposition, very close to the modern value. However, the first accurate determination of the absolute distance scale of the solar system was not made until 1771 (by J. Lalande), using observations of the Venus transits in 1761 and 1769.

138 According to the notes by F. Hammer (KGW vol. IV, p. 515), there are no extant records of these observations by Kepler in April/May 1611 other than these remarks.

139 Modern values for the orbital periods of the four Galilean satellites are: 16.69 days (Callisto), 7.16 days (Ganymede), 3.55 days (Europa), and 1.77 days (Io).

to be as large as 30 minutes [of arc], fills barely six or seven minutes; and within a period of twelve of our years, having traversed the zodiac, it will be seen again among the same fixed stars. Thus, as the creatures dwelling on that globe of Jupiter contemplate those very fast laps of the four moons among the fixed stars, and as they observe the rising and setting of both the moons themselves and the Sun every day, they would swear by the Jupiter Stone[140] *(for I have recently returned from those regions) that their globe of Jupiter remains motionless in one place, while the fixed stars and the Sun, which truly are at rest, revolve around their domicile with a multiplicity of motions, no less than the four moons. From this example, the supporter of the Samian philosophy will now have learned, much more than from the previous example of our Moon, how one can reply to someone who objects that the teaching of the Earth's motion is absurd, citing the testimony of our sight. Oh, Perspicillum of great knowledge, more precious than any sceptre: is not the one who holds you in his right hand ordained as king and lord of the works of God? Truly,*

Thou makest all that is above his head, the great spheres with their motions, bow before his genius.[141]

If someone who is a little more favourably disposed towards Copernicus and the torches of the Samian philosophy now hesitates only because of ***15*** *doubts about how it could be possible for the Earth, as it is treading a path through the aethereal fields in the midst of the planets, to have the Moon adhere to it so consistently, like an inseparable companion that also orbits the globe of the Earth itself, in the manner of a faithful little dog that now runs ahead of its travelling master, now strays around him on either side: then let him behold Jupiter, who, as demonstrated by this perspicillum, has not just a single companion as the Earth does, according to Copernicus, but in fact draws no less than four companions with him, which never abandon him, while each of them continually traverses its own circular path. But*

140 The pun is a reference to a solemn oath, said to have been sworn by the Romans when signing the first treaty of Carthage: "In the case of the first treaty the Carthaginians swore by their ancestral gods and the Romans, following an old custom, by Jupiter Lapis [Stone]" (from *The histories of Polybius*, Book III, 24; p. 61 in vol. II of the translation by W. R. Paton). The remark in parentheses may anticipate the *Somnium*, although Kepler there describes a journey to the Moon, not Jupiter.

141 This is a line from the hymn included at the end of the *Mysterium Cosmographicum*. I have adopted Duncan's translation from *The Secret of the Universe* (p. 225).

enough has been said about these matters in the Conversation with the Starry Messenger. It is time for me to turn to those things which have been revealed through the use of the perspicillum after the publication of the Starry Messenger and after my Conversation with it.

A year has now passed since Galileo wrote to Prague that he had observed something new in the heavens in addition to what had previously been discovered. And lest there be someone who, out of a spirit of disparagement, would claim to have made the same observation earlier, he allowed a period of time for anyone to divulge whatever new things they might have seen; meanwhile, he himself described what he had found with letters transposed in the following manner:

S m a i s m r m i l m e p o e t a l e u m i b u n e n u g t t a u i r a s.

From these letters, I composed a semi-barbaric verse, which I included in my Narrative in the month of September of the previous year:

Be greeted, twinned middle-held, offspring of Mars.[142]
[Salve umbistineum geminatum Martia proles.]

But I strayed very far from the meaning of the letters, which contained nothing about Mars. And lest I detain you, reader, behold the revelation of the riddle, in the words of its author, Galileo himself:[143]

Di Firenze li 13 di 9bre 1610.

Ma passando ad altro già che il S. Keplero hà in questa sua ultima narrazione stampate le lettere che io mandai à V. S. Ill.ma trasposte, venendomi anco significato, come S. M.a ne desidera il senso: ecco che io lo mando a V. S.

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142 The oddness of the English translation reflects that of Kepler's original "semi-barbaric" verse, and in particular the fact that *umbistineum* is not a real Latin word. Kepler might have derived *umbi-* from *umbo*, "knob", or from *umbilicus*, "navel / middle" and *(s)-tineum* perhaps from *teneo* – "hold, keep". Carlos suggests that "this is some German word with a Latin ending", but this seems unlikely, considering that Kepler got the riddle from Galileo. Caspar renders *umbistineum geminatum* as "double knob" while Koestler has "burning twin", possibly from *uro*, "burn". I am grateful to Vincent Hunink and Frank Verbunt for their suggestions and ideas here.

143 Galileo's letters are reproduced in Italian here, as in the original text, followed by English versions of Kepler's translations.

Ill.ma per participarlo con S. M.a col S. Keplero, et con chi piacera à V. S. Ill.ma bramando io che lo sappi ogn'uno. Le lettere dunque combinate nel loro vero senso dicono cosi. Altissimum planetam tergeminum observavi, 16
questo è, che Saturno con mia grand.ma ammiratione ho osservato essere non una stella sola, mà tre insieme, le quali quasi si toccano; sono tra di loro totalmente immobili, et costituite in questa guisa **oOo.** *Quella di mezzo è assai più grande delle laterali, sono situate una da oriente, et l'altra da occidente nella med.ma linea retta à capello; non sono giustamente secondo la drittura del Zodiaco, mà la occidentale si eleva alquanto verso Borea, forse sono parallele all' Equinotiale: se si riguarderanno con un OcchiaIe, che non sia di grand.ma multiplicazione, non appariranno 3 stelle ben distinte, mà parrà che Saturno sià una stella lunghetta in forma di una uliua, cosi* **O***. Ma seruendosi di un Occhiale, che multiplichi piu di mille uolte in superficie, si uedranno li 3 globi distintissimi, et che quasi si toccano, non apparendo trà essi maggior divisione di un sottil filo oscuro: Hor ecco trouata la corte à Gioue, et due servi à questo vecchio, che l'aiutano à camminare, ne mai se gli staccano dal fianco: intorno à gl' altri Pianeti non ci è nouità alcuna. Etc.*

Although this differs only slightly from a Latin wording, I will translate so the reader is not delayed. Therefore, here is what he writes:

But I now come to the next point, regarding the letters which I had transposed and sent to Your Most Illustrious Lordship,[144] *and which were printed by Mr. Kepler in his recent Narrative. As it has also been indicated to me that His Majesty desires to be informed about the meaning of the letters, behold, I am transmitting it to Your Most Illustrious Lordship in order to communicate it to His Majesty and to Mr. Kepler and to whomever you wish.*

The meaning of the letters, connected in the correct order, is this:

I have observed the highest planet triple.
[Altissimum planetam tergeminum observavi]

144 That is, Julian de Medici, the Tuscan ambassador.

With the utmost astonishment, I have discovered, beyond any doubt, that Saturn is not one single star but three stars very close to each other, so close indeed that they almost touch each other. They are completely immobile with respect to each other and arranged in this manner: oOo. The middle one of them is much larger than the outer ones. They are situated with one of them towards the east, the other towards the west, in a straight line within a hair's breadth. However, they are not exactly aligned with the longitude of the zodiac: the western one rises somewhat towards the
17 *north, and perhaps they are parallel to the celestial equator. If you observe them through an eyeglass that does not magnify much, the stars will not appear clearly distinct from each other, but the star of Saturn will appear somewhat elongated, in the shape of an olive, like this: O. But if you use an eyeglass that magnifies the surface more than a thousand times,*[145] *the three globes will be seen most distinctly, almost touching each other, and they will be reckoned to be separated from each other by no more than the width of the thinnest and barely visible thread. And behold the discovery of the courtiers of Jupiter and the two servants of this decrepit old man, who assist his movement, never leaving his sides. As for the other planets, I have discovered nothing new.*

These are the words of Galileo. But if I have a choice, I will not make an old dotard out of Saturn and servants out of his companion globes, but rather a triple-bodied Geryon[146] *out of those three joined globes, Hercules out of Galileo, and his club out of the perspicillum. Armed with this, Galileo conquered the highest of the planets and revealed to the eyes of us all what he had extracted*[147] *from the innermost sanctuaries of nature and brought to our lands. It is indeed pleasing, having discovered this nest, to contemplate what kind of birdlets occupy it; what kind of life, if any life, exists between these globes that almost touch each other pair by pair, where not only*

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145 That is, a linear magnification of more than about 30 times ($\sqrt{1000}$).

146 The reference is to the ancient Greek fable about Geryon, as told e.g. by Apollodorus, *The Library*, Book II (p. 211 in the translation by J. G. Frazer): As his "tenth labour", Hercules had to travel to the end of the world, steal the cattle of Geryon the three-headed monster, and bring it back to Greece. As part of this ordeal, Hercules used his club to kill Geryon's two-headed watchdog, Orthrus.

147 *extractum* could be read as referring specifically to Saturn itself, but I have chosen to retain the slight ambiguity to preserve the equivalence with the storyline of the fable - where Hercules brings back the cattle, not Geryon himself.

"the open space of Sky is three yards and no more",[148] *but where a gap of scarcely a nail's width exists around the circumference. Or we may wonder whether the astrologers rightly assign to Saturn the guardianship of metal miners, who, like moles, are accustomed to living underground and rarely draw in free air under the sky. And yet the darkness is somewhat more tolerable there, because the Sun, which appears as large to the dwellers of Saturn as Venus does to us on Earth, perpetually casts its rays through the divisions between the globes, so that those who stand on one of the globes, being covered by the other as if by a ceiling, are illuminated by the projections of this ceiling of theirs, extended into the light of the Sun, as if by some smouldering embers from above. But the reins of the mind must be tightened as it seizes the free plains of the ether, for later observations might reveal something different from that previous account and changed by the progress of time.*

It seemed to Galileo that he had completed his accounts of the planets and the new observations of them by the end of this letter. However, that ever-attentive factitious eye, I mean the perspicillum, soon uncovered ***18***
more. Read the following letter from Galileo about this:

Di Firenze li 11 di Xbre 1610

Sto con desiderio, attendendo la risposta a due mie scritte ultimamente, per sentire, quello che hauerà detto il S. Keplero della strauaganza di Saturno. In tanto gli mando la cifera di un altro particolare osseruato da me nuouamente, il quale si tira dietro la decisione di grandissime controuersie in astronomia, et in particolare contiene in se un gagliardo argomento per la constituzione Pythagorea et Copernicana; et à suo tempo publichero la deciferatione et altri particolari. Spero che hauerò trovato il metodo per definire i periodi dei quattro pianeti Medicei, stimati con gran ragione quasi inesplicabili dal S. Keplero, al quale piacera, etc.

Le lettere trasposte sono queste.

Haec immatura à me jam frustra leguntur o y.

148 Virgil, *Eclogues III*, l. 105. Here, I adopt the translation by J. W. Mackail (1950), p. 273. Kepler would have known his Virgil well, as this was one of the main subjects taught in the school. He himself was, in fact, asked to teach Virgil at the school in Graz when, one year, there were no students in his mathematics class (KGW vol. XIX, Letter 1.16 from the school inspectors in Graz, Jan 3rd, 1596).

In Latin thus: I await with eagerness the response to the last two [letters], so that I may learn what Kepler has to say about the marvel of the Saturnian star.

Meanwhile, I send him a riddle concerning a certain new and extraordinary observation, which enables the settlement of great controversies in astronomy and, in particular, contains within itself a beautiful argument for the Pythagorean and Copernican arrangement of the world. In due time, I will reveal the solution to the riddle and some other singular matters. I hope to have discovered a method for determining the periods of the four Medicean stars, which Kepler, not without good reason, considered unsolvable, etc.

These are the transposed letters:

Haec immatura à me jam frustra leguntur o y.
("These immature things are now read by me in vain o y.")

Thus far, Galileo. But if this letter has filled you, the reader, with a desire to understand the meaning contained within that riddle,[149] *then go ahead and read the next letter from Galileo.*

First, however, I would like you to notice in passing what Galileo means by the Pythagorean and Copernican arrangement of the world.
19 *For he points a finger at my* Mysterium Cosmographicum*, published 14 years ago, in which I took the dimensions of the planetary orbs from the astronomy of Copernicus, who makes the Sun stable in the middle, with the Earth moving both around the Sun and around its own axis. I showed, in fact, that the separations between those orbs*[150] *correspond to the five*

149 To bypass the confusion that might be caused by the dual meanings of "letter" [epistola/litera] in English, I have here substituted "riddle" for "letters" [literis].

150 This is the central point in the *Mysterium Cosmographicum*: that, to a reasonable approximation, one can fit the cube in between the orbits of Jupiter and Saturn, the pyramid between those of Mars and Jupiter, the dodecahedron between the Earth and Mars, the icosahedron between Venus and the Earth, and the octahedron between Mercury and Venus (*Mysterium Cosmographicum*, chapter IX). Kepler believed that this explained why there are exactly six planets, and not some other number. However, as Kepler also realised, this scheme works only to within a rough approximation. In the revised version of the *Mysterium Cosmographicum* (1621), Kepler wrote: "almost every book on astronomy which I have published since that time could be referred to one or another of the important chapters set out in this little book, and would contain either an illustration or a completion of it." (From the translation by A. M. Duncan, p. 39).

regular Pythagorean bodies, which were already distributed among the elements[151] *of the world by that author long ago, through a pleasing more than a felicitous or genuine effort, and for the sake of which bodies Euclid wrote his entire geometry.*

Accordingly, in that Mysterium *a certain combination of astronomy and Euclidean geometry is found, and through this, the consummation and most absolute perfection of both. This was the reason why I was waiting, with great eagerness, to find out what evidence Galileo would bring forth for this Pythagorean arrangement of the world. Therefore, Galileo's letter with this evidence now follows.*

Ill.^{mo} *et Reuer.*^{mo} *sig.*^{re} *mio col.*^{mo}

E tempo che io deciferi à V. S. Ill.^{ma} *et R.*^{ma} *et per lei al S. Keplero le lettere trasposte, le quali alcune settimane sono, gli inuiai; è tempo dico già, che sono interissimamente chiaro della verità del fatto si che non ci resta un minimo scrupulo, ò dubbio. Sapranno dunque come circa 3 mesi fà vedendosi Venere vespertina la cominciai ad osseruare diligentemente con l' occhiale per ueder col senso stesso, quello, di che non dubitaua l' intelletto. La ueddi dunque sul principio di figura rotonda, pulita et terminata, mà molto piccola; di tal figura si mantenne sino che cominciò ad auuicinarsi alla sua massima digressione, tutta uia andò crescendo in mole. Cominciò poi à mancare dalla rotondita nella sua parte orientale et auersa al sole, et in pochi giorni si ridusse ad essere un mezo cerchio perfettissimo, et tale si mantenne senza punto alterarsi sin che incominciò à ritirarsi verso il sole allontanandosi dalla tangente: hora và calando dal mezo cerchio, et si mostra cornicolata, et anderà assottigliandosi sino all' occultazione riducendosi allora con corna sottilissime, quindi passando ad apparizione mattutina, la uedremo pur falcata et sottilissima et con le corna auerse al sole, anderà poi crescendo sino alla massima digressione, doue sarà semicircolare, et tale senza alterarsi si manterrà molti giorni: et poi dal mezo cerchio passera presto al tutto tondo, et cosi rotonda si conseruerà poi per molti mesi, mà è il suo diametro adesso circa cinque volte maggiore di quello che si mostraua nella sua prima apparizione vespertina: dalla*

151 I.e., earth (the cube), water (the icosahedron), air (the octahedron), fire (pyramid), and the ether (dodecahedron). See also *Harmonice Mundi*, Book II, prop. XXV, and the *Timaeus* 55e-56b (p. 1258-1259 in *Plato, Complete Works*).

quale mirabile esperienza hauiamo sensata et certa dimostrazione di due gran questioni state sin qui dubbie tra' maggiori ingegni del mondo. L'una è che i pianeti tutti sono di loro natura tenebrosi (accadendo anco à Mercurio l'istesso che a Venere). L'altra, che Venere necessariissimamente si uolge intorno al Sole come anco Mercurio, et tutti li altri pianeti, cosa ben creduta da i Pittagorici, Copernico, Keplero et me. Ma non sensatamente prouata, come hora in Venere et in Mercurio. Haueranno dunque il Sig. Kep. et gli altri Copernicani da gloriarsi di hauere creduto et filosofato bene, se bene ci è toccato, et ci è per toccare ancora ad esser reputati dall' uniuersalità de i filosofi in libris, per poco intendenti, et poco meno che stolti. Le parole dunque, che mandai trasposte, et che diceuano

Haec immatura à me jam frustra leguntur, o. y.

ordinate

Cynthiae figuras aemulatur mater amorum

ciò è che Venere imità le figure della luna.

Osseruai 3 notti sono, l'eclisse, nella quale non ui è cosa notabile, solo si uede il taglio dell' ombra indistinto, confuso, et come annebiato, et questo per deriuare essa ombra da la Terra lontanissimamente da essa ☽.

Voleua scriuere altri particolari. Ma sendo stato trattenuto molto dà alcuni gentilhuomini et essendo l'hora tardissima, son forzato à finire. Fauoriscami salutare in mio nome i SS. Kep. Asdale et Segheti, et à V. S. Ill.^{ma} con ogni reu^{a} baciolemani, et dal S. Dio gli prego felicità. Di Firenze il primo di Gennaio Anno 1611.

Di V. S. Ill.^{ma} et Reu.^{ma}
Ser.^{re} Oblig.^{mo}

Galilaeo Galilaei

21 *That is Galileo's letter, of which the main points, in Latin words, are these:*

It is time for me to explain how to read the letters I sent transposed several weeks ago. Now, I say, is the time, after I have become absolutely certain about the matter and no longer have even the slightest doubt. Let it *be known, therefore, that about three months ago when the star of Venus became visible, I began to diligently observe it through the eyeglass, so that I might also comprehend with my senses what I held in my mind as beyond doubt. At first, Venus appeared with a perfectly circular shape, enclosed within an exact and evident boundary, but very small. Venus retained this shape until it began to approach its greatest separation from the Sun, and meanwhile, the apparent size of its body continuously increased. From then on, the roundness began to subside on the eastern side, which was turned away from the Sun, and within a few days, the entire figure was contained within a most perfect semicircle; this shape was maintained with no change at all until Venus began to approach the Sun after leaving the tangent of its epicycle. At this time, it is now subsiding more and more from the semicircular shape, which will continue to diminish until the occultation, when it will fade into the thinnest crescent. Having completed that crossing, it will appear to us as merely sickle-shaped at the morning appearance, with the thinnest of horns turned away from the Sun. Afterwards, the horn will increasingly be filled in until the greatest separation from the Sun is reached, at which point a semicircular shape will be apparent, and this shape will last for many days without notable change. Then, gradually, the entire circle will be filled, and that perfectly circular shape will be maintained for a good many months. Moreover, the diameter of the body of Venus presently appears about five times larger than at the first evening appearance. From this remarkable observation, we have the most certain demonstration, clearly evident through sense perception, of two major questions that were disputed by the greatest minds on either side up to this day. One is that all planets are inherently dark bodies by nature (let us conceive the same for Mercury as for Venus), and the other is that the greatest necessity urges us to assert that Venus (and also Mercury) is carried around the Sun, like all the other* ***22*** *planets. This belief was indeed held by the Pythagoreans, Copernicus, and Kepler, but it was never before confirmed through experience, as it has now been for Venus and Mercury. Kepler and the other Copernicans, therefore, have reason to boast that they have philosophised well and that their belief is not without merit: even if it may have happened to them, and may happen again, that those philosophers of this time who base their philosophising on books, may consider them by universal consensus stupid and little less than fools.*

I plainly accept the argument of the author that the locations of the orbs of Venus and Mercury around the Sun are as in the Copernican and Pythagorean arrangement of the world. Nor do I add anything, except to congratulate Pena, who previously showed the same using another weaker argument.

The words that I sent in transposed letters, which read: "Haec immatura à me jam frustra leguntur o. y." *sound like this when rearranged in their correct order:* "Cynthiae figuras aemulatur mater amorum" *which means "Venus [the mother of passions] imitates the shapes of the Moon [Cynthia]."*[152]

Three nights ago, I observed a lunar eclipse, during which nothing unusual occurred. The cone of the shadow merely appeared indistinct and confused, as if obscured; this is because the shadow rises from the Earth, far from the body of the Moon.

I had other unusual things, but I am prevented from writing about them, etc.

Thus far, Galileo.

What now, dear reader, shall we fashion from our perspicillum? Perhaps the caduceus of Mercury, supported by which we may journey across the pure ether and join Lucian[153] *in leading a colony to the desert of the Evening Star, drawn by the charm of that region? Or rather, Cupid's arrow, so that when it has slipped through our eyes and pierced our most intimate soul, the love of Venus may flare up within us? For what shall I not say about the astonishing beauty of this globe, which lacks her own light but is given such splendour even just by the borrowed light of the Sun? Neither Jupiter nor the Moon, which enjoys similar proximity to the Sun as Venus, possesses such splendour; and although the light of the Moon is greater compared with that of Venus due to the larger apparent size of the Moon's body, it seems dull, lifeless, and lead-like. Oh truly golden Venus! Will anyone doubt henceforth that the entire globe of Venus is crafted from the purest unmixed gold, so smoothly wrought, and with a surface which radiates such vibrant splendour when placed before the Sun? Let me also refer here to the experiments regarding the twinkling of the light of Venus with each*

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152 This passage is necessarily somewhat awkward to translate exactly, given that Kepler here gives a translation into Latin of Galileo's original Italian explanation of the Latin meaning of the riddle.

153 A reference to the story *A true history* by Lucian of Samosata that involves the colonisation of the Morning Star, "because the country was desert and had nobody dwelling in it." (translated by F. Hickes, p. 33 in the 1894 ed.). Kepler explains, in the *Somnium,* that he had chosen the text as his "means of mastering the [Greek] language" as a student in Tübingen, and that a German translation was available in Prague in the early 17th century.

blink of the eye, which I recounted in the Optical Part of Astronomy:[154] *reason can gather nothing else but that the star of Venus whirls around her axis with swift rotation, in turn exposing different parts of her surface, of which some are more receptive to the solar light than others.* **23**

It is indeed also a pleasure to marvel with delight at the cleverness of the astrologers, who for so many centuries have held it as established that the love and pride of mistresses, and the customs and tricks of lovers, are governed by this star of Venus. Should then Venus herself not be horned, she who causes horns to be put on so many every day, whenever she suddenly disappears from the eyes and unimpeded view of her beloved, hastening back into the haughty rays of the Sun, as if to another man, to receive the caresses she craves, while the desires of lovers are frustrated? It would indeed be strange if Venus herself should not also, like the Moon, be reborn, when the Venusian passions are the only and unique cause of procreation. Behold, then, how this most beautiful of stars, having relinquished the perfect circle of her appearance like some mature offspring, descends to the deepest point of her epicycle near the Earth, empty and plain within her horn, as if for the sake of conceiving new progeny. And after she has been united with the Sun and has subjected herself to the Sun, as if to her husband, in a lower position, as is the custom and nature of women, she then gradually rises upwards again on the other side, swelling more and more as if impregnated. Then finally, in the tenth month after conception (for that is the time between two conjunctions of the Sun and Venus), she brings her full womb, I say, the full circle of her appearance, to the summit of her epicycle above the Sun, and once again joined with him she brings her offspring home as if to its true father.

But enough of my reasonings. Let us now, as an epilogue, also hear

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154 Chapter 6, sect. 12, “On the light of other heavenly bodies”. Here Kepler argues that “.. by ocular experience, twinkling is added to the planets either by some internal alteration of the body, perpetual and continuous, which you might say is like a paroxysm, or by an external revolution of the body, belonging to the parts and to the surfaces, proceeding by the unfolding of some parts after others, which Tycho in the Progymnasmata favoured.” (from Donahue’s translation). It is now understood that any twinkling of the light of stars or planets, visible to the naked eye, is in fact caused by turbulence in the Earth’s atmosphere, and not due to any intrinsic variation of the light received from these objects.

Indeed, Newton wrote, in his *Opticks* (1730): “For the Air through which we look upon the Stars, is in a perpetual Tremor; as may be seen by the tremulous Motion of Shadows cast from high Towers, and by the twinkling of the fix’d Stars” (4th ed., Book I, prop. VIII, prob. II, p. 98).

Galileo's reasoning based upon all of those observations with the perspicillum that he has reported. Thus he speaks again:

Ill.^{mo} et Reu.^{mo} Sig.^{re} Col.^{mo}

Ho riceuuto gusto et contento particolariss: nella lettura dell' ultima di V. S. Ill.^{ma} et Reu.^{ma} delli 7 stante, et in particolare in quella parte doue ella mi accenna la fauoreuole inclinazione dell' Ill.^{mo} Sig. Cons. Wacker verso di me; la quale io infinitamente stimo et apprezzo; et poi che quella hà principalmente origine dall' hauere io incontrate osseruazioni necessariamente dimostranti conclusioni per auanti tenute vere da sua
24 *Sig. Ill. per confermarmi maggiormente il possesso di grazia tanto pregiata da me, prego V. S. Ill.^{ma} à fargli intendere per mia parte, come conforme alla credenza di sua Sig.^{ria} Ill.^{ma} ho demostratione certa, che si come tutti i Pianeti ricevono il lume dal Sole essendo per se stessi tenebrosi et opachi; cosi le stelle fisse risplendono per loro natura, non bisognosè della illustrazione de i raggi solari, li quali, Dio sa, se arrivano a tanta altezza, piu di quello, che arrivi a noi il lume di una di esse fisse. Il principale fondamento del mio discorso è nell' osseruare io molto euidentemente con l'occhiali, che quelli pianeti di mano in mano che si trouano piu vicini a noi, ò al Sole, ricevono maggiore splendore, et piu illustremente ce lo riverberano; et percio Marte perigeo, et a noi viciniss: si vede assai piu splendido che Gioue: benche a quello di mole assai inferiore, et difficilmente se gli può con l'occhiale leuare quella irradiazione, che impedisce il uedere il suo disco terminato, et rotondo; il che in Gioue non accade, vedendosi esquisitamente circolato; Saturno poi per la sua gran lontananza si vede essattamente terminato, si la stella maggiore di mezo come le due laterali piccoliss: et appare il suo lume languido et abacinato, senza niuna irradiazione, che impedisca il distinguere i suoi 3 piccoli globi terminatissimi. Hora poiche apertissimamente veggiamo, che il sole molto splendidamente illustra Marte vicino, et che molto piu languido è il lume di Gioue (se bene senza lo strumento appare assai chiaro, il che accade per la grandezza, et candore della stella) languidissimo et fosco quello di Saturno, come molto piu lontano: quali doueriano apparirci le stelle fisse lontane indicibilmente piu di Saturno, quando il lume loro deriuasse dal Sole? Certamente debolissime, torbide e smorte. Ma tutto l' opposito si vede, però che se rimireremo per essempio il Cane, incontreremo un fulgore viuissimo, che quasi ci toglie la vista con una vibrazione di raggi*

tanto fiera, et possente che in comparazione di quello rimangono i pianeti, e dico Gioue et Venere stessa, come un impurissimo uetro appresso un limpidissimo et finissimo Diamante: Et benche il disco di esso Cane apparisca non maggiore della cinquantesima parte di quello di Giove, 25
tutta uia la sua irradiazione è grande et fiera in modo che l' istesso globo trà i proprij crini si implica et quasi si perde, et con qualche difficultà si distingue, doue che Gioue (e molto piu Saturno) si veggono et terminati, et di una luce languida, et per cosi dire quieta. Et per tanto io stimo che bene filosoferemo, referendo la causa della scintillazione delle stelle fisse, al vibrare, che elle fanno dello splendore proprio et natiuo dall' intima loro sustanza, doue che nella superficie de i pianeti termina piu presto, et si finisce la illuminazione, che dal Sole deriua et si parte. Se io sentiro qualche particolare questione ricercata dal medesimo S. Wackher, non resterò di affaticarmici intorno per dimostrarmi, quale io sono desiderosiss: di servire un tanto Signore, et non già con speranza di aggiugnere al termine consequito dal suo discorso, perche benissimo comprendo che a quanto sià passato per il finiss: cribro del giudizio suo, et del S. Keplero, non si può aggiugnere di esquisitezza, ne iò pretenderei altro che col dubitare, e mal filosofare eccitargli al ritrouamento di nuoue sottigliezze. Gl' ingegni singolari che in gran numero fioriscono nell' Alemagna mi hanno lungo tempo tenuto in desiderio di vederla, il qual desiderio hora si raddoppia per la nuoua grazia dell' Ill.mo Wackher la quale mi farebbe diuenir grande ogni piccola occasione, che mi si presentasse. Ma hò di souerchio occupata V. S. Ill.ma et Reu.ma degnisi per fine di offerirmi et dedicarmi deuot.mo ser.re all' Ill.mo S. Wackher, salutando anco caramente il S. Keplero, et a lei con ogni reuerenza bacio le mani et dal Sig.re Dio le prego somma felicità. Di Firenze li 26 di Marzo 1611.

Gallileo de' Gallilei

Written with a Latin pen, the meaning is this:

Your recent letter was a wonderful delight to me, especially where it testifies to the friendly opinion of me held by the most Illustrious and Esteemed Imperial Counsellor, Mr. Wackher, which I indeed value very highly. Since this inclination of his is born out of the fact that I demonstrated, by reasoning based upon some observations, certain inevitable conclusions 26

which he had also held to be true for a long time, I ask that you bring the following message to him, so that I may strengthen this possession of goodwill that is so precious to me: I have the most certain demonstrations at hand, showing clearly, just like he himself also maintains, that all of the planets indeed receive their light from the Sun, while they themselves are naturally opaque and dark bodies. The fixed stars, instead, shine by their own natural light and require no illumination by the rays of the Sun. Whether the latter even reach the highest region of the fixed stars with a brightness as trifling as that by which the rays of the fixed stars come down to us from there, God only knows. My reasoning is founded chiefly on my observations through the eyeglass, which clearly show that as any one of the planets is closer to us and the Sun at whatever time, it accordingly receives a greater splendour and reflects it back more brilliantly. Thus, Mars at perigee, when it is at its smallest distance from Earth, leaves the splendour of Jupiter behind by no small distance, although it yields to Jupiter, by a large margin, by the size of its body. The brightness makes it difficult to observe the light of Mars through the eyeglass, for the glare is so intense that it hinders clear vision and prevents the eye from discerning a round boundary of the disc of the body of Mars. This does not happen with Jupiter, which appears exquisitely circular. After these planets, Saturn, on account of its very large distance, appears most distinctly terminated, both the large globe in the middle and the two smaller balls on the sides. For it shines with a weak and subdued light, without any such irradiation as might be strong enough to impede the distinct perception of the very well-defined termination of its three globes. When therefore, we see Mars very splendidly illuminated by the Sun from a short distance, while the light of the more distant Jupiter is much weaker (it does appear quite bright when no instrument is used, but this is due to the size and whiteness of its body), and the light of the most distant planet Saturn is extremely weak and almost watery, how then, do you think, would the light of the fixed stars, which are immeasurably more distant from the Sun than Saturn, appear if they were only illuminated by the Sun? Altogether feeble, gloomy, and lifeless. But instead, we experience the exact opposite. Let us examine, for example, the Dog Star:[155] *it shines with a most vigorous brilliance that almost stings*
27 *the eye and with a very rapid twinkling of the rays, of such vigour that*

155 Sirius

the planets, like Jupiter, or even Venus itself, are just as overwhelmed and subdued in comparison as if we were to compare the cheapest and dirtiest glass with the purest and most brilliant diamond. And although the disc of the Dog Star appears no larger than a fiftieth part of Jupiter's disc, its glare is nevertheless powerful and extremely violent, so much so that the image of its disc seems to hide within the rays of its own glare, entangling itself and almost vanishing into them, and can only be discerned with some difficulty from the surrounding hair-like rays. On the contrary, Jupiter, and much more so Saturn, are clearly bounded, and their light is dull and, as it were, tranquil. Therefore, I believe we shall philosophise correctly if we attribute the twinkling of the fixed stars to a vibration of their own inherent and native splendour, innate to their substance, whereas, in the case of the planets, we may say that that illumination terminates near their surface, which is derived from the Sun and is spread throughout the world.

These are the scientific matters in Galileo's letter; I omit the rest.

You now see, studious reader, how that most ingenious mind of Galileo's, by Hercules, a most outstanding philosopher indeed, climbs the last and highest walls of the visible world, using this perspicillum of ours as a ladder of sorts, then takes stock of everything brought into view, and from there looks down at our humble dwellings, I mean the planetary globes, with the keenest reasoning, comparing the outermost to the innermost and the highest to the lowest with firm judgement.

Now, since fervour or disparagement between nations is never lacking in philosophy, and many here in the German lands will seek testimony from Germans, let me present to them a letter about these same matters from one German, Simon Marius, a renowned Franconian astronomer.[156] *From this, it will also become clear that Galileo did well to communicate his discoveries to us in Prague in a timely manner through riddles while he was busy with his own matters.*

This is what Marius wrote to a common friend of ours:[157]

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156 "Simon Marius, a renowned Franconian astronomer" is missing in the original 1611 edition, added as an erratum.

157 Communicated to Kepler by Nikolaus Vicke (KGW vol. XVI, Letter #618).

Meanwhile, I am undertaking another work, in which I first affirm the immobility of the Earth, entirely disregarding personal matters[a],[158] *but only examining arguments against the Copernican reasoning which in our time has been endorsed by Kepler, together with Galileo, the mathematician*
28 *from Padua, and which he earnestly holds to be true*[b]*. I take the arguments for my claim from sacred sources*[c]*, with support also from physics*[d] *and astronomy. Then the opinion of those who believe the celestial bodies to be of such monstrous bulk will be refuted, and I will provide a new and more plausible determination of their size, in which matter I was greatly assisted by the Dutch instrument that is commonly known as the perspicillum. Thirdly, I will demonstrate that Venus is illuminated by the Sun in the same way as the Moon and that it is rendered as crescent-shaped, bisected, etc., as I have seen and observed many times and very carefully from the end of the previous year*[e] *until April of the current one with the help of the Dutch perspicillum, when Venus was closest to the Earth, both in the west and in the east. Fourthly, I will discuss the new Jovian planets,*[159] *which are carried around Jupiter like the other planets around the Sun, albeit with different separations and periods. I have already established the periods of the two outermost ones and have constructed tables so that it can easily be known at any*

a) He freed Kepler from fear, who indeed feared greatly for his reputation if Marius had intervened in the motion of the Earth with reference to his name.
b) The first omen of victory before the battle, because Marius, by inexperience with people, confines the importance of that school to two men, although it is now already almost public - unless every flower of learned men be enclosed within the confines of the Academies.
c) Resist, theologians, he is undertaking an irrelevant business. He is going to abuse the authority of Scripture.
d) Let us be judged through our actions.
e) At that same time, Galileo in Florence wrote to Prague about the Mother of Passions, and had already described beforehand the things that would appear to Marius in this order.

158 Marius evidently did not appreciate these marginal notes (essentially a condensed version of more detailed remarks in Kepler's letter to Vicke, #619). At the request of the imperial counsel, Eisen, Kepler sent a conciliatory letter to Marius dated Nov 10th 1612 (KGW vol. XVII, Letter #640). The letter apparently had the desired effect, and Marius replied (Aug 16/26, 1613; KGW vol. XVII, Letter #662) that "Your letter from the previous year [...] duly reached my hands. Having seen and read it, I was wonderfully delighted […] I fully accept what you have conveyed".

159 Marius' observations of Jupiter's moons were published in his *Mundus Iovialis* in 1614 (English transl. by A. O. Prickard). His first recorded observations of Jupiter's moons took place the day after those of Galileo (J. M. Pasachoff 2015). While Galileo referred to the four large moons of Jupiter as the Medicean Stars, the names now generally used (Io, Europa, Ganymede, Callisto) were proposed by Marius, who acknowledged that they "were suggested to me by Kepler, Imperial Astronomer, when we met at the Regensburg fair in October 1613".

time by how many minutes they are separated from Jupiter to the right or left. These last two points are entirely unheard of until now. Perhaps other things will also arise in the meantime as I work.

Thus far, Marius.

From this, dear reader, you then have the trustworthiness of the perspicillum for observations of the new celestial phenomena confirmed by the testimony of one more German. What should then prevent me from composing a geometric panegyric to this most excellent instrument in this booklet, and you, dear reader, from joining me, for the sake of respect, with present spirit and the attention of no ordinary mind while I recite it? Through this effort, you will both sharpen your mind and emerge more skilled in philosophy through understanding of causes, and also be more adept at mechanics and the invention of useful and delightful things, and you will ultimately be more wary and safer from the thousands of ways in which common people are often led into error. Farewell, and consider this prelude fair and good counsel.

1

DIOPTRICS,

or

DEMONSTRATION OF THOSE THINGS THAT OCCUR TO VISION and visible objects by means of *Conspicilla*, that is, glasses or transparent crystals.

I. DEFINITION

The inclination above a surface is obtained as the angle between a normal to the surface and any other ray that intersects the normal in a point on the surface.

II. OPTICAL AXIOM

Rays entering a denser medium at an inclination are refracted, and within the body, the refracted rays approach the normal raised above the surface of the denser medium at the point of incidence. The same rays are refracted when leaving the denser medium, and outside the dense body, the refracted rays withdraw from the normal.

III. OPTICAL AXIOM

The refraction of rays is the same whether they naturally enter or leave, or are considered as such.

IV. PROBLEM

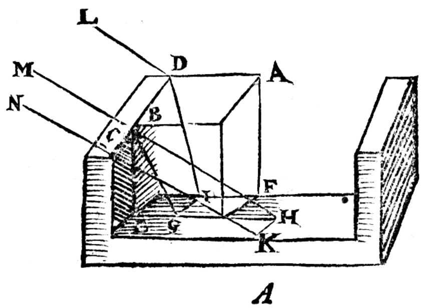
To skilfully measure the refraction in a transparent hard body for every inclination of the rays.

Let AE *be a transparent hard body. Let it be bounded by one perfectly plane surface* DE, *against which two other plane surfaces* BA *and* EF *are raised parallel to each other and at right angles to the former. Prepare a frame for the body made of any material, such as wood, with its surfaces,*
2 *especially the inner ones, well smoothed. Let two sides be rising from the*

bottom H *at right angles so that* BEH *and the other angles are right, and let the transparent body fit snugly with one edge protruding into the hollow angle of the frame, filling it completely. Let the side* DC *of the frame extend beyond the end of the side* DB *of the transparent body by some small amount* BC, *with the height* BE *being equal on both sides, so that the top is like one joined surface of both the transparent and opaque bodies.*

With this done and the bodies joined, let the side DC, *which has the part* DB *in common with both bodies, be placed perpendicular to the rays of the Sun, for whatever inclination of the plane* BA *with respect to those same rays.*

Let LD, MB, *and* NC *be rays of the Sun. Any rays between* MB *and* NC *will extend straight past* BC *in the direction of* MBH *and* NCK *since they encounter no transparent body other than the air. Therefore,* CB *will cast a shadow* HK *on the bottom of the frame and sometimes on its opposite side.*



The difference between the direction towards the Sun and the normal to the surface BA *is now obtained from the proportion of the height* BE *to the shadow* EH. *For as* BE *is to* EH, *so is the total sine*[160] *to the tangent of the [angular] distance between the Sun and the normal to the plane* BA, *i.e. the angle* EBH.

But the rays of the Sun between MB *and* LD, *which are incident upon the denser transparent surface* BA, *will be refracted towards the normal* BE, *and* MB *will thus be refracted into* BG, *and* LD *into* DI. *And* BD *will cast a shorter shadow* GI *through the crystal. The length of the shadow can, furthermore, be judged by eye if the bottom of the frame is first divided into known parts with ink. For the body which covers the bottom is transparent.*

Again, therefore, as the height BE *is to the shadow* EG, *so is the total sine to the tangent of the angle* EBG.

Moreover, subtracting the angle EBG *found here from the angle* EBH

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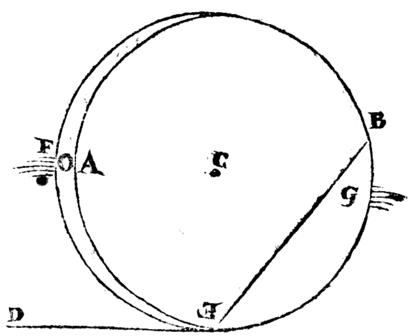
160 That is, BE/EH = sin(90°)/tan(EBH). Today we would just write EH/BE = tan(EBH), with the implicit convention that sin(90°) = 1.

found previously, the angle GBH *remains, which is the magnitude of the refractive angle at this inclination* EBH.

V. PROBLEM

To determine the refractions at greater inclinations and, at the same time, the previous ones more conveniently in another way.

3 *Let a cylindrical body be made from a transparent plate of sufficient thickness, that is, about half an inch thick. Let* AG *be the body with thickness* FA. *Let the plate be pierced along a line following the diameter of the circumference so that there will be a long hole at* FA, *passing through the centre at* C, *and emerging at* G. *Instead of a hole, a rule may be attached on top of the cylindrical body along the line* ACG, *with dioptras*[161] *of equal heights at* A *and* G. *Let the circular edge be divided into 360 parts, starting at* E, *so that* AE *is one quadrant. Then let the hole or dioptra* AG *be aligned towards the Sun so that the sunlight entering through* A *will be clearly visible on an opposite surface or a wall beyond* G. *Since the entire semi-circumference, of which one quadrant extends from* A *on either side, is now illuminated at once, it is evident that a line* DE *touching the surface of the cylinder at* E *will be parallel to* AG, *and thus arrives from the Sun as the outermost of the rays which fall onto the semi-circumference of the cylinder.*



Now, lead an opaque stylus around the cylindrical surface from AF *to* E, *and observe where its shadow falls on the opposite edge near the parts* GB. *Suppose that when the marker is placed at* E, *the shadow falls at* B. *Then half of the part* EB *of the circumference measures the angle of refraction of the ray* DE, *which has the greatest deviation from the normal because it strikes the cylindrical surface of the crystal tangentially at* E.

161 This evidently refers to a very basic type of *dioptra*; see also *Optics* chapter 5, sect. 5.

VI. AXIOM

The refractions of crystal and glass are nearly the same.

VII. AXIOM

Within the limits of sense perception,[162] the refraction by crystal is proportional to the inclination up until the thirtieth [degree of] inclination.

IIX. AXIOM

The angle of refraction by crystal is, up to the stated limit, as nearly as 4
possible, a third of the inclination in air.[163]

IX. AXIOM

The maximum refraction by crystal is approximately 48 degrees.[164]

X. OPTICAL AXIOM

Inclination causes refraction, and equal inclinations of rays within the same medium cause equal refractions or angles of refraction. Greater

162 Here adopting Donahue's translation of *ad sensum*.

163 As Plehn points out, Kepler defines the angle of refraction differently from what we now normally do. For him, it is the change in the direction of the ray, while we now define the angle of refraction θ_r between the normal and the ray in the refracted medium, hence Snell's law:

$$n_r \sin(\theta_r) = n_i \sin(\theta_i)$$

Plehn argues that this may have hindered Kepler from independently discovering Snell's law. For visible light, the refractive index of crown glass is about $n_r = 1.52$, while that of air is close to $n_i = 1.0$. Quartz crystal has a slightly higher $n_r = 1.55$. Hence for small angles, Snell's law is:

$$\theta_r \approx 2/3\ \theta_i$$

If we denote the angle of the "Keplerian" deflection θ_d, so that $\theta_d + \theta_r = \theta_i$ then we have

$$\theta_d \approx \theta_i - 2/3\ \theta_i = 1/3\ \theta_i$$

hence Kepler's approximation is indeed good for small angles, such as those considered in the *Dioptrice*.

164 By Snell's law, we find a maximum $\theta_r = \sin^{-1}(1/1.55) = 40.2°$, or $\theta_d = 49.8°$, or 48.9° for crown glass, so quite close to Kepler's 48°.

inclination also causes greater refraction.[165] No inclination, no refraction; that is, a perpendicular ray is not refracted.

XI. OPTICAL AXIOM

Rays from different luminous points, incident upon the same point on the surface of a denser medium, mutually intersect, and the order of the incident rays is inverted in the refracted rays no less than if the intersection had occurred without refraction.

Proven in Optics[166] *by means of X.*

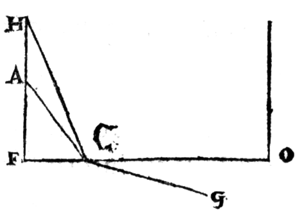
XII. PROPOSITION

The refractions, accurately considered, are not proportional to the inclinations in air.

For when the inclination is 30°, the refraction is 10°, as per VIII. Multiply both by three. Then, in this proportion, the refraction should be 30° for an inclination of 90°, but experiment gives 48°, as per IX.

XIII. PROPOSITION

Within a body of crystal, no ray that is inclined by more than 42° above a surface of the body can penetrate that surface.[167]



In the diagram, let AC *be a ray*[168] *in a body of crystal with a plane surface* FCO. *Let* AC *be inclined by more than 42° so that the angle* FCA *is less than 48°. But if* AC *passes into the air, it will be refracted, and in the air, it will*

165 See also chapter 4, sect. 6, prop. 1, of the *Optics*, referring to Book X, prop. 14 of Witelo's *Perspectiva* (Risner 1572).

166 Proposition 12 in chapter 5 of the *Optics*.

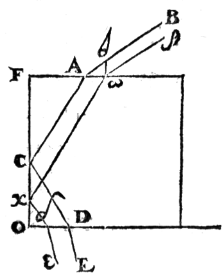
167 Here is Kepler's description of total internal reflection.

168 The text reads: .. *sit corpus crystalli AC..*, "let AC be the body of crystal", although AC seems to refer to the ray in the drawing, so I have translated this a bit freely.

either touch the surface along CO*, or it will not touch the surface but be elevated above it, like* CG*. But neither is possible. For by IX, the refraction of a ray touching* CO *is 48°, and the refracted ray* CH *belonging to* OC *is, therefore, more inward than* CA *since* FCA *is assumed to be less than 48°. Since* OC *is thus refracted into* CH*, and not into* CA*, then nor will* AC *be refracted into* CO*, as per III. But nor is* GC *refracted into* CA*. For by XI, since* GC *and* OC *pass through the same point* C *and intersect there, the ray* GC*, which lies above* OC*, will be lower than* CH *when refracted and, therefore, will not lie above it like* CA*. Therefore,* AC *cannot pass through* C*.*

XIV. PROPOSITION. PROBLEM.

Projecting shadows against the Sun.



This can be accomplished with a crystalline cube. For let FO *be a cube and* Bβ *the Sun. Let* Aω *be a small body on the surface* FA *of the cube. The rays* BA *and* βω*, which form a shadow as they pass around the small body, are then refracted into* AC *and* ωκ*. And* CA *and* κω *will necessarily be raised by more than 48° above the points* Aω *on the surface, as per IX. Now, since the angle* AFC *of the cube is right, and* CAF *is greater than 48°,* FCA *will be less than 42°. Therefore,* AC *and* ωκ *are inclined by more than 48°, and thus also by more than 42°, with respect to the normal to the surface* CF*. Therefore, per XIII,* AC *and* ωκ *will not penetrate the surface* FC*. Consequently, according to optical principles, they will be totally reflected onto the surface* OD*, with the angles* ACF *and* DCO*, in fact, being equal. And because the angle* COD *of the cube is right and* DCO *(equal to* ACF*) is less than 42°,* CDO *will therefore be greater than 48°. Thus, [the ray CD] will be inclined by less than 42° with respect to the normal to the surface* DO*, and it can, therefore, emerge towards* E;[169] *and similarly* κδ *towards* ε*. Thus, the shadow of* Aω *falls in the opposite direction towards* Eε*, and it will occur closer to the Sun than the body* Aω *if* DE *and* δε *are extended further.*

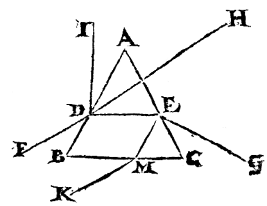
169 The lines DE and δε are incorrectly drawn (corrected in the figure in the KGW edition).

6 *In the same way, it can be demonstrated that if an upright turret,* ωϑ, *is placed at* ω, *the peak* E *of the shadow will be projected against the Sun.*

XV. PROPOSITION

Rays can penetrate the vertex angle[170] of a triangular equilateral prism made of glass or crystal.

Let ABC *be an equilateral section within the prism. Parallel to* BC, *draw* DE *which shall represent some ray. I say that the ray will be able to pass into the air on both sides, both at* D *and at* E. *For the angle* ABC, *and hence* ADE, *is 60 degrees. Therefore, the complement of this angle, or the distance*[171] *between the ray* DE *and the normal to the surface* DA *at the point* D, *is 30°, which is less than 42°. Consequently,* ED *will emerge into* DF. *Similarly,* DE *will also emerge from the region towards* G.



XVI. SENSORY AXIOM

The most delightful colours of the rainbow arise when there is such great refraction, and this is so whether the eyes are looking through [a prism] or the Sun is shining through it.[172]

XVII. PROPOSITION

When the Sun illuminates a prism, three varieties of rays are produced: pure, with the colour of the glass, and with the colours of the rainbow.

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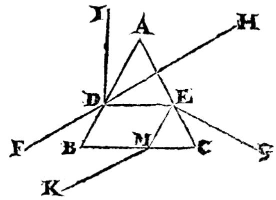
170 *angulum linearem.*

171 *distantia*. Kepler means the angle between the ray and the normal to the surface.

172 Between 1606 and 1609, Kepler exchanged several letters with Thomas Harriot about refraction and the nature of colours. On Oct 2nd 1606 (KGW vol. XV, Letter #394), Kepler wrote to Harriot: "I am eager to learn from you, who are engaged in chemical studies, about the origin of colours and their essential characteristics". Harriot, alas, could not fully satisfy Kepler's curiosity.



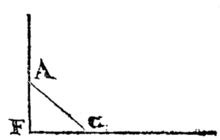
For let F *be the Sun. Let it radiate onto* D. *Here, the substance of the solar ray is divided, as it were, and a small part of it is reflected into* DI *at the angle* ADI, *which is equal to* BDF *by which the ray approaches. The pure, but weak, ray, is propelled along* DI *towards* I. *It is pure because it is not tinged in the glass, the body of which it does not enter.*[173]

However, the greater part of its substance penetrates D *and is refracted into* DE. *At* E, *it is again divided into fractions of its substance. For the* 7
greater part passes through E, *and owing to the double great refraction, it casts the colours of the rainbow towards* G.

The remaining, very weak part of DE *is reflected from the surface* AC *towards* EM. *But if* DE *falls slightly more obliquely upon* AE, *it will be reflected more obliquely towards* EM *than here. For if you diminish* DEA, *then* MEC *must also be diminished according to the law of reflection. And thus,* EM *will eventually fall perpendicularly onto* BC, *so that no refraction will occur at* M. *However, since* FD *has now passed twice through the body of glass, once along* DE, *and again along* EM, *and then emerges straight through* M, *a ray tinged with the colour of the glass is thrust towards* K, *but coming more directly from the region located towards* A itself. *For we are taught in Optics that clear rays are tinged in coloured media.*[174]

XIIX. PROPOSITION

If a right angle of a crystalline or glass body is placed between the eye and a visible object, then it will not transmit rays from the visible object to the eye, but the surface of the crystal facing the visible object will be reckoned as opaque and coloured with the colour of the body.



For let CA *be a ray within the body. It will either be equally inclined above the surfaces* FC *and* FA *or unequally. If equally, it will thus be inclined by more than 42°, namely, 45°. Consequently, it can*

173 Cf. chapter 1, prop. 24 of the *Optics*.
174 Chapter 1, prop. 25 of the *Optics*.

pass through neither one nor the other surface, according to XIII. But if unequally, it was demonstrated in prop. XIII that it cannot pass through one of them. Therefore, no ray passes through both surfaces of a right-angled crystalline body simultaneously.

XIX. OPTICAL AXIOM

The location of an object is reckoned from the direction in which the sight ray first emerges from the eye,[175] no matter how this direction is changed due to refraction of the ray along the path between the object and the eye. For the eye cannot perceive what happens to rays through
8 encounters with media outside itself but assumes that they continue in the same direction in which they had begun.

XX. PROPOSITION

If the apex of the prism is facing upwards, objects on the opposite side are seen raised, and if it is facing downwards, they are seen lowered, facing right to the right, and facing left to the left.

Let the previous sketch from prop. XVII be considered again, and let A *be the upwards-facing apex and* F *the eye. Therefore,* FD *is carried into* DE*, and at* D*, it deviates by 20° (by XII) from the path* DH*. Furthermore,* DE *is carried into* EG*, deviating by another 20° from the path* DE *and, thus, by 40° from the path* FDH*, which is almost half of a right angle. And the eye at* F *thinks it sees objects that are lowered at* G *as if they were raised at* H*, according to XIX.*

So much about a plane crystal; now about curved surfaces.
First about light.

175 Plehn has rewritten this Axiom as if the ray travelled from the object to the eye (which is, of course, what physically happens, as Kepler was well aware). I have kept Kepler's actual formulation; see Kepler's remarks on emission/reception of rays in the preface.

XXI. DEFINITION

The motion of light towards a place is expressed by the [Latin] term *vergere*.[176] Rays are said to *converge* when they approach each other more and more while advancing from the source. They are said to *diverge* when they separate more and more from each other while advancing from the source. Accordingly, rays that [initially] converge will diverge as they proceed onwards after intersecting at the point of concurrence.

XXII. DEFINITION

Radiant points are said to be distant or remote when their distance is so great that the diameter of the eye's pupil vanishes when compared to the distance. They are considered nearby when the proportion of the pupil's diameter to the distance is perceptible.

XXIII. POSTULATE

Although some distant point of a visible object radiates in every
direction,[177] nevertheless, when considering an eye or a *perspicillum*, the **9**
diameters of which have no perceptible proportion to the distance, those
rays touching the outermost parts of the eye or of the *perspicillum* can be
considered parallel, and only one of these can be perpendicular when it
meets the curved surface.

XXIV. DEFINITION

Rays from a single point of a nearby visible object diverge towards the pupil of the eye, but the individual rays from several points of any visible object converge towards the centre of sight. And this is the case if the rays travel freely. It must be carefully noted, therefore, whether a beam of rays from a single point, or the mutual arrangement of beams from multiple points, is being considered.

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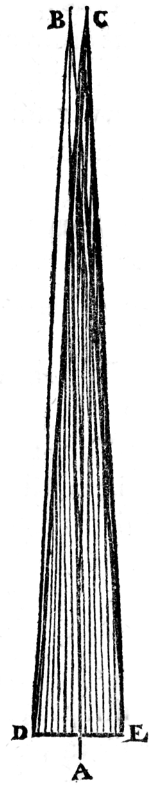
176 From the *Latin Dictionary* by C. T. Lewis & C. Short: to bend, turn, incline, verge.

177 Chapter 1, prop. 2 of the *Optics*. The notion that light is emitted in all directions from each point of an object dates back to Al-Kindi and Alhacen (Lindberg, *Theories of Vision*, p. 30, 63, 73).

CD, CA, *and* CE *diverge towards the eye* DE. *This is also true for* BD, BA, BE, *and all rays in between. But* BA *and* CA *converge towards the centre of the eye* A.



About the lens.

XXV. DEFINITION

A lens is a glass or crystal shaped like a circular disc, wider than it is thick.

XXVI.

A convex lens is one which is either convex on both surfaces or on only one and plane on the remaining surface.

Understand the same for a concave lens. Both may also be called "pure" with a common term.

XXVII.

A mixed lens is one for which one surface is convex and the other concave, with both surfaces defined by perfect circles. This case is contrary to that of a pure lens.

10 XXIIX.

Something convex, concave, or mixed, in the neutral gender [*convexum*, *cavum*, *mixtum*],[178] is understood as referring to a *perspicillum*, glass, body, etc., and expresses the same as a convex, concave, or mixed lens [*lens convexa*, *cava*, *mixta*], etc.

178 Later in the text, Kepler often uses these terms without a specific noun attached. In the translation, this will usually be rendered as "convex body" or "convex surface", etc.

XXIX.

The magnitude of the lens as such is one thing, and the magnitude of the convexity or concavity of the lens is another. The former refers to the size of the body, the latter to the shape.

XXX.

The magnitude of the body itself has a twofold meaning. It can be either absolute, as when the spheres or discs of actual lenses are considered and compared with each other, or it can be relative to the circle of their convexity, that is, how large a part the lens is with respect to its circle of convexity.

XXXI.

"Convex or concave defined by a small or large circle", or "convex or concave of a small or large circle", are understood to refer not to the body, but to the figure and shape.

XXXII.

The convexity or concavity of a small circle is large; of a large circle, it is small.

XXXIII. POSTULATE

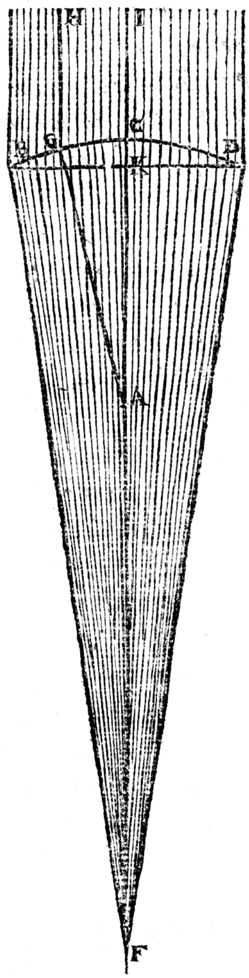
That each surface of a convex, concave, or mixed lens has the centre of its circle on the same line that passes through the centre of the lens.

Concurrence by a lens.

XXXIV. PROPOSITION

Suppose that a point emits parallel rays towards a perpendicularly situated convex lens that is a smaller part than 30° [of its circle of convexity][179] and that nothing else happens to the rays besides refraction upon entry: then, only that ray which passes through the centre of the sphere and falls perpendicularly upon its surface will remain unrefracted, while the other rays, having undergone refraction, will concur with the perpendicular ray after approximately one and a half diameters[180] of the sphere.

11 *Let there be some distant point that irradiates the part* BD *of a crystalline sphere, and let* BCD *be less than 30°. The beam will then be parallel, as per XXIII. Of these rays, only* IC *will be perpendicular, since it passes through the centre* A.



Besides the perpendicular IC, *let one of the parallel rays in the air be selected; let this be the ray* HG. *Now, since* HG *falls obliquely upon the surface* BGC, *it will be refracted towards the normal* GA *at the point of incidence* G, *as per II, so that* IC *and* HG *are no longer parallel below* G. *They will therefore concur. Let the concurrence be at* F, *and let* HG *be refracted into* GF. *For it is assumed that nothing else happens to* HG *after* G. *I then say that* AF *is twice* CA, *and therefore, equal to the diameter of the sphere* BCD. *For* HG, *which is parallel to the perpendicular ray* IC, *is inclined by the size of the angle* GAC. *But if the refraction were equal to the inclination, then* HG *would be refracted into* GA, *that is to say, towards the centre itself. But the refraction is not equal to the inclination, and not equal to three-thirds of the inclination, but one-third, per VIII, and therefore, the refracted ray* GF *deviates from* GA *by two-thirds of the inclination* GAC. *Therefore,* FGA *is two-thirds of* GAC. *But the*

179 Cf. XXX.
180 *Sesquidiameter.*

sum of the angles AGF *and* AFG *equals* GAC. *Therefore,* GFA *is one-third of* GAC *and half of* FGA. *Now, from the properties of triangles, the sine of* GFA *is to the sine of* FGA[181] *as* GA *is to* AF. *But the sines of angles less than 15° are approximately proportional*[182] *to the angles or the arcs. Therefore, the sines are approximately in a double ratio. Therefore,* GA *or* CA *is also to* AF *as one to two or as the semidiameter to the diameter, and* CF *is thus approximately one and a half times the diameter.*

XXXV. PROPOSITION

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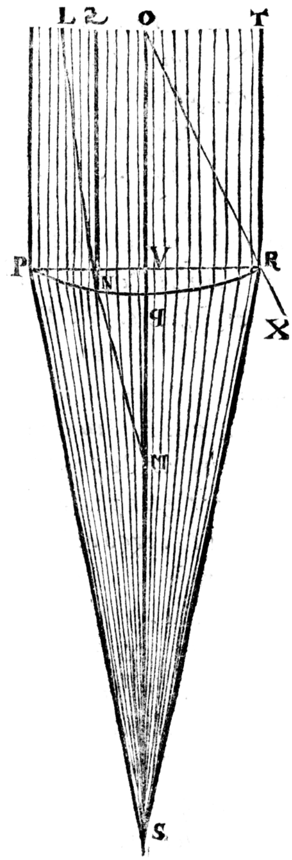


Suppose that parallel rays are propagating within the body of a convex crystal: then, outside the body, they will concur with the perpendicular ray at approximately the diameter of the convexity after the convex surface, provided it is a smaller part than 30°.

Let the body of the crystal POR *be terminated by the convex surface* PQR, *and let a number of parallel rays propagate through this body, of which the middle and perpendicular [ray] is* OQ. *Let* TR *be one of the others. I first say that* TR, *outside the body, is refracted into* RS *by an angle of refraction that is less than the inclination by half, so that, since* SRX *and* TRO *are the inclinations of the rays* SR *and* TR, *the angle* TRO *therefore has two such parts of which* SRX *has three. For the angle of refraction is a third part of the inclination, as per VIII. Accordingly, since* SR *will be refracted into*

181 Literally translated: *the sine of the half-angle GFA is to the sine of the double-angle* FGA. Kepler is using the law of sines here.

182 The small-angle approximation: in modern terms, sin θ is approximately equal to θ for small angles θ (measured in radians).

RT *when entering the body,* RT *is also refracted into* SR *when leaving, as per III. Therefore, the refraction of* TR *is half of the inclination* TRO *since the ray emerges from the dense body. Furthermore, I say that* RS *will concur with* OQ *after approximately the whole diameter of the circle* PQR. *For* RSO *is the amount of refraction, and thus half of* TRO *or* ROS, *and a third part of* XRS. *Now, the sine of the angle* XRS *is to the sine of the angle* RSO *as* OS *to* OR. *But the ratios of the sines of such small angles are nearly the same as those of their arcs. Therefore, the sine of* XRS *is approximately triple the sine of* RSO. *Therefore,* OS *is also triple* OR *or* OQ. *Consequently, since* OQ *is the semidiameter,* QS *will be approximately the diameter.*

XXXVI. PROPOSITION

If the rays are not parallel within the dense body but converge towards the convex boundary of the dense body, they will meet at a point located
13 beyond the convex surface at a distance that is smaller than the diameter of the convexity.

For let OQ *and* LN *converge towards* QN. *And let the ray* NZ *be parallel to* QO *and refracted into* NS. *Therefore,* LN *and* ZN *mutually intersect. Now,* LN *is exterior to* ZN, *and after refraction, it therefore becomes interior to* NS, *the refracted ray belonging to* ZN, *as per XI. Therefore, it concurs with* QS *above* S, *suppose at* M. *And* QM *is shorter than the diameter* QS.

XXXVII. PROPOSITION

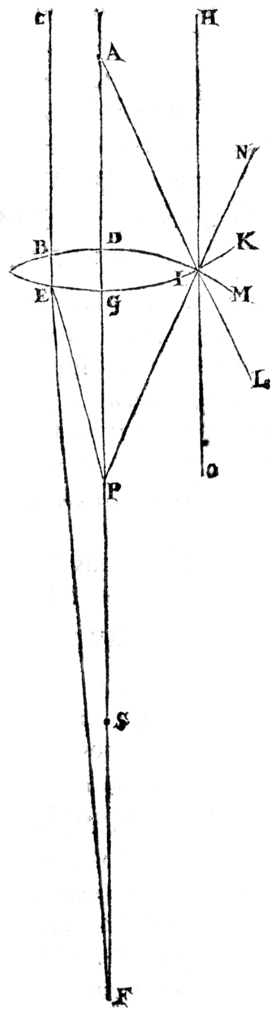
If a radiant point is closer to a convex surface than the diameter of the convexity, then the refracted rays coming from that point will not become parallel within the dense body but will diverge.

For with QS *given as the diameter of the convexity, let* M *be a radiant point closer to the lens than* S, *and let* MN *and* MQ *be diverging rays. Therefore, their refracted rays* NL *and* QO *also diverge towards* LO *as in the previous proposition, as per XI, although it is true that they diverge slightly less.*

So far, only a single convex surface of a lens was considered. Now, about a whole lens.

XXXIIX. PROPOSITION

Parallel rays coming from one radiant point, incident upon a glass or crystal lens that is convex on both sides and placed perpendicularly with respect to the rays, will concur after the lens at a point that is closer to the lens than the diameter of the circle defining the posterior[183] surface, and closer than one and a half times the diameter of the anterior surface.



Let the lens DG *be convex on both sides and* ADGF *a perpendicular line drawn through the centres of the convexities. Let any number of parallel rays* AD *and* CB *come from a distant radiant point. Since* AD *and* CB *and any others are nearly parallel in the air, as per XXIII,* DG *and* BE *converge towards* EG *within the crystal, as per XXXIV, as if they were going to concur at* F.[184] *Therefore, by XXXVI, the point* F *at which* EF, *the refracted ray belonging to* BE, *concurs* 14
will be closer to G *than the diameter* GS *of the convexity* GE. *In the same way,* DG *and* BE *would have concurred at one and a half times the diameter of the convexity* BD *after* D, *as per XXXIV, if indeed they had undergone no further refraction after* B. *However, they are now refracted a second time at* E *towards the perpendicular* GF, *indeed away from the normal*

183 That is, the surface facing the radiant point. The Latin terms used by Kepler are *superficies aversa/obversa* – turned-away or turned-towards (with respect to the radiant point).

184 As noted by F. Hammer (KGW vol. IV, p. 517), the label F is used to refer to two different points, first to the focal point of the first surface BD, and then, mistakenly, to the focal point of the whole lens, which should lie within S.

at E, *as per II. It is, therefore, clear that they concur after less than one and a half times the diameter of* BD. *This latter case had to be demonstrated separately. For it does not follow that if the rays concur closer than* GS, *the diameter of* EG, *they will then also concur closer than* DF, *one and a half times the diameter of* BD. *For the former diameter may be greater than one and a half times the latter.*

XXXIX. PROPOSITION

If both convexities are defined by the same circle, then, with the same assumptions as before, the concurrence will happen after the lens at a point which is separated by about one semidiameter from the anterior convex surface, that is, at its centre.

In the previous diagram, let BD *and* EG *be equally convex, and let* A *and* P *be the centres of their circles. Let the circles intersect at* I, *and let* GI *be extended to* K, *and* DI *to* M. *Let also the normals* AL *and* PN *be drawn from the centres through the intersection point* I, *and let* HO *pass through* I *parallel to* AE. *Since* BD *and* EG *differ little in the previous proposition, they may be considered equal, and instead of them, let* DI *and* GI *be taken to be exactly equal. Now, the ray* HI *is inclined above* DIM *and deviates from the normal* IN *by the angle* HIN, *which is equal to* OIP *or* IPD, *and after refraction, the ray HI will, therefore, deviate by a third part of* OIP *from* OI *towards* IP *within the convex body, as per IIX. But* LIO *is equal to* NIH, *since* AI *and* IP *are equal, and* HIO *is parallel to* AP. *Therefore, the refracted ray travelling within the dense body will fall upon the posterior surface* KIG *(whose normal through* I *is* AL*) at an angle which is one-third part greater than* LIO. *Within the body of the crystal, that refracted ray, therefore, has four parts of inclination against the posterior surface. But when it emerges into the free air through point* I, *it must acquire an inclination in the air that is one-half times greater because a ray which enters the convex body from the air at an inclination loses one-third of that inclination within the body, as per VIII. Therefore, in the air, after passing through the lens, the inclined ray has six of those parts of which the angles* NIH *or* LIO *have three. Thus, the angle of that inclination is twice the angle* LIO. *But since* LIO *and* OIP *are equal,* LIP *is also twice* LIO. *Thus,* IP *is that refracted ray coming from* HI, *which is indeed refracted twice,* *once when entering through the convex surface* DIM *at* I*, and then again when emerging at* I *through the convex surface* GIK*. If the convexities are equal, the parallel rays* CB, AD, *and* HI *will, therefore, concur at* P, *the centre of the anterior convexity* BDI. *Compare with XXXIV, XXXV, and XXXIIX. Thus, for the sake of memory: three semidiameters after the anterior convex surface, two after the posterior, and one after both.*

XL. PORISM[185]

From this it is evident that if the convexities are unequal, the distance from the lens to the point of concurrence will be intermediate between the semidiameters of the two convexities. For the distance will be greater than the semidiameter of the smaller circle because the other surface is defined by a larger circle, but if that surface had been defined by an equal circle, it would have been the measure of the semidiameter within the range. The distance will be less than the diameter of the smaller circle because the surface defined by the smaller [circle] is not alone. Finally, it will be less than the semidiameter of the larger circle, for if the surface defined by the smaller circle had been equal, it would have been the measure of the larger semidiameter within the range, but now it is not equal, but smaller.

XLI. PROPOSITION ***16***

Rays from a distant point on a visible object concur closer to the lens, and the concurrence of rays from a closer point occurs farther beyond the lens.

For by XXXIV, XXXV, and XXXIIX and their three diagrams, the concurrence is at F, S, *or* P *if the point is infinitely distant. When instead the radiant point approaches the lens,*[186] *so that instead of being distant it comes close and is collocated with* F, S, *or* P, *the concurrence of the rays is extended towards infinity as per the same propositions and per III. However, with the extremes given, the intermediates are also given, for as the radiant point moves beyond* F, S, *or* P, *the concurrence of rays occurs*

185 Here, a *porism* is something like a deduction or corollary. The usage of this term by Euclid is discussed by J. L. Heiberg (1882) and T. L. Heath (1956).

186 Reading *ad lentem* instead of *ad rem*.

closer than infinity but still at a great distance as long as the visible object is very close. As the visible object, in turn, recedes into the distance, the point of concurrence approaches F, S, *or* P. *Finally, by XXXV,*[187] *if the lens is equally convex on both sides, and the distance of the radiant point from the lens is equal to one diameter, the point of concurrence will also have a distance of one diameter, with the rays being parallel within the lens.*

Effects of a lens by itself.

XLII. DEFINITION

When any convex lens gathers the rays from a single luminous point at a specific point, the latter point will shift farther past the centre if the luminous point is nearby than if it is distant, as per XLI. Accordingly, whenever the point of concurrence is mentioned *simpliciter*, with nothing added,[188] it is understood to be that point at which the rays from a distant point, that is, parallel rays, are gathered and concur.

XLIII. PROBLEM

To depict visible objects on a white wall with a convex lens.

In a camera obscura, *let a convex lens be placed in the only opening.*[189] *Let a sheet of paper be placed at the point of concurrence. For a point of*
17 *a visible object is again assembled into nearly a single point on the paper from all the rays that it radiates into the lens. However, visible objects consist of an infinite number of points. An infinity of such points, that is, the entire surface of the visible object, will thus be depicted on the paper.*

XLIV. PROPOSITION

The picture[190] produced by a lens is inverted.

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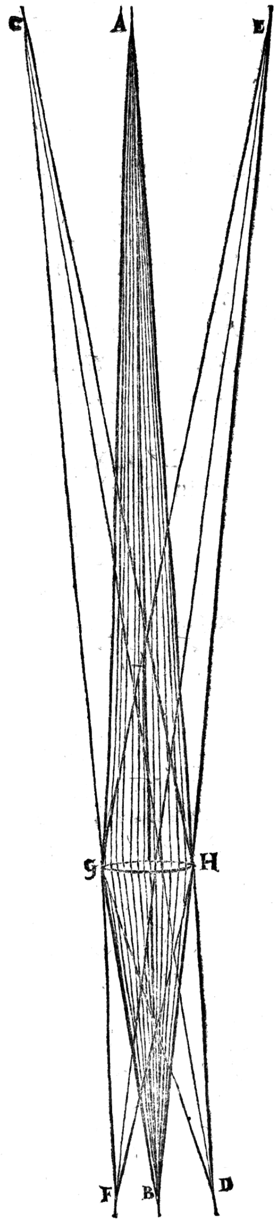
187 The KGW edition and Plehn both refer to XXXVIII, while the original text refers to XXXV.

188 As pointed out by Plehn, the point of concurrence "simply mentioned" is equivalent to the modern concept of the focal point.

189 As noted in the KGW, this setup is described by della Porta in the *Natural Magick*, Book 17, chapter 6.

190 Note the very specific sense in which the word "picture" (*pictura*) is used. See the definition, chapter 5, sect. 3 of the *Optics* (Donahue's translation): "Since hitherto an Image has been a



For the lens serves as a base upon which two cones rest, one on either side, with the vertex of one cone at the visible point and the vertex of the other at the point in the picture on the paper.

XLV. DEFINITION

We shall call such a pairing, for the sake of instruction, a pencil [*Penicillum*].

Now, all the pencils of all the points concur in the lens as if on a common base of the cones and diverge again upon passing through the lens, thus acquiring the opposite order. In the diagram, there are three pencils, AB, CD, *and* EF, *concurring in the convex lens* GH *as if on a common base.*

XLVI. PROPOSITION

As the diameter of the picture is related to its distance from the lens, so is the diameter of the object being viewed also related to its distance from the lens, approximately.[191] For the axes of the pencils (the straight lines drawn from the visible point to the corresponding point in the picture) intersect each other in
approximately one point near the centre of the *18*
lens. Therefore, with the vertical angles[192] being equal, according to proposition XV of Euclid's first book, the bases are also proportional to the legs on both sides, as per proposition IV of Euclid's sixth book.

Being of the reason, now let the figures of objects that really exist on paper or upon another surface be called pictures".

191 See also chapter 2, prop. 3 of the *Optics*.

192 Kepler is referring to the Greek "κατά κορύφην" from Euclid's *Elements*, Book I, prop. XV. Heath's translation of *The Elements* has this as: "If two straight lines cut one another, they make the vertical angles equal to one another."

XLVII. PROBLEM

To compendiously find the semidiameter of convexity if the lens is convex on both sides with equal convexity.

Place a sheet of paper where distant objects are depicted with the greatest degree of distinction. For by XLIII, the paper will be at the point of concurrence. Therefore, by XXXIX, the distance of the paper from the lens will be equal to the semidiameter of its convexity.

XLIIX. PROBLEM

To find the same if the lens is convex on one side and plane on the other.

Turn the plane side of the lens towards a distant visible object, placing it perpendicularly so that the rays enter at a right angle and are not refracted. And place the paper where the visible object is depicted distinctly. Then, as per XLIII, the paper will be at the point of concurrence, and per XXXV, its distance from the lens will be approximately the full diameter of the convexity.

XLIX. PROBLEM

For a lens of equal convexity on both sides, to measure how large the diameter of the convexity is using a nearby visible object.

Place the lens in the middle between a sheet of paper and a visible object, perpendicularly and precisely. Then, increase or decrease the distance of each from the lens by equal increments until the picture on the paper becomes maximally distinct.

For because the visible object is depicted on the paper, the paper is at the point of concurrence of the rays from the visible object, as per XLIII. Moreover, since the visible object and the paper are equally distant from the lens, the parts of the rays that are within the lens will be parallel. For if they were not parallel, no part of any ray (except those close to the centre of the lens, passing perpendicularly through its middle) would fall upon

either of the equal surfaces equally inclined and, therefore, no part would **19**
be refracted equally, as per XIIX. Therefore, no ray would concur with the perpendicular ray at equal distances from the lens. Since the rays are thus parallel within the body, the concurrence occurs at a distance of one diameter from the lens, as per XXXV.

L. PROBLEM

To set fire with a lens that is equally convex on both sides.[193]

Place the lens perpendicularly towards the Sun, and place the combustible at the point of concurrence, which is at a distance of one semidiameter of the convexity, as per XXXVIIII, because the rays from the centre of the Sun are parallel, as per XXIII.

LI. PROBLEM

Accomplish the same with a lens that is plane on one of the two sides.

It happens at approximately one diameter of the convexity after the lens, as per XXXV.

LII. PROBLEM

To illuminate letters with a convex lens at night with the assistance of one bright star, so that they can be read.[194]

Let the star radiate perpendicularly upon the lens. Let the paper with the letters to be read be placed behind the lens. If the lens is equally convex on both sides, let the distance be one semidiameter, as per XLIII and XXXIX, but if one side is plane, let it be the diameter, as per XXXV. But if the convexities are unequal, the distance will contain more than the semidiameter of the smaller [circle] but less than the diameter, as per XL.

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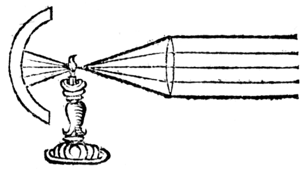
193 The next few problems may have been inspired by della Porta's *Natural Magick*, e.g. in Book 17, chapter X: "How with a convex crystal lenticular to kindle fire".

194 Cf. the *Natural Magick*: "In a dark night read a letter by a Lenticular Crystal".

LIII. PROBLEM

To project light very far at night with a convex lens.

Let the light be placed behind the lens at the point of concurrence of parallel rays. The light rays diverging towards the lens will then emerge parallel after refraction, as per XXXIV, XXXV,
20 *XXXIX, and XL. It is advantageous to place this light at the centre of a concave mirror so that rays travelling in the opposite direction are reflected back onto the light and pass through it into the lens. But if you withdraw the light from the lens, the strongest illumination will approach the lens from infinity, and you will thus be able to regulate it and illuminate any place as distant as you wish, as per XLI.*



LIV. PROBLEM

To measure the distance of a visible object from a single station with a lens that is equally convex on both sides.

If the visible object is depicted on a sheet of paper placed at a distance from the lens that is greater than the diameter of the convexity, then the visible object will be less distant than the diameter of the convexity. If the paper is at a distance of one diameter, the visible object is also at a distance of a diameter, as per XXXV. Furthermore, if the paper is at a distance of less than a diameter, the visible object will be at a distance of more than a diameter, as per XLI. Finally, if the picture on the paper is perfect and the distance of the paper is equal to the semidiameter of the known convexity, the object will be so far away that its distance can no longer be measured by the picture, as per XXXIX.

LV. PROBLEM

To achieve the same with a convex lens by another method: if the size of the visible object is known.

This is done by means of XLVI. For as the length of the picture is to its distance from the lens, so is the known length of the visible object to its distance from the lens.

LVI. NOTE

Giovanni Battista della Porta presents the problem of igniting combustion at an infinite distance with a burning line,[195] which he states can be done with a mirror, although others hold that it should be done with a convex lens. Whichever approach you follow, you will attempt the impossible. The science of optics opposes it.

Firstly, combustion occurs due to the intersection of rays. An intersection is a point, not a line. Secondly, if the line combusts at infinity, it, therefore, also does so on the very surface of the lens whence it emerges. The lens would, therefore, be destroyed. Thirdly, if a ray acquires the power of combustion, this power is acquired from the accumulation of many rays into one. But this is impossible. For one ray also falls upon one single point. **21** *But for one point on any surface, there is one and only one refraction of each ray passing through that point. Therefore, also after that point, there is only one ray, not many distinct rays with distinct inclinations that may be gathered into one by refraction. However, I will say more about this matter below when I join concave lenses with convex ones.*

So much about the convex lens and its uses without regard to the eye.
Now, about those uses that it has in aiding vision.
And first about vision itself.

195 Book 17, chapter XVII: "A parabolical section that may burn to infinite distance".
".. This Glass doth not burn for ten, twenty, a hundred, or a thousand paces, or to a set distance, but at infinite distance […] the burning line proceeds from the Centre of the Glass of any Longitude, and it burns all it meets with in the way." As noted by della Porta in the introduction to Book 17, Archimedes is purported to have defeated the Roman forces at the Siege of Syracuse using concave mirrors to focus sunlight onto their ships and thereby setting them on fire. Whether or not that story is true, military applications have often been a driver of invention in optics, as in other fields.

LVII. PHYSICAL AXIOM

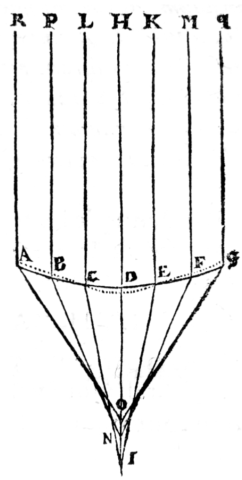
The axes passing through the centres of the pupil and the humours of the eyes are parallel when moved naturally, or even more so, when at rest, but can be voluntarily twisted in order to observe nearby objects.

LIIX. DEFINITION

Vision is said to be distinct when the subtle parts of an object shine forth and come into view. It is said to be confused when the larger parts remain apparent while the smaller parts are hidden, as if erased or smeared, with the boundaries between them being blurred. Strong or clear vision is when an object appears as if in bright light; weak or obscure vision is when an object appears as if in faint light, of the kind that occurs during a solar eclipse or when the Moon is shining.

LIX. PROPOSITION

The surface of a dense body, which causes parallel rays passing through the body to concur perfectly after refraction, resembles a hyperbola.[196]



Let ABCDEFG *be part of a circle with centre* H, *and let the perpendicular line* HD *be sufficiently extended. Let* RA, PB, LC, KE, MF, *and* QG *be parallel to it.*

22 *If all angles of refraction were proportional to the angles of incidence, then all parallel rays would concur at the same point, such as* I, *after refraction, as per XXXV. However, by XII, the angles are not proportional but increase further beyond this measure at large inclinations. Therefore,* LC *and* KE *do indeed concur at* I, *but the next rays,* PB *and* MF, *concur above this point at* N, *and the last ones,* RA *and* QG, *still higher, at* O.

196 See also chapter 5, prop. 24 of the *Optics*.

Thus, in order for the points O, N, *and* I *to coincide at* N, *it is necessary for the refractions to be smaller at* A *and* G *and greater at* C *and* E. *But the refractions at* A *and* G *will be smaller if the inclinations of* RA *and* QG *above the surface are smaller in those points, and they will be greater at* C *and* E *if the inclinations of* LC *and* KE *are greater*.

But the inclination of RA *with respect to* AB *will be smaller if* AB *approaches* R *at the boundary* B, *that is, if there is some surface which intersects the circular surface* ABC *at* A, *while rising above* ABC. *Similarly, if* BCD *again intersects at* C, *the inclination of* LC *above it will be greater. Likewise at* E *and* G. *The new curve thus intersects the old one at four points. And a hyperbola does the same. An ellipse does not do so. For an ellipse only intersects an arc smaller than a semicircle at two points. While a parabola does do the same, this does not make it similar to the required surface. For it aligns itself with no specific angle. But the required surface must align itself with a specific angle,*[197] *namely 96°, because the maximum refraction is 48°, as per IX, of which the double is 96°.*

LX. PROPOSITION[198]

The crystalline humour of the eye is a convex lens shaped like a
hyperbola, and the retiform tunic,[199] filled with spirit and located behind
the crystalline, is like paper, and visible objects are depicted on it with ***23***
a real picture. That the crystalline humour is a very clear convex lens is
evident from investigations of anatomists. These also bear witness that
the rear part is hyperbolic,[200] that the retiform is extended in a circle
or hollow sphere all around the crystalline at a fixed distance from the
crystalline, and that it moreover has a pale reddish colour, like paper.

197 What Kepler means here is that the angle between the asymptotic tangents to the hyperbola must approach a maximum of 96 degrees.

198 Kepler's full account of the "Means of Vision" is presented in chapter 5 of the *Optics*. While Platter had suggested that the retina, and not the "crystalline humour", is the primary organ of vision, Kepler's analysis was the first to demonstrate how the refraction of rays passing through the various humours of the eye leads to a real, inverted image on the retina. See Lindberg (1976), who also notes that Kepler's new theory of vision introduced a much clearer distinction between the optical/geometrical part of the problem (prop. LX) and the physiological/neurological part (prop. LXI).

199 The retina.

200 See also chapter 5, sect. 1, of the *Optics*.

With these points established, it follows, as per XLIII, that a picture of visible objects arises on the retiform. And because it is obtained through a figure similar to a hyperbola, it is consistent with LIX that this is done in order to bring about a perfect and pure sharpness of the pencils so they produce the most distinct picture.

LXI. PROPOSITION

Vision is a perception of the stimulated retiform filled with visual spirit. In other words, to see is to perceive the stimulated retiform insofar as it is stimulated.

The retiform tunic is painted by the coloured rays from visible objects. This picture or illustration is a certain kind of effect[201] *that we undergo, which is not merely superficial, as when chalk is rubbed onto a wall or when light shines upon it, but also qualitative and penetrating the spirits. I first prove this from the nature of light, which can ignite fire if strong and concentrated, as per L. But if the subtle glow flowing into the retiform has the same proportion to the most subtle tenuousness of the spirit in the retiform as exists outside in the air between the most concentrated scorching light and the densest bulk of those things which are being burned, then the penetrating action of the glow in the retiform, and the effect on the retiform and the spirit, will follow no less than the burning (action) by the light and the destruction (effect) of the material that is burned follow outside in the air. Secondly, I prove it from experience. If the eyes are staring at a strong light, they are stimulated so strongly that even after they have been turned away from the glare of what was seen, they retain its image and carry it with them, sometimes for quite a long time. Therefore, that picture on the retiform is a penetrating effect. But the picture does not yet bring about complete vision, unless the* species *experienced by the retiform passes into the brain through a continuity of spirits and is placed at the threshold of the faculty of the soul. And it happens in the following way.*

24 *Just as every external sense is accomplished through reception and impression, and indeed through an effect, when the* species *of the external*

201 The Latin word is *passio*, as a contrast to *actio*. Donahue translates *passio* as "receiving", while Aiton et al. use "passive experience" in *The Harmony of the World* (e.g. Book IV, chapter I). Here I adopt "effect" from the *Oxford Latin Dictionary.*

object is imprinted into that which perceives, and this effect is called a perception, so also there is something within the brain, whatever it may be, which is called the sensus communis, *into which is imprinted the* species *of the stimulated visual instrument, that is, of what was depicted by the light of the visible object. Thus, what happens to the instrument outside the seat of the* sensus communis *flows from the stimulated or painted instrument as an incorporeal* species *and is brought to the threshold of the* sensus communis *and imprinted into that* sensus communis. *But that imprinting happens by hidden means, and it cannot even be safely stated that the* species *is carried inwards by a passage through the optic nerves, which intersect each other in a crosswise manner. For another use of those optic nerves appears more evident, namely to supply each eye with the visual spirit from both parts of the brain,*[202] *and for that reason, they are crossed so that if one sinus of the brain is injured or the nerve is blocked, the other eye will not immediately be deprived of the spirit coming from that part. Since the optic nerves thus have an evident purpose, it is unclear whether they also serve to bring the* species *from the stimulated instrument inwards into the brain or whether there are some other spirits, more tenuous than that corporeal one which is spread through the retiform, which do not require corporeal movement and roam freely through the whole body, where they receive the stimulations of the body parts and communicate these to the faculty of the brain, called the* sensus communis. *Perhaps it happens in such a way that the* species *of the stimulated instrument is transferred from the retiform to the brain by passage through the optic nerve, not insofar as there is some corporeal movement, but insofar as it is full of spirit from the seat of the* sensus communis *all the way to the retiform optic nerve.*[203] *In this way, the continuity of the spirit may be the cause of the transfer of stimulation from the eye to the brain, just like a pebble thrown into a pool of water produces motion which propagates all the way to the shore, as long as the surface of the water is continuous.*

One could say that, in the same way that the Sun illuminates everything by pellucid straight lines, so also do the instruments illuminate the faculty

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202 This notion, that the sensitive part of the eye is supplied with "visual spirit" from the brain through the optic nerve, is also mentioned by Alhacen (Lindberg 1976, p. 69), and brings to mind Galen's concept of "pneuma" (*On the doctrines of Hippocrates and Plato*, Book VII, sect. 5).

203 The original edition only has "optic nerve" with "retiform" inserted in the Errata. One might conjecture that this should simply be "retiform".

of the soul, which is in the brain, by lines of the spirit, however bent and curved, as long as they are continuous. For just as a clear sky does not help
25 *us if something opaque intervenes between the Sun and us, so also will the spirit which awaits in the retiform be of no use if that highest and innermost spirited conduction is interrupted in the head for whatever reason and ceases to be continuous. Hence, that sudden extinction of light in illnesses occurs not through the retreat of spirits but through their interruption and interception by a constricted, obstructed, or severed passage.*

So much for the other effect, which belongs to the sensus communis *and is caused by the* species *perceived by the instrument, which is its object.*

LXII. PROPOSITION

When both instruments are stimulated similarly, we seem to see one *species*. But when the retiform tunics inside the two eyes are differently stimulated or painted, visible objects are shown to us as two instead of one.

For the sense of the instrument is not within the sensus communis *insofar as it is a mere instrument. Or if it is, it is perpetual and not suited for bringing about any new perception. But there is an instrumental sense, insofar as it is stimulated, as per LXI.*

Now, if they are similarly stimulated, there will also be a similar impression or effect from each stimulated instrument in the sensus communis, *since they are one and the same. For the footprint, so to speak, that the right eye imprints on the* sensus communis *with its stimulation will also be imprinted by the left eye with its own, as far as bringing about a new perception in the brain is concerned. The last part of the proposition follows from LXI. For if vision is a perception of the stimulated instrument, as stimulated, and there are, in fact, two instruments which are stimulated separately in whatever manner, then two imprints will be made on the* sensus communis, *and there will thus be two perceptions of the same thing.*

The crossing of the optic nerves inside the brain, therefore, does not serve to recognise the unity of a thing seen by the two eyes.[204] *For the fact*

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204 This is also discussed by Pena in his preface, where he refers to Witelo and others who claimed that the crossing of the optic nerves causes a single object, seen with both eyes, to appear as one. Pena himself, like Kepler, argues against this idea.

that they are always crossed also speaks against this; on the contrary, we do not always seem to see a thing as one, even if we see the same thing with both eyes.

LXIII. PROPOSITION

It is not possible for the retiform to be distinctly painted by both nearby and distant objects while maintaining the same position in the eye. ***26***

For by XLI, the rays from distant points concur at a shorter distance behind the lens than those from nearby points. Now, by XLIII, an accurate picture is formed at the point where the rays come together, and the picture, therefore, becomes confused except at the point of concurrence, which leads to indistinct vision, as per LX. And thus, where nearby objects are accurately depicted, there is no concurrence of rays from distant points, and distant objects are therefore depicted confusedly in that place, and vice versa. Consequently, in that position of the retiform tunic with respect to the crystalline in which we see distant objects distinctly, we see nearby objects confusedly.

LXIV. PROPOSITION

There are those who see distant things distinctly and nearby things confusedly, whom Aristotle calls presbyopes, and there are those who see nearby things distinctly and distant things confusedly, who are myopic, according to Aristotle. Then there are those who see both nearby and distant things confusedly, and finally, those who see both distinctly.

This proposition is physiological and almost medical. Those who see both nearby and distant objects confusedly have a disorder of the eye, being either dim-sighted or completely blind. For a damaged constitution of the eye leads to this condition.

Those who see both nearby and distant objects distinctly have a healthy eye with a flexible shape. For while the retiform cannot be painted equally in the same position in both cases, as per LXIII, it is, in fact, painted equally in those who see both distinctly, as per LX and LXI. For them, therefore,

either the crystalline humour is moved with respect to the location of the retiform or the retiform is moved with respect to the location of the crystalline humour. And it is plausible for a healthy, lively, and youthful eye that, just as it has an evident natural movement in the front whereby the pupil constricts in bright light and dilates in faint light, it also has a similar ability in the retiform tunic behind the crystalline, so that it may widen the eyeball, thereby causing the back to approach the crystalline if remote objects are to be seen, or it may constrict the eyeball in order to make the back recede if looking at nearby objects. Or perhaps this natural movement
27 *belongs rather to the arachnoid membrane, or arachnoid tunic, which has the lens of the crystalline humour fixed at its centre and connects it with the uvea through black rays extending on all sides. For these black rays, called ciliary processes, are seen to be arranged in a comb-like manner such that each is like a single muscle on its own. When all of them simultaneously retract into themselves and are shortened, the midsection of the eye, so to speak, is narrowed and the sides of the eye are contracted, and this produces a somewhat oblong or ellipsoidal shape of the eye, whereby the back or the cavity of the retiform tunic recedes from the crystalline humour. However, when the ciliary processes in the arachnoid membrane are attenuated and thus stretched lengthwise, the circle that goes around the sides of the eye widens, and the eye takes on a more lenticular shape, and the back of the retiform approaches the crystalline, by the function of the same uvea which also tightens and loosens the pupil. For this purpose, the humours, except for the crystalline, are fluid and can be compressed.*[205]

Those who, instead, only see distinctly in one of the two cases still have a healthy eye, but one that is already becoming hardened, habituated, and somewhat old. For it is false that only the elderly do not see nearby objects distinctly or that only the young do not see distant objects clearly. These things happen indiscriminately to both, according to the condition of their bodies or the exercise of their youth. For those who, from childhood, are devoted to hunting, bird-catching, sailing, or travelling, accustom their eyes to distant objects; but because they must also now and then grab hold of

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205 In reality, the accommodating mechanism of the eye involves no change in the shape of the eyeball in humans. Kepler is correct in supposing that the ciliary processes are involved, but their function is to allow an adjustment of the shape of the lens itself, not of the relative positions of the retina and the lens (see Land, 2015, for a historical overview).

food and converse with people, the eye remains in the habit of also looking at nearby objects. However, with time, the exercising weakens, and it thus happens that those who suffer no defect of vision in their youth see only distant objects distinctly in old age. For it is more natural to keep the eyes parallel than to twist them towards nearby objects, as per LVII. And in old age, the eye becomes tired so that, to retain its natural direction, it disregards those things which are perceived with effort. And this defect usually occurs slowly, in very old age.

In contrast, those leading a sedentary life from childhood, confined within walls and dedicating themselves to studies and delicate manual crafts, quickly become accustomed to nearby objects, and they are never drawn away from them with advancing age, but rather see distant objects more and more poorly.

People of the first type are also more prone to drunkenness, drowsiness, ***28***
idleness, and contemplation; that is, they usually pay no attention to things happening before their feet and under their hands, and their eyes are therefore directed as much as possible in a parallel position in which only distant objects are seen distinctly.

People of the second type are instead sober, vigilant, industrious, and attentive towards the present.

Thus, the former individuals are generally tall in stature, as they have their eyes farther removed from the ground and can see further into the distance, while the latter are generally short; however, this is not permanent. For it is said that a natural constitution of the body can, to some extent, be reclaimed here.

The anatomical plate 49 of the esteemed Felix Platter[206] is inserted on page 177 of my *Paralipomena to Witelo*, or the *Optical Part of Astronomy.*[207] In this plate, a figure of the arachnoid membrane is shown, marked by the number X, in the middle of which the crystalline humour is suspended, with its own figure marked by the number XIII. The crystalline is positioned within the eye in number I, marked by the

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206 From Platter (1583), *De corporis humani structura et usu*.

207 The following remarks are included after the main text in the original edition but are inserted here as in the KGW edition. Platter's figure appears on p. 188 in Donahue's translation of the *Optics*, and is reproduced in this book on p. 169.

letter A,[208] where the rays of the arachnoid membrane are represented by KK. The ends of the rays, which are surrounded by a circle in number X, are to be considered as continuous with the uveal tunic on the inside. In number VII, you see that tunic turned around and near the letters O. O. imprints of those rays cut off from the uveal tunic. In the same figure, the letter N indicates the pupil. Since both this tunic and the aforementioned rays of the arachnoid membrane are made of the same substance and are mutually continuous bodies, also having the same black colour, it is then highly probable that they also share the same nature of movement. There is thus a natural constriction of the parts around N, or an opposite augmentation. Therefore, it also seems natural that the rays at number X are sometimes shortened as they retract into themselves, and in this way, the circle by which they are surrounded and the imprints at number VII near O. O. are narrowed, and at the same time the crystalline at O. O. recedes from the back at P. Conversely, the stretching lengthwise of the rays in number X, which occurs by the attenuation of each one individually, enlarges the circle encompassing their extremes and exhibiting the imprints of the cut-off rays above O. O. in number VII. By this means, the widened circle at O. O. brings the back at P closer to the crystalline suspended in the middle of the circle at O. O.

The explanation of the remaining parts of the eye, useful to propositions 60, 61, 62, 63, and 64, can be found in Platter's work and in my *Optical Part of Astronomy*.

LXV. PROPOSITION

If, for whatever reason, rays from a single luminous point are converging towards the eye, it is impossible to achieve distinct vision.

For every eye is made to see either distant objects distinctly or nearby ones. Distant objects radiate nearly parallel rays, as per XXIII. Nearby objects send diverging rays into the eye, as per XXIV. Accordingly, no distinctly visible point radiates in such a way that its rays converge where they touch the eye.

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208 The figure in the 1604 edition of the *Optics* (and in Donahue's translation) has a small 'a'.

So much about the eye and vision. Next follow the benefits of lenses in relation to the eye.

LXVI. OPTICAL AXIOM

An object of known distance and unknown size is perceived as large when unexpectedly seen as subtending a large angle of sight and as small when seen as subtending a small angle.

This is proven from XIX[209] in *Optics*.

LXVII. OPTICAL AXIOM

The distances between the eye and a small object are in inverse proportion to the angles of sight: that is, the farther any object recedes, the smaller the angle it is perceived to subtend.

LXVIII. *29*

An object of known size and unknown distance, such as the face of an adult human, is perceived as nearby if unexpectedly seen as subtending a large angle of sight with one eye and as distant if subtending a small angle, as per LXVII.

This is the reverse demonstration of the previous one. The vision must indeed be performed with only one eye because the duality and distance between the eyes (and no less a movement of the head, which provides a sequence of multiple eyes separated from each other) renders the unknown distance of an object known if the proportion is perceptible.

LXIX.

Since all remote objects are thus reckoned to be at the same distance,

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209 F. Hammer, KGW IV p. 519, suggests that this should actually be a reference to Witelo Book IV, prop. 19: "All things seen as subtending the same angle, the distances of which with respect to each other are not perceived, appear equal". Prop. LXVI here is similar to prop. 20 in Witelo: "Every object which is seen as subtending a larger angle, appears larger, and what is seen as subtending a smaller angle, smaller".

which is indeed unknown, but is nevertheless perceived as known because it is very large (for the sake of illustration, we conceive of one surface of the heavens upon which all stars are situated, regardless of any differences in the distances), remote objects of unknown size seen as subtending a larger angle are perceived as larger, and those seen as subtending a smaller angle as smaller, absolutely. From LXVI.

If, for any reason, the angle by which the Moon is seen were widened, we would consider the Moon itself to have become larger. For regarding the distance of the Moon, we conceive of nothing other than this: that for whatever angle it is seen to subtend, it remains in the same sky.

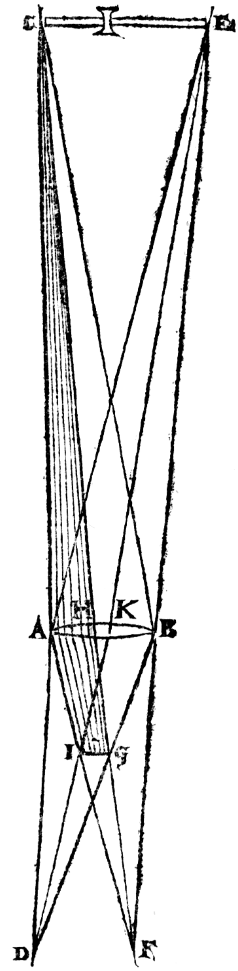
LXX. PROPOSITION

A visible object viewed through a convex lens is shown in its actual orientation if the eye is placed within the propinquity[210] of the point of concurrence of rays coming from the point of the visible object. For example, the object is shown as upright if it is indeed upright, and so forth.

Let AB *be a lens. Let the visible object* CE *be not just a single point but extended. Let the extreme points of the visible object be* C *and* E. *Let the beam from point* C *be represented by* CBF, CHF, CAF, *etc., and let* F *be the point of concurrence. Similarly, let the beam from point* E *be*
30 *represented by* EBD, EKD, EAD, *etc., and the point of concurrence by* D. *Let now the eye be placed at some intermediate location between the points of concurrence* D *and* F *and the lens* AB, *such as at* IG, *with* IG *being the size of the aperture of the pupil. Therefore, with the eye positioned in this manner, it does not admit the entire pencil* EADBE *from point* E, *but only the part* EKIDGBE, *of which the intersection with the lens is in the part* KB. *Likewise,* IG *does not admit the entire pencil* CAFBC *from point* C, *but only the part* CAIFGHC, *of which the intersection with the lens is in the part* AH. *Thus, each ray between* KI *and* BG *shows the right point* E *to the right. And each ray between* AI *and* HG *shows the left point* C *to the left. Therefore, the parts* AHGI *and* KBGI *of the pencils flow towards*

210 That is, closer to the lens than the point of concurrence.

the eye GI *in the same manner in which the vertices* C *and* E *of the pencils or the actual visible points are situated.*



LXXI. PROPOSITION

Every upright representation of distant upright objects through convex lenses is necessarily confused, and the more confused, the farther the convex lens is from the eye.

For by the above propositions XXXIV to XL, the rays CA, CH, *and any others from any point of a distant visible object (let it be point* C *in the previous diagram), which are parallel (as per XXIII*[211]*) until they reach the convex lens, will now converge towards the eye* IG *after refraction has occurred in the convex lens. But, by LXV, when rays from a single point converge towards the eye, distinct vision becomes impossible. And since convergence is a cause of confusion, greater convergence will be a cause of greater confusion. Now, greater convergence occurs when a greater part of the pencil is intercepted by the eye; that is, when the eye is* ***31***
farther from the lens. Therefore, there will be greater confusion of the upright vision if the lens is farther from the eye.

LXXII. PROPOSITION

In some cases, the upright representation of nearby objects through convex lenses is distinct for the presbyopes.

Those who see distant objects distinctly but are dim-sighted for nearby objects are called presbyopes by Aristotle; see LXIV. Thus, such an

211 The original edition refers to prop. XXI, while the KGW edition refers to XXIII, which I have kept here.

individual has accustomed the eyes to parallel rays from each point, as per XXIII. Now, by XXXV and XXXIX, there is some point beyond the lens or perspicillum,[212] *such that rays from a point on a visible object placed there will be parallel as they proceed towards the eye after passing through the lens. Therefore, the visible object is shown as distinct to such individuals through the convex lens.*

Observe also that the demonstration delicately defines the limits of the matters. Nature certainly deviates in both directions with no great inconvenience to vision, except when it deviates excessively.

LXXIII. PROPOSITION

The eye, when placed at the point of concurrence of parallel rays, still sees nearby objects upright.

For an eye located at the point of concurrence of parallel rays (that is, those coming from a remote and distant point, as per XXIII) is still within the boundaries of concurrence of rays from a nearby visible point, as per XLI. Therefore, by LXX, the visible object will still be shown upright.

LXXIV. PROPOSITION

The eye, when placed at the point of concurrence of rays flowing from the point of an object, does not see that radiant point distinctly through the lens but most confusedly of all.[213]

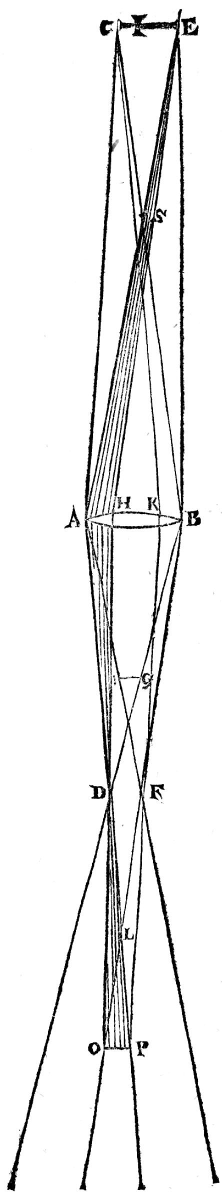
32 *For the rays from a single point, after refraction by the lens, converge towards the point of concurrence. Accordingly, if the eye is at the point of concurrence, they converge towards the eye. By LXV, however, when these rays are converging, their source and origin is not seen distinctly. Since the convergence at that point is the greatest out of all those that can occur through one lens, the confusion there will be the greatest of all.*

212 Clearly the word *perspicillum* is not used in the meaning "telescope" here.

213 See also chapter 5, sect. 3, of the *Optics*.

LXXV. PROPOSITION

The eye, when placed outside the point at which the rays from a point of a visible object concur, sees the points of that visible object in an inverted position through the convex lens.[214]



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I do not claim that, at any distance from the point of concurrence of the rays from a single point, the eye will see the entire visible object inverted. For to see a large part of the visible object, a large distance is required. However, generally speaking, I say that the transition past the point of concurrence of rays from a specific visible object is accompanied by the inversion of that visible object.

For suppose, in the diagram of proposition LXX, that the eye is not at IG *within the points* D *or* F *of concurrence but is placed at* OP *outside these points, at such a distance that the entire visible object* CE *can be seen, and with the left-most ray* AD *from the right point* E, *and the right-most ray* BF *from the left point* C *extended to the point of concurrence (let it be* L*) and beyond; and let the pupil of the eye OP be beyond this concurrence.*

Accordingly, the right point E, *by the ray* EADLP *and its neighbours (which fall upon points near* A *towards* H *on the left side of the lens, and, having been refracted concur at* D, *then diverge again towards the width of the eye* PO*), by these rays, I say, coming from the left part* A *of the lens, the right point* E *irradiates the eye at* OP. *Conversely, the left point* C *radiates into the eye at* OP *by the ray* CBFO *and its*

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neighbours towards K, *which converge towards* F *and afterwards diverge again towards the eye at* OP, *as per XXI. Thus, the left point* C *of the visible object radiates from* BK *on the right side of the lens. However, since the eye does not perceive what happens to the rays in the lens itself, but rather reckons any part of the visible object to be located in the direction whence its rays enter the eye, as per XIX, the visible object* CE *is therefore shown inverted to the eye at* OP.

LXXVI. PROPOSITION

The point of inversion, or the point of intersection of two lines, coming from two points of a visible object and meeting at the centre of the eye, that point, I say, is between the visible object and the lens.

For it will be proven, as above in proposition LXXV, that the right parts of the lens correspond to the left parts of the visible object, and conversely. Hence there is no intersection of the visual cones between the eye and the lens, but between the lens and the visible object. What is true for the whole cones must also necessarily be true for the middle lines of the cones, which fall upon the centre of the pupil, and also for those that fall upon the extremities of the pupil. See the diagram of prop. LXXV, where *the lines* EADLP *and* CBFLO, *which touch the extremes* P *and* O *of the pupil, intersect at the point S. The intersection at* L, *however, is part of the concurrence of the cones* ODP *and* OFP *in* OP, *which is no longer considered here, because this did not invert the position of the object in prop. LXX above. There the cones were* IACHG *and* IKEBG.

LXXVII. PROPOSITION

The presbyopic eye sees almost no objects that are inverted by a convex lens distinctly.

Since the presbyopes, by LXIV, have accustomed the eye to parallel rays, that is, coming from a distant point, it is therefore not adapted for seeing
34 *distinctly when the rays diverge perceptibly from a single point. When there is an inversion of a visible object, all points of the visible object have rays that diverge again towards the eye* OP *after the points of concurrence* D

and F*, as per XXI, such as* DO *and* DP*, as well as* FO *and* FP*. Therefore, the eye of a presbyope does not see distinctly at* OP *except if the width of the pupil* OP *is longer a perceptible and proportionate fraction of the length of* DO*, such that* DO *and* DP *are effectively parallel.*

LXXVIII. PROPOSITION

The myopic eye sees any object inverted by a convex lens distinctly, whether it is near or far, provided the eye is at a specific distance from the concurrence of rays coming from a single point of that visible object.

Myopes, according to Aristotle, are those who see nearby objects distinctly but distant ones dimly. See prop. LXIV.

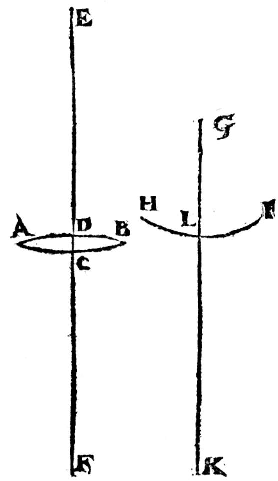
Their eyes are thus accustomed to rays diverging perceptibly from a single point. But, by LXXV, the inversion happens beyond the point of concurrence. By XXI, the rays from a single luminous point C*, which diverged towards the lens* KB *and then converged towards the point of concurrence* F *after passing through the lens, now, having passed through that point, diverge again towards the eye* OP*. Therefore, they become adapted to this eye for distinct vision of that point C.*

I say, however, that the eye must be at one specific location with respect to the points of concurrence D *and* F *of the rays from the visible object* CE *that is to be seen. For the abilities of different eyes are distinguished according to greater and smaller divergence, as per LXIV. At a greater distance of the pupil* OP *from the points of concurrence* D *and* F*, there will be less divergence because the angle* ODP *or* OFP *will be smaller if the base* OP *remains the same but the legs* OD *and* PD *are longer. Therefore, each eye is accommodated by a specific distance from the points of concurrence* D *and* F.

LXXIX. PROPOSITION

A single convex surface defined by a small circle is equivalent, for the purpose of gathering rays into a point, to a lens with two convex surfaces defined by circles twice as large. **35**

Let AB *be equally convex on both sides defined by circles* ADB *and* ACB, *the centres of which are* F *and* E. *Accordingly, by XXXIX, the point of concurrence is at* F. *Take* GL *to be half of* DF *or* CE. *And around the centre* G, *let a circle* HLI *with radius* GL *be drawn, which by itself would cause the refraction of parallel rays coming from the region around the centre* G. *Let* GL *be extended to* K, *and let* LK *be twice* GL, *hence equal to* DF. *Then, by XXXV, parallel rays refracted at* H, L, *and* I *concur at K. Thus, the single convexity* HLI *defined by a small circle produces the same effect that two convexities defined by circles twice as large produce in* AB *because the point of concurrence is equally distant from the dense body in both cases, that is, by the equal distances* DF *and* LK.

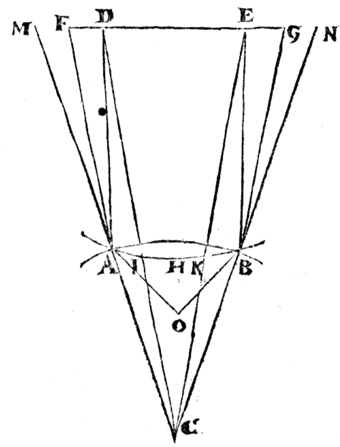


XXC. PROPOSITION

Every image of a visible object seen upright through a convex lens is necessarily larger than its proper size.

For, by LXX inverted, if the image is upright, the eye is within the propinquity of the point of concurrence of the rays flowing from one point of the visible object, and there is no intersection of the cones coming from the visible points into the pupil, or of the lines coming from the same points into the centre of the eye, between the visible object and the eye, as per LXXVI. Let now AB *be the lens,* C *the eye, and* DE *the visible object. When now several points of the visible object are considered, then of the individual lines descending from each point into the centre of the eye, or the other way around, either one only of these will be perpendicular to the lens, or none. Therefore, either all of these are refracted in the lens, or all except one, as per X.*

36 *Now, by LXXIX, two convexities of a lens provide the same refraction as one that comprises both in itself. Therefore, to avoid confusion from the duplicity of convexities, let there be a single convex surface equivalent*



to both, denoted as AHB. *If points* D *and* E *are connected with* C *with straight lines, intersecting the dense convex body at* I *and* K, *it will be clear from what has been stated that these lines will not become the sightlines of points* D *and* E *since they remain straight. For the optical laws require that* CI *deflects from* ID *at the surface* I *and approaches the line which is perpendicular to the surface at point* I, *thus falling inwards from* D *towards* E. *Similarly,* CK, *after refraction, will not continue along* KE *but will fall inwards from* KE *towards* D. *Consequently, the lines* CI *and* CK, *and the angle* ICK *by which the visible object* DE *could have been seen without the lens, do not enclose the visible object* DE *now that the lens is inserted but something smaller, which will be perceived as having the size of the whole object* DE.

In order to enclose all of DE, *rays exterior to* CI *and* CK *must come from the eye, such as* CA *and* CB. *If these are separated from* CI *and* CK *by an appropriate space, they will enclose* DE *after refraction at* A *and* B, *as the sightlines will be* CAD *and* CBE. *However, since the angle* ACB *is greater than* ICK, *through which the visible object is seen with the lens removed, the visible object* DE *will be perceived as larger than it actually is, as per LXVIII. For, by XIX, the eye does not know what happens to the rays* CA *and* CB *as they pass through* A *and* B, *and it assumes they continue in a straight line, as if they were* CAF *and* CBG, *so that* FG, *the imagined size, is larger than* DE.

XXCI.

As the eye is farther removed from the convex lens towards the point of concurrence, it sees a narrower part of the hemisphere through the lens and reckons that part to be smaller.

For since both the lens and what is seen through it on both sides [of the 37
diameter] are discerned by the same angle and are indeed discerned as smaller when the lens is farther removed than when it is close, it follows

that the part seen through the lens when it is farther away is perceived as smaller, as per LXVII. Moreover, a smaller part is actually discerned through it when it is farther away. For in the previous diagram, let the lens AB *be farther from the eye at* C *than from the eye at* O, *and let straight lines be drawn from* O *to* A *and* B. *Since the rays* OA *and* OB *are more interior than* CA *and* CB, *their refracted rays will be more exterior after the intersections at* A *and* B, *as per XI. Let the exterior refracted ray of* OA *be* AM *and the exterior refracted ray of* OB *be* BN. *It is evident, then, that a larger portion of the hemisphere is cut off by the refracted rays* AM *and* BN *coming from the nearby eye* O, *whereas a smaller portion of the hemisphere is cut off by the refracted rays* AD *and* BE *coming from the more distant eye* C. *This will be even more evident if, for the same inclinations of the refracted rays, the eyes at* O *and* C *come together into one, and the lens assumes various positions.*

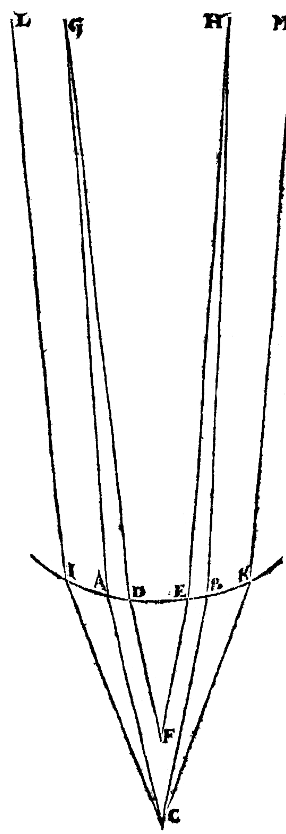
XXCII. PROPOSITION

Suppose that the eye is placed near a lens and observes a distant visible object: when it then recedes back from the lens towards the point of concurrence, it will see the same object as larger than when it was near the lens.

This proposition seems contrary to the previous one; therefore, an explanation is provided. For note that all the objects visible through a distant lens are discerned jointly at a smaller angle, as per LXXXI. However, individual objects, when viewed through either a nearby or distant lens, are seen at a larger angle when viewed through the distant lens. For the angle by which the lens as a whole is seen and the angle by which a certain object is viewed through a particular part of the lens undergo contrary changes. As the lens is farther removed, the former angle decreases while the latter increases, and together with it, the portion of the lens through which that object is viewed also increases, firstly to apprehend the same visible object and then to show it as larger. Indeed, when the eye is at the actual point of concurrence, a single point of the visible object is discerned through the entire lens, whereas from a place closer to the lens, it was discerned through a particular part of the lens, smaller or certainly not larger than the pupil of the eye.

Now to the demonstration. Let, therefore, as above in LXXIX, the power 38 of a lens that is convex on both sides be collated into the surface AB of a dense body that extends up to the visible object. Let that surface be oriented towards the eye. And let the eye be placed at a closer point F and a more distant point C. Now, let the points D and E be on the surface AB, and let the lines FD and FE be drawn from the closer eye at F to these points, forming the angle DFE by which angle and lines the visible object is enclosed. I say that the more distant eye at C requires a larger angle to enclose the same visible object if the object is distant.

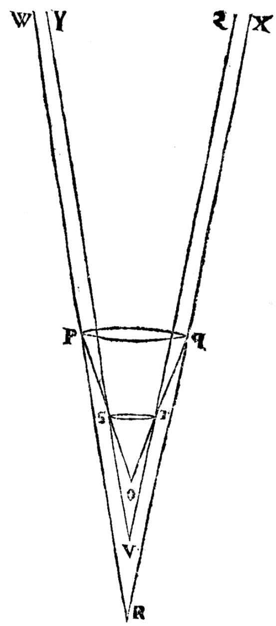
For let the refracted rays DG and EH be drawn from D and E all the way to the visible object. But if the distant visible object is not seen as subtending a larger angle from C, then let it be seen as subtending an equal angle, and let the lines CA and CB be drawn parallel to FD and FE from C until the surface, such that the angles ACB and DFE are equal. Since CA and CB are more inclined above the surface AB than FD and FE, then CA and CB will be refracted more than FD and FE, as per X. Therefore, the refracted rays of CA and CB (because of this and per XXXIV) will concur with the refracted rays of FD and FE, respectively, because CA and FD are parallel, as are CB and FE. Let them concur, and let the points of concurrence be G and H. And let the refracted rays of CA and CB be AG and BH. Therefore, since it is assumed that the visible object is seen as subtending the angle ACB, it will be seen and enclosed by the refracted rays AG and BH. However, it is also seen and enclosed by the refracted rays DG and EH. Therefore, the boundaries of the 39 visible object must necessarily be G and H. Thus, the visible object will not be distant but nearby, which is contrary to the assumption. Therefore, the eye at C will not see this visible object by the rays



CA *and* CB *and an angle* ACB *equal to* DFE, *but by more exterior lines, such as* CI *and* CK, *and by an angle* ICK *that is greater than* ACB *or* DFE. *This will allow the refracted rays* IL *and* KM *of* CI *and* CK, *being virtually parallel to* DG *and* EH, *to be extended and enclose the endpoints of the distant visible object.*

XXCIII. PROPOSITION

If the eye observes the same distant visible object through two individual convex lenses separately, then the visible object will appear with the same size through each lens separately, provided the distance of each lens from the eye is in the same proportion to the diameter of its convexity. But if the proportion is changed, the object will appear larger through that lens whose distance is greater in proportion.



Let O *be the eye and* PQ *a large lens described by the centre* R. *Let the points* P *and* Q *be connected with* O, *and at some points on these lines, let there be a smaller lens* ST, *which is described by the lines* SV *and* TV, *drawn parallel to* PR *and* QR *from their point of concurrence*[215] V *through the points* S *and* T. *Let* OP *and* OQ *be refracted into* PW *and* QX.

Since VS *and* RP *are parallel, and likewise* VT *and* RQ, *the straight lines* OP *and* OQ *intersecting them will form equal angles* OPR *and* OSV, *and likewise* OQR *and* OTV. *But* VTS *and* RQP, *between the lenses and their semidiameters, are equal as well. Thus,* OTS *and* OQP *are also equal since equal angles are removed.*[216] *Therefore,* OT *and* OQ *are*

215 To clarify: R is then the centre of curvature of one of the convex surfaces of the large lens PQ, while V is the centre of curvature of the smaller lens ST. Since the lenses are biconvex, these are also the "points of concurrence".

216 That is, the equal angles OTV and OQR are subtracted from VTS and RQP, which are also equal, to form OTS and OQP: VTS-OTV = RQP-OQR and therefore OTS = OQP.

inclined equally above the lenses TS *and* QP. *Consequently, the refractions* **40**
will be equal in both cases. Therefore, the refracted rays from S *and* T, *denoted as* SY *and* TZ, *respectively, will be parallel to* PW *and* QX. *And since they are parallel, they enclose the same visible object, as per XXIII, subtending the same angle* POQ *or* SOT, *within the limits of sense perception. Hence, the object will be perceived to be of the same size, as per LXVI. Furthermore, the semidiameter* VS *of the lens* ST *is to its distance* SO *from the eye, as the semidiameter* PR *of the lens* PQ *to its distance* PO *from the eye, and vice versa. Thus, the first part of the proposition is demonstrated. Now, for the second part.*

I now say that if there is one proportion between the distances and another between the semidiameters, such as if the eye O *is at a distance* SO *from the lens* ST, *and the eye* V *is at a distance* PV *from lens* PQ, *then the visible objects will appear larger through the lens* PQ, *which has a greater distance from the eye* V *in proportion to the semidiameter* PR, *than the distance* SO *of the lens* ST *from the eye* O *in proportion to its semidiameter* SV. *Indeed, since* OS *is to* SV *as* OP *to* PR, OP *is shorter than* VP.

For by XXCII, with the lens PQ *being considered, visible objects appear larger to the eye* V *than to the eye* O. *But as previously demonstrated, visible objects appear equal to the eye* O *through lenses* ST *and* PQ *in this position. Therefore, visible objects appear larger to the eye* V *through the lens* PQ *than to the eye* O *through the lens* ST.

XXCIV. PROPOSITION

The farther the eye moves beyond the point of concurrence, the smaller the inverted object will appear to it.

The demonstration of this proposition is better understood through explanation and comparison with previous propositions.

For let us start from XXXVII inverted, and let the eye replace the radiant point, since this is equivalent, as per III. Therefore, if the eye is very close to the lens, then its rays diverge as they pass through the lens and also afterwards as they continue towards the visible object, and what was demonstrated in LXX will happen, namely that the object appears upright. As the eye recedes slightly farther away from the lens, the visible objects increase in size, as per XCII, while their number decreases, as per XXCI.

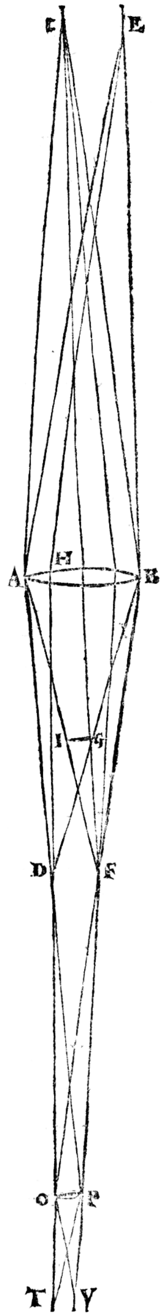
Then, as the eye comes close to the point of concurrence, its rays become
41 *parallel after entering the lens, as per XXXV, inverted. If you move the eye by a nail's width further away from the lens, all rays from the eye refracted through the lens begin to concur, first after the visible object if they were to continue, then in an actual single point of the distant visible object. At that moment, nothing but a single point is discerned of the visible object, and it appears as large as the lens and extremely blurred. If you withdraw the eye slightly further from the lens, the point of concurrence of the rays or lines coming from the eye (refracted in the lens) now shifts away from that visible object and approaches the lens. But since the concurring rays intersect each other and proceed beyond the point of concurrence, as per XXI, these lines drawn from the eye through the lens and continued past their intersection will fall upon the visible object in reverse order as per LXXVI, first capturing the smallest particle of it and a point nearest to it. Then, what was demonstrated in prop. LXXV starts to happen so that some parts of the visible object appear inverted.*

As the eye is withdrawn still farther from there, that intersection descends more and more towards the lens per XLI, and the angle of the intersection becomes greater, enclosing more of the visible objects until the eye is withdrawn to a very great distance. There, the lines from its centre are nearly parallel as they approach the lens, and it happens, as in prop. XXXIV that they meet at a specific and well-determined point on the other side of the lens. Therefore, a portion of the hemisphere corresponding to the size of the angle BFD *in the diagram of prop. XXXIV appears in an inverted position. For as* BF *and* DF *advance further, they intersect again and fall onto the visible objects in that manner.*

However, visible objects located closer to the lens than the intersection after the lens of lines from the centre of the eye are always exempt from this inversion. Hence, it can happen that distant objects appear inverted while closer objects appear upright, both subtending the same angle.

With these things thus established, it appears, firstly (per LXVII), that the angle which the lens is discerned to subtend becomes smaller as the lens is farther from the eye, and with it, the entirety of what is seen inverted through the lens. Secondly, as the lens moves away from the eye, more of the visible hemisphere is enclosed by it, as was just explained. Therefore, in a more distant position of the eye, a greater part is seen jointly, while
42 *appearing smaller, than the smaller part seen in a closer position. Hence, on*

two accounts, individual inverted objects also become smaller if the lens is farther removed from the eye.



XXCV. PROBLEM

To show visible objects as distinct with one convex lens but inverted and smaller.

Let the eye be placed beyond the point of concurrence at a specific point according to its ability. For, by LXXVIII, the myopic eye will see distinctly, but, per LXXV, in an inverted position, and per XXCIV, smaller than the proper size, just as if the eye required some distant point of distinct vision.

So much about a single convex lens.
Now about convex lenses joined together.

XXCVI. PROBLEM

To show visible objects as larger and distinct but in an inverted position, with two convex lenses.[217]

Let two convex lenses be arranged in such a way with respect to the eye that the more distant lens by itself would send an inverted image to the eye; not, however, a distinct image, but with the eye being placed closer to the lens than the point at which objects are shown as distinct, as per LXXIIX. As if, in the diagram of prop. LXXV, the divergence of the rays DO *and* DP *from one point and their angle* ODP *were too great for the eye, and the eye at* OP *were outside the points of concurrence* D *and* E. *Then let a closer lens be inserted between that first lens and the eye, in such a position that the eye is within the point of concurrence of the*

217 Here we have the description of the astronomical (or "Keplerian") telescope.

closer lens as if the eye were at IG *in the diagrams of prop. LXX and prop. LXXV. In this way, the eye will see objects upright but likewise confused through the latter lens by itself but for the opposite reason, as per prop. LXXI. Therefore, since the divergence from the more distant lens is too great, the contrary convergence from the closer lens will now remedy that excessive divergence so that it is corrected and approaches the eye amended in order to furnish distinct vision.*

43 *And although the image of a visible object is inverted by one lens, the closer lens does not invert again what it receives from the more distant one but transmits it to the eye as it was received, as per the assumption. And because it receives an inverted image with respect to the visible object, it, therefore, also transmits to the eye an inverted image with respect to the visible object.*

And because the inverted image appears larger than the object itself near the point of concurrence, of equal size when more distant, and smaller when still more distant, as per XXCIV, this inverted image, when magnified by the closer lens, will, therefore, in the first two cases certainly emerge larger than the object itself, and in the last case either larger, equal, or smaller, according to the proportion of the lenses with respect to each other,[218] *which is at the discretion of the craftsman. However, the image will certainly be larger than that received by the lens closer to the eye from the more distant lens, as per XXC.*

XXCVII. PROBLEM

To show visible objects as upright and distinct, but smaller, with two convex lenses.

These two convex lenses must have a sufficient difference between their convexities. Let then the eye be placed outside both points of concurrence, closer to one point of distinct vision and farther from the other point of distinct vision, so that the inverted images would not be perceived distinctly in either of the individual positions. For if the lenses are placed in this arrangement with the eye along the same line, their opposite

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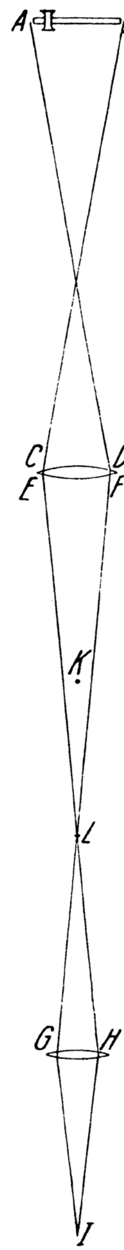
218 Expressed in modern terms, for focal lengths F and f of the objective and ocular lens, respectively, the magnification m is given by $m = F/f$.

defects will cancel each other, and distinct vision will follow.

However, in order for the image to also be upright, it must be inverted twice. To achieve this, the closer lens itself must also be placed beyond the points of concurrence of the more distant lens.

For let AB *be the visible object, and* CD, EF *the lens* 44
farther from the eye. Let K *be the point of concurrence. Now, if the image of* AB *is inverted by this one lens, then the point where the image appears inverted will be farther removed from the lens, beyond* K, *as per LXXV. Let that position be* L. *Because the* species *of the lens* EF, *along with the inverted image of* AB, *must be inverted again by the other lens* GH, *and the inverted image of the object* AB *is enclosed by the lines* ADFL *and* BCEL, *the lens* GH *must therefore be placed beyond* L, *as per LXXVI.* L *was, however, withdrawn from the lens* EF *beyond the point of concurrence* K. *The second lens* GH *will, therefore, be placed much farther beyond* K, *its point of concurrence, so that the lines* FLG *and* ELH, *coming from the extremities of the object and having undergone a second refraction at* G *and* H, *will eventually be united again and be gathered into the eye at* I.

Finally, this image is smaller than the visible object. Firstly, the species *of the lens* EF *itself (and of those objects seen through it), inverted by the lens* GH *and appearing distinct, will be smaller at* I, *as per XXCV. But by the same proposition, with the eye placed at* L, *the visible object* AB *inverted by the lens* CD *would also appear to occupy less space in the lens than its actual size. This is because* L *cannot be very close to the point of concurrence* K *so as to avoid excessive confusion. For* L *must be near the point of distinct vision, just as* I. *Thus, on two accounts, the visible object* AB *is shown as small.*



XXCIIX. PROPOSITION. PROBLEM

To depict visible objects on paper in an upright position with two convex lenses.

The solution to this problem has been sought for a long time.[219] *Let the convex lenses be arranged as in prop. XXCVII; that is to say, such that the*
45 *lens closer to the paper is placed beyond the point of concurrence* K. *For the pencils ending in a sharp point near* K *are widened again beyond* K *and diverge from each other. Accordingly, the second convex lens, receiving these pencils, refracts them anew and sharpens each one individually, and causes them all to converge mutually again towards a new intersection.*[220] *Having passed this point, they diverge mutually once more and thus fall upon the paper with their vertices in the original order. For in the diagram of prop. LXXXVI this is no different than if the visible object* CE *were now transferred to a picture at* DF[221] *and* OP *were no longer an eye but a second lens below it. But if the lens* OP *is placed just below the picture* DF, *the picture* TV *requires the paper to be distant, and the picture will be large.*

XXCIX. PROBLEM

To show visible objects as upright, distinct, and larger with three convex lenses.

Let two convex lenses and the eye be arranged such that the same as was stated in prop. XXCVII will happen, except for this single difference: the eye is closer to the point of distinction and sees confusedly. For the third convex lens, applied as was done in prop. XXCVI with the second lens in that proposition, that is, such that the eye is closer to the lens than its point of concurrence, will make the species *(which was inverted twice and is now upright, and therefore reduced in size) larger again. If the proportions of the lenses are correct, the magnification will exceed the previous reduction produced by the two lenses alone in XXCVII. Distinctness will follow from the same causes as those stated in XXCVI.*

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219 In the *Natural Magick (*Book XVII, chapter VI), della Porta writes about the problem of making images appear upright when using the camera obscura as a painter's instrument: "This is a great secret: many have tryed it, but none could obtain it." He suggests that it can be achieved by combining a convex glass and a concave mirror.

220 This intersection of the pencils corresponds, in modern terminology, to the exit pupil. See e.g. H. G. J. Rutten & M. A. M. van Venrooij (1988), sect. 3.3 and fig 3.10.

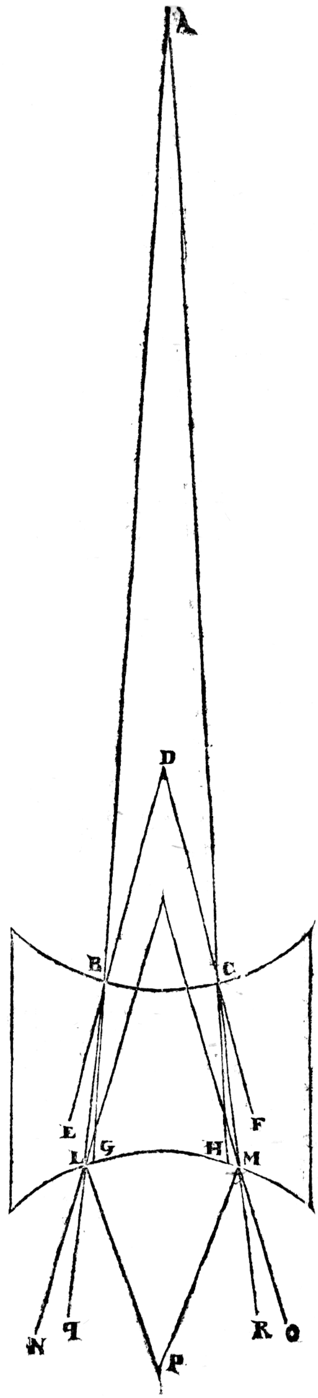
221 Here, Kepler comes very close to using "picture" not just to describe something that "really exists on a surface" but also independently of the existence of a physical surface in the focal plane of a lens.

So much about convex lenses.
Now about the concave ones.

XC. PROPOSITION

If rays from a single luminous point, whether they are parallel or diverging, pass through the concave surface of a denser medium (assuming that the luminous point is outside the centre of [curvature of] the surface), they will diverge more as they pass through the body of the dense medium.

Let the diverging rays AB *and* AC *descend from the luminous point* A *to the concave surface* BC *of the denser medium, the centre* 46
D *of which is contained between* AB *and* AC. *I say that after undergoing refraction at* B *and* C, *the rays* AB *and* AC *will diverge more after* BC. *For let the normals* DB *and* DC *be drawn from the centre* D *to the surface, and let them be extended somewhat further to* E *and* F. *Let also* AB *and* AC *be extended to* G *and* H. *Now, since* AB *is inclined above the surface of the denser medium, it will be refracted at* B *towards the normal* BE, *as per II. Let* BL *be the refracted ray. Similarly,* AC *will also be refracted at* C, *and the refracted ray* CM *will deflect from* CH *towards the normal* CF. *But* DBE *and* DCF *diverge more than* AG *and* AH *because they are drawn from a closer point, while* AG *and* AH *are drawn from a more distant one through the same points* B *and* C. *And* BL *and* CM *approach those more diverging lines whilst they recede from the less diverging lines* BG *and* CH, *and they, therefore, diverge more than* AB *and* AC, *and this is within the denser body.*



XCI. PROPOSITION

If the luminous point is closer to the lens than the centre of the concavity, the diverging rays, after refraction, will diverge less within the dense body.

For let now A *be the centre of the circle and* D *the radiant point.* ABG *and* ACH *are thus the normals and* DB *and* DC *the rays, which, instead of continuing along the paths* BE *and* CF, *are refracted at the points* B *and* C
47 *and approach the normals* BG *and* CH, *thus becoming* BL *and* CM, *which diverge less than* BE *and* CF.

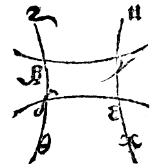
XCII.

Diverging rays passing through a denser body towards its concave boundary will diverge even more upon crossing it.

Let the rays BL *and* CM *diverge towards the concave boundary* LM *of the denser body, with the centre at* P, *from which the normals* PL *and* PM *extend to the points* L *and* M. *And let* BL *and* CM *be extended beyond the intersections at* L *and* M *to the points* Q *and* R. *Because the rays* BL *and CM travelling within the denser body will fall obliquely upon the surface* LM *of the more tenuous substance at* P, *or, which is the same, upon the boundary of the denser body in which they are, they will be refracted away from the normals* PL *and* PM, *and the refracted rays will not be* LQ *and* MR *but more exterior, as per II. Let* LN *and* MO *represent these refracted rays. And since* BLQ *and* CMR *diverge,* LN *and* MO *diverge even more.*

XCIII. PROPOSITION

If rays travel through a denser body parallel to each other, they will diverge after crossing its concave boundary.

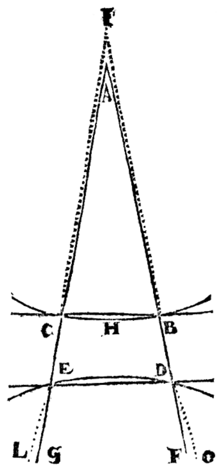


Let $\beta\delta$ *and* $\gamma\varepsilon$ *be parallel rays, of which not more than one can be perpendicular to* $\beta\gamma$, *while the rest, arriving obliquely, will be refracted away from their normals, as per II. Therefore, as before, the rays* $\beta\zeta$

and γη diverge when emerging outside, and rays emerging through the other surface δε will similarly diverge into δθ and εκ.

XCIV. PROPOSITION

Rays diverging towards a lens, regardless of the position of the radiant point with respect to the lens, always diverge more after passing through the lens, whether the lens is concave on both sides in whatever way or even plane on the other side.

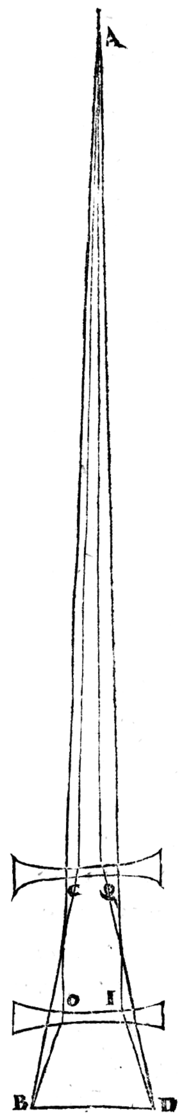


For if this were not true, it would not hold true for a position of the radiant point within the centre of the concavity because, in this case, as per XCI, the divergence within the body is less. Likewise, it would not be true if the lens were plane on one side. And it would least of all hold true if both 48 *conditions were met simultaneously. However, it is true when both conditions are met. For let there be a dense parallelepiped* CB, ED, *with rays* EC *and* DB *inclined towards each other within it at equal angles* CED *and* BDE*: these will be refracted at the points* C, E, B, *and* D. *The refracted rays* EG *and* CA *will be parallel as per III, as will* DF *and* BA, *because* CB *and* ED *are parallel. Therefore, the divergence of* AC *and* AB *is equal to that of* EG *and* DF. *Now, let* CB *be hollowed out by the circle* CHB. *The inclination of* EC *above the concave surface will then be diminished, and the refraction will, therefore, also be reduced. The refracted rays will thus lie more outwards, like* CI *and* BI. *Therefore,* IC *and* IB *will now diverge less than* EG *and* DF. *And much less if* ED *is also hollowed out, because* CE *will then be more inclined above the new surface. And the refracted rays will then diverge more than* EG *and* DF *do now, like* EL *and* DO.

XCV. PROPOSITION

Distant objects are shown as distinct if a sufficiently concave lens is placed at one point before a myopic eye.

For distant points radiate parallel rays, as per XXIII. Since myopes are accustomed to seeing nearby things, they are thus accustomed to diverging rays, as per XXIV, and therefore see distant objects confusedly. But concave lenses make parallel rays diverge, as per XC. Therefore, they cause the points of their parallel rays to be seen distinctly. Not, however, for every position of the concave lens. For the same point A, *radiating into the pupil of the eye* BD *through the concave lens* CE *located farther from the eye* BD, *utilises a small portion of the lens* CE, *inasmuch as what is radiated into a wider area strays around the eye by excessive divergence. Conversely, the same point* A *utilises a larger portion* OI *of the nearer lens* OI *to spread the rays from* A *across the entire pupil* BD. *But the small part* CE *is closer to the perpendicular drawn from* A *to the lens than the large part* OI, *and the inclination of the rays* AC *and* AE
49 *above the surface is, therefore, smaller than that of the rays* AO *and* AI, *and the refractions* ACB *and* AED *are therefore less than* AOB *and* AID, *as per X, and the divergence of* CB *and* ED *is thus less than of* OB *and* ID. *However, a particular divergence benefits each eye and, therefore, a particular position of each lens.*



XCVI. PROPOSITION

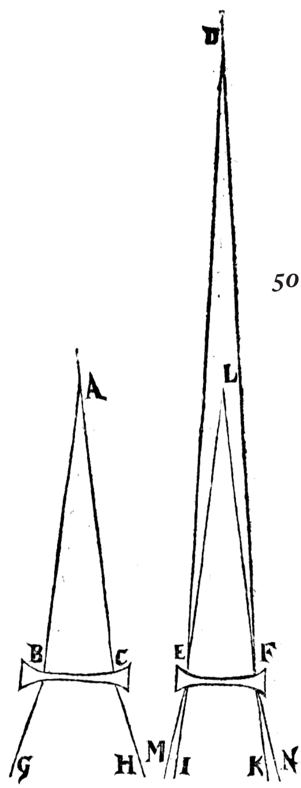
Visible objects are shown as smaller through concave lenses.

For in the previous diagram, let now BD *be the visible object and* A *the centre of the eye. Now, since the rays from* A *are refracted outwards in the lens* CE, *as per XCIV, it is clear that if the lines* BA *and* DA *were drawn, the angle* BAD, *by which the visible object would be seen by the free eye, would be greater than the angle* CAE, *by which* DB *is seen through the lens* CE, *and therefore is perceived as smaller, as per LXVI. For the eye does not know what happens to the rays* AC *and* AE *at points* C *and* E, *and it therefore* *assumes that they continue straight, as per XIX. But if this were to happen, they would, in fact, only intercept a part of the visible object* BD. *However, the refracted rays capture the entire visible object. Therefore, the* species *of the whole equals a part of the whole, and therefore, it is smaller than the whole itself.*

XCVII. PROPOSITION

If a concave lens is farther removed from the eye, fewer visible objects will come through the concave lens to the eye.

Let A *be an eye and* BC *the closer lens. Let again* D *be an eye and* EF *the more distant lens that is equal to the other lens* BC. *Therefore, the base* EF *is equal to the base* BC, *but the sides* DE *and* DF *are longer than the sides* AB *and* AC. *The angle* BAC *is, therefore, greater than the angle* EDF. *Now, let the rays be refracted and let the refracted rays be* BG, CH, *and* EI, FK. *By XCIV,* BG *and* CH *will always* 50
diverge more than EI *and* FK. *For let the triangle* ELF *be equivalent to* BAC. *Therefore, since* DE *and* LE *descend from* D *and* L *to the same point* E *on the surface of the denser body, they will intersect each other upon refraction at* E, *and the inner ray* LE *emerges as the outer ray* EM. *In the same way,* LF *emerges as* FN, *as per XI. Thus,* EM *and* FN *diverge more than* EI *and* FK *and thereby intercept more of the hemisphere. Therefore,* BG *and* CH, *being refracted by the closer lens, also intercept more than* EI *and* FK, *which are refracted by the more distant lens.*

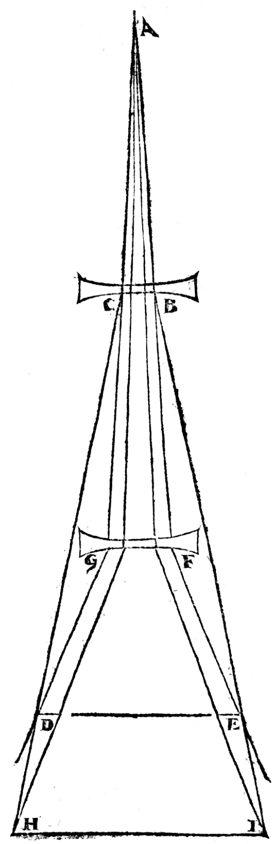


XCIIX. PROPOSITION

If a concave lens is farther removed from the eye, visible objects are shown as smaller as long as the lens is not closer to the visible object than to the eye.

For within the limits of sense perception, the apparent size of the lens decreases equally with its distance, as per LXVII. But it does not equally receive fewer visible objects at a greater distance. For even though it does receive a decreasing number of objects, as per XCVII, this reduction is an exceedingly small part of the whole when the objects are far removed because the refractions remain almost unchanged with greater distance since the inclinations (in the previous diagram) of the rays LE, DE, *etc. above the lens* EF *hardly change with greater distance. Therefore, more is subtracted from the apparent size than from the quantity of objects seen through the lens. Consequently, all objects are discerned jointly at a smaller angle, and therefore also individual ones.*

51 *Put differently: Let* A *be the eye and* ABF *and* ACG *straight rays, enclosing the angle* FAG. *Let them intersect a nearby lens* BC *and a distant one* FG. *Therefore, they will be refracted outwards at points* B *and* C, *as per XCIV. Let* BE *and* CD *be the refracted rays. But since* AF *and* AG *intercept a larger part of the lens* FG, *the refraction in the points* F and G *will also be greater than in* BC, *as per XI. Hence, the refracted rays emerging at* F *and* G *will diverge more than those emerging at* B *and* C *and will thus concur with them. Let them concur and let the points of concurrence be at* E *and* D, *and let the refracted rays be* FE *and* GD. *Since* FE *and* GD *become more exterior than* BE *and* CD *after the concurrence and intersection, no visible object (except one whose ends are at the very points of concurrence* E *and* D*) will be seen at the same angle* BAC *or* FAG *through both the nearby and the distant lens. For visible objects that are farther removed than* ED, *such as the visible object* HI *enclosed by the rays* BI *and* CH, *which are refracted by the closer lens, will not be enclosed by the refracted rays* FE *and* GD, *which approach the eye at the same angle* FAG, *but by those interior to* FG, *which come to the eye at* A *at a smaller angle. Therefore, objects*

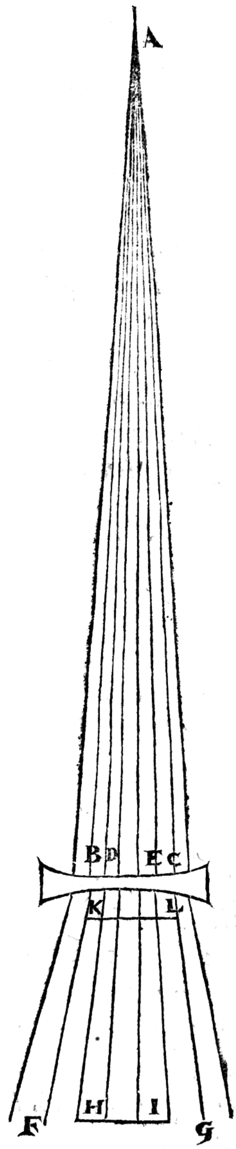


appear smaller through the distant lens GF *than through the nearby lens* BC, *as per LXVI.*

XCIX. PROPOSITION 52

If a concave lens is placed near the eye or at a certain distance common to all people, as when spectacles are riding on the nose, then a lens adapted to each individual is required in order to bring about distinct vision.

For, by XCV, any concave lens has a specific distance, according to the ability of the eye, for distinct vision. Therefore, if the choice of distance is removed, a choice of lenses must be granted to the eye, or it will see distant objects confusedly. For either the lens will not be sufficiently concave, and then it will not remove confusion arising from the parallelism of rays, or it will be too concave, inducing too much divergence and confusion that is contrary to the previous one.



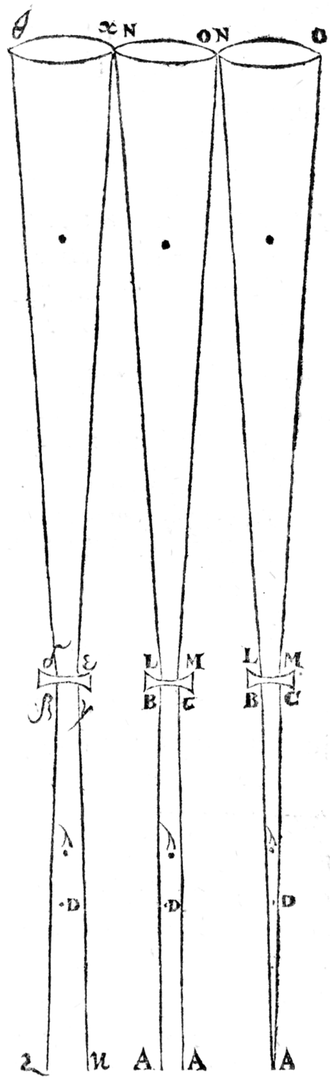
C. PROPOSITION

Lenses which, due to excessive concavity, render objects near the eye confused, will render them distinct from a certain distance, and contrariwise.

This is as if prop. XCV were reversed. For let the visible point A *radiate into the concave lens* BC*: then all rays will diverge from each other after refraction, as per XCI and XCIV, and therefore diverge more the farther they are from each other. Let the diversion of the rays* AB *and* AC *into* BF *and* CG *be too great for the eye. Let instead the rays* AD *and* AE *diverging into* DH *and* EI *be*

suitable for the eye. Let the width of the pupil be HI *and its position be at* HI, *where it encloses the diverging rays suited for it. If it had enclosed the diverging rays* F and G, *faulty and confused vision of the point A would*
53 *have resulted. But the width* HI *of the pupil, when placed near the lens at* KL, *already encompasses and intercepts the excessively diverging rays* F *and* G. *Therefore, point* A *will be seen confusedly if the eye is located at* KL, *but distinctly if the eye is located at* HI.

So far separately about convex lenses and separately about concave ones. Now about concave and convex lenses joined together.



CI. DEFINITION

Let the word *Tube* be used for an opaque hollow cylinder whose two openings are closed by transparent glasses; that is, for that ocular instrument by which we observe distant things as if close at hand.

CII.

One of its openings, with its glass, is turned towards the eye in a suitable position, the other towards the visible object.

CIII. POSTULATE

In order for a line to pass through the centres of the convexities and concavities of each glass in the tube, it should be one and the same. That is, so that the glasses are parallel and the tube rests on them at right angles.

CIV.

If the rays from a single point converge after they have undergone refraction by passing through a convex lens and are intercepted by a 54
concave lens before they would have reached their point of concurrence, then either the point of concurrence will be extended into the distance or the rays will proceed in parallel onwards, or they will diverge again.

For let NL *and* OM *converge towards the concave lens* LM *as if they were going to concur at the point λ. Then, after refraction has occurred at* L and M, *the refracted rays* LB *and* MC *converge less as they advance through the dense body towards* BC, *the other concave surface, as if they were going to concur at point* D, *as per XCII inverted. In the same way, after* LB *and* MC *have undergone a second refraction at* B and C, *the refracted rays* BA *and* CA *converge still less and finally concur at* A [right]. *The concurrence at* A *has thus been extended, for it should have occurred at λ.*

But if the refraction is slightly greater, then the last refracted rays BA *and* CA *will extend infinitely before concurring, as per XC inverted* [middle].

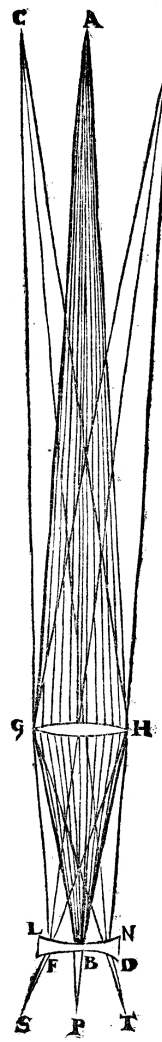
Finally, if the first refraction is so great that the rays θδ and κε, converging towards δε, become the parallel rays δβ and εγ in the dense body, then by XCIII inverted, they diverge again into βζ and γη [left].

CV. PROBLEM

To depict visible objects on paper with a concave and a convex lens, making them appear larger than they would with only a single convex lens, but inverted.

In the diagram of prop. XLIV, let GH *be a convex lens, with the points of concurrence, or the apexes of the pencils, at* F, B, *and* D. *Let the concave lens* LN *be inserted slightly above* FBD. *Then, the visible object* CAE *will first be depicted above the concave lens near* DBF, *but slightly more confusedly because the concave lens intercepts the apexes of the pencils; and it will be depicted in an inverted position because the intersection of the pencils has already occurred in* GH *and the apexes of the pencils have already been spread out from each other, each one being narrowed into itself. Therefore, as the individual pencils pass through the concave lens, as*

per CIV, they either terminate into sharp points at a greater distance at S, P, *and* T*, and then the picture depicted on the paper there will be distinct, or the rays of each individual pencil proceed in parallel, and then the picture*
55 *remains in the slight confusion in which it first entered the concave lens, or finally, they diverge and the pencils spread out, and then the picture becomes more and more confused as the paper is moved away from the concave lens. However, the picture* SPT *is rendered larger than the picture* FBD *by the convex lens* GH *alone because the pencils* F *and* D, which *are refracted in the concave lens* LN, *curve outwards into* S and T*, XC, and the outer ones always more than the inner ones, as per II.*



CVI. NOTE

When Giovanni Battista della Porta professes to first collect the rays of the Sun and then to send them out to infinity and thus combust, it seems that this should be understood as referring to lenses,[222] although he speaks of mirrors, for he intentionally concealed the meaning.[223] But if it is understood as referring to lenses, the trick will be none other than to first collect many rays with a convex lens and then receive the collected rays very close to the point of concurrence with a concave lens, which makes parallel rays out of the converging ones, as stated in prop. CV. Accordingly, note those points in prop. LVI that are stated against this. To these, I now furthermore add that, even if you correct in Porta's words the part about the infinite burning line so that it is equivalent to a burning cone, stretched out as much as you wish, so the burning can still be

222 *perspicillis*.

223 The "burning line" is indeed part of della Porta's discussion of parabolic mirrors, but the following chapter (XIX) of Book 17 of the *Natural Magick* has the title "Fire is kindled more forcible by refraction", and further down, "By refraction, kindle fire afar off".

sought through the intersection of the rays at the end of the cone; then, nevertheless, nothing will be accomplished. For if intersection causes burning, a strong intersection will produce strong burning, and a weak intersection will produce weak burning. But at the vertex of an extremely long cone, the intersection will be extremely weak. ***56***

CVII. PROPOSITION

If a concave lens placed near the eye by itself shows visible objects as confused, then any convex lens defined by a larger circle, placed at one specific distance from the concave lens, will distinguish and magnify the visible objects.

For by C, concave lenses defined by a too-narrow circle will render objects confused if applied near the eye due to excessive divergence of the rays. But by LXXI, the rays from a single point passing through a single convex lens will cause confused vision to an eye placed closer to the lens than the centre of concurrence, owing to the convergence of the rays.

And by CIV, the former excess of divergence and the latter convergence cancel each other out when the lenses are placed in a tube. Therefore, with the convergence removed and the excessive divergence corrected, distinct vision will follow. However, the excess of divergence in any concave lens placed near the eye is [only] removed when the convex lens is at a specific distance from the eye. For when the convex lens is near the eye, the correction of the excess divergence (by convergence) is small, as when a concave lens is placed at IG *in the diagram of prop. LXX, and the extreme rays* AI and HG *cut off a portion of the concave lens* IG *and converge at a small angle* IFG. *On the other hand, if the convex lens is farther removed from the eye, it will provide a large amount of correction, as when the concave lens, together with the eye, is placed slightly above* F. *The extreme rays from a single point* C *will then be* AF *and* BF, *cutting off the same part of the concave lens with a larger angle* AFB.

A convex lens defined by a larger circle is required, for if the circle of convexity were equal to the circle of concavity, so that the convexity of the former might be placed within the concavity of the latter, and the remaining
convexity of the former were about parallel to the remaining concavity of the latter, then the closely joined lenses would nearly compensate each other ***57***

and one would abolish the effect of the other so that the eye struggling with confusion when seeing distant objects would have no, or only a worthless, remedy available in the excess of [concavity in] the second lens. However, if the connected convex lens were separated from the concave one, the rays incident upon the concave lens would converge still more and thus could not even be made parallel by the concave lens, let alone diverge. The same applies even more to a convex lens defined by a smaller circle. Therefore, the only adequate convexity that remains is that of a larger circle.

Finally, I say that the species *of visible objects is enlarged if the circle of the convexity is larger. For by XXC, a single convex lens magnifies visible objects. While it is true, as per XCVI, that a single concave lens reduces the apparent size of visible objects, and it is also true that both the convex lens itself and the objects seen through it appear larger if the convex lens is alone than when the concave lens is interposed, nevertheless, per XXCII and XCIIX, this enlargement and reduction is greater if the lenses are more distant. Therefore, since the concave lens is near the eye, the reduction caused by it will be almost negligible, and since the convex lens is farther removed from the eye, it will cause a greater enlargement.*

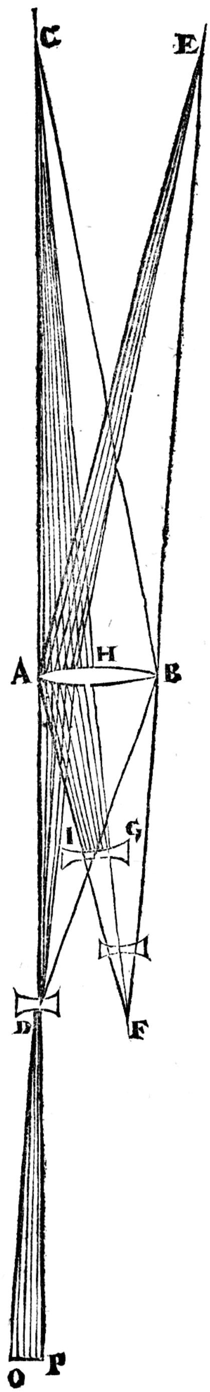
CIIX. PROPOSITION

If a convex lens is placed at any distance from the eye, then any concave lens which would show visible objects as confused if applied to the eye by itself, and which has a concavity defined by a smaller circle than what the convex lens uses, will show visible objects as distinct when placed at a specific distance and position between the eye and the convex lens.

This is nearly the reverse of the previous proposition but less constrained. In the former, the position of the concave lens near the eye was given and,
58 therefore, unique, while the position of the convex lens could be chosen. Here, however, the position of the convex lens is given but is not unique, as it can indeed vary both quantitatively and qualitatively, while the position of the concave lens can be chosen.

Let first this quality of the position of the convex lens be given: that the eye is within the point of concurrence. Then the affinity of this proposition with the previous one is greater and more appropriate for consideration of the ocular tube.

Then, in the adjacent diagram, adopted from prop. LXX, the position of the concave lens and the eye will be between the convex lens AB *and the points of concurrence* D *and* F, *for example at* IG, *and the convergence of the rays* AI *and* HG *will be measured by the angle* IFG. *To avoid hindering distinct vision, this convergence must at least be removed, for the presbyopic eye, so that the rays become parallel, or a divergence must, furthermore, be induced for the myopic eye. But by CIV, both can be achieved by placing the concave lens at some point before the points of concurrence. However, the concavity of that lens should be defined by a smaller circle than that used for the convex lens, which is demonstrated as in prop. CVII. But also the concave lens by itself, when placed near the eye, should render visible objects confused. Because that which compensates confusion caused by the convex lens must also cause confusion by the opposite cause.*



Secondly, let that quality of the eye's position whereby it is placed outside the points of concurrence be considered so that in the adjacent diagram, adopted from prop. LXX and prop. LXXV, the eye is at OP, *beyond* D *and* F. *Then, with the concave lens placed within the point of concurrence* D *or* F, *it can be brought about, as per CIV, that there will be no concurrence but that the rays diverge again and thus reach the eye at* OP. *However, in this case, many circumstances are required. Firstly, the concave lens must be defined by a small circle. For if it were defined by a large one, all the rays between* AD *and* BD *would intercept only a small part of the circle, close to the perpendicular, and refraction would thus be of little effect and not enough to remove the convergence. This is common to this case and the previous one. Secondly, if the concavity of the lens is defined by a small circle, so that it could induce*

divergence, it still does not send all those diverging rays to the eye, which is
59 *placed far beyond the points of concurrence* D *and* F. *For if the rays diverge, they stray around the eye placed at a distance. Therefore, only a few rays remain, transmitted right through the middle of a very narrow portion of the convex lens (or through some other point, according to the position of the concave lens) towards the very lowest part of the concave lens* D, *near the perpendicular, and these rays have almost no divergence and can therefore be considered parallel. Hence, they only serve the presbyopic eye. Thirdly, this configuration will transmit a very small part of the visible object to the eye, owing to the distance of the eye,* OP, *both from the convex lens* AB *(by what was mentioned), and from the concave lens to be placed above* D *or* F, *as per XCVII, and whatever is transmitted is moreover [seen] at a very small angle, as per XCIIX.*

CIX. PROPOSITION

In instruments that show visible objects as larger and distinct, the concave lens is never placed very far from the points of concurrence that are located after the convex lens.

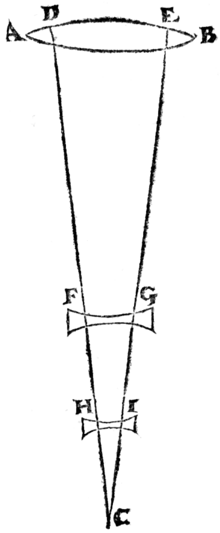
For in order to show the visible objects with the highest possible magnification, the concave lens must be placed near the eye, as per XCIIX. However, the convex lens must be placed far from the eye, as per XXCII. Therefore, it must also be far from the concave lens; yet the position of the concave lens, as per CIV, is between the convex lens and its point of concurrence. So, while the convex lens is placed far from the concave lens, the point of concurrence will be near the concave lens.

CX. PROPOSITION

For a given convex lens, out of any concave lenses to be placed near the eye, that lens of which the concavity is defined by the smallest circle must be placed farthest from the convex lens and closest to the point of concurrence.

Let the part DE *of the convex lens* AB *transmit rays coming from the same point, and let the concurrence be at* C.

Then, since the convergence of the rays DC *and* EC *is the same through a given convex lens* AB *(or the same given part* DE *of it), one and the same remedy must be applied, namely [the same] divergence through the concave lenses. But divergence is caused by refraction, and refraction of the same ray, such as* DC, *can be the same only in similar parts of unequal concave lenses. Accordingly, let* FG *and* HI *be concave lenses. And since any parts* FG *and* HI *are similar with respect to the concavity of their own lens, they must indeed be cut by the same rays* DC *and* EC. *Thus, as the part* FG *of the larger concavity is to the similar part* HI *of the smaller concavity, so is its larger distance* FC *from the concurrence to the smaller distance* HC. *But if* HI *is less distant from* C *than* FG, *then the same* HI *is more distant from* DE *than the lens* FG *of which the concavity is defined by a larger circle.* ***60***



CXI. PROPOSITION

If one and the same concave lens is placed near the eye in order to show visible objects as distinct with different convex lenses, it must be placed at the same distance from the points of concurrence of all of them.

One concave lens provides only one remedy and, therefore, corrects only one single convergence of rays. But at the same distance of the concave lens from the concurrences of any convex lenses, the convergence of rays that are actually intercepted by the same concave lens is the same. For even if one of the convex lenses is much wider, and its outermost rays converge more, they will nevertheless stray around the concave lens or around that part of the lens whence the refracted rays can enter the pupil of the eye.

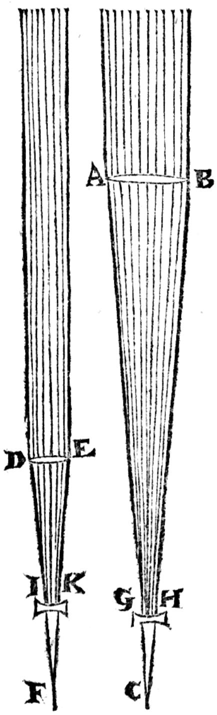
CXII. PROPOSITION

For a concave lens placed near the eye, convex lenses defined by a large circle require a large distance from the concave lens and the eye, while those defined by a small circle require a short distance. ***61***

For by CIX, the eye is near the point of concurrence, and by CXI, the concave lens, which is always of the same kind, is at the same distance from the concurrences of all convex lenses. But the concurrences are unequally distant from their respective convex lenses. For they are far from lenses with convexities defined by a large circle and less so from those defined by a small circle, as per XXXIX. Now, when equal quantities are subtracted from unequal quantities, the remainders will still be unequal, and the distance between the concave and convex lens is always less, by a constant distance between the concurrence and the concave lens than the distance between the convex lens and the point of concurrence. The concave lens (and the eye) will, therefore, be farther removed from a convex lens defined by a larger circle than from one defined by a smaller circle.

CXIII. PROPOSITION

For a given convex lens, a concave lens defined by a smaller circle will show visible objects as larger, and one defined by a larger circle will show them as smaller.[224]



For by CX, concave lenses defined by a small circle, together with the eye with which they are closely joined, must recede farther away from the convex lens in order for distinct vision to be achieved. But by XXCII, the farther the eye recedes from the convex lens towards the point of concurrence, the larger it will see visible objects. Therefore, through a concave lens defined by a smaller circle, the eye will see distinctly seen objects as larger than through a concave lens defined by a larger circle.

224 As in the case of two convex lenses, the magnification m is given by the ratio of the focal lengths F of the objective (convex) lens and f of the ocular (concave) lens: $m = -F/f$ (where the negative sign accounts for the negative focal length of a concave lens). Here, as previously, one may substitute the radii of curvature for the focal lengths. Several of the following propositions follow from this simple relation.

CXIV. PROPOSITION

If the distance between a concave lens and a convex lens is increased by a very small amount, visible objects are greatly enlarged.

For by CIX, in the following diagram, GH *and* IK *are very close to the points* C *and* F. *And if the very short space* IF *is traversed and the eye* IK
is placed at F, *then the size of a single point, from which all the rays flow* **62**
onto DE, *increases to some magnitude equal to the entire lens* DE *so that the point is seen by the angle* DFE, *which is an infinite multiplication. Therefore, a slight movement has a great effect.*

CXV. PROPOSITION

For a given concave lens placed near the eye, a convex lens of which the convexity is defined by a smaller circle will show visible objects as smaller, while one defined by a larger circle will show them as larger.

Let AB *be a convex lens with a larger semidiameter*[225] AC *and* DE *a lens with a smaller semidiameter* DF. *The points of concurrence will then be at C and F, as per XXXIX. In either case, let a concave lens* GH *or* IK *be placed near the eye, causing a certain divergence of parallel rays with a certain part of the lens. Therefore, since the concave lens placed at GH and* IK *is one and the same, it will be in the same position relative to the points of concurrence* C *and* F, *as per CXI. The eye is near the lens in both cases, as per the premise. Thus, with equal apexes* GC *and* IF *removed from the unequal parts* AC *and* DF, *the remaining parts* AG *and* DI *will be in a greater proportion. Therefore, in terms of proportions, the convex lens* AB *will be farther removed from* GH *and the eye than the lens* DE *will be from* IK *and the eye. And* GH, *together with the eye, is closer to* C *in proportion to* ABC *than* IK *with the eye is to* F *in proportion to* DEF. *Therefore, visible objects are shown as larger by* AB *and* GH *than by* DE *and* IK, *as per XXCIII. And thus, with the slightest alteration of the proportion, much larger, as per CXIV.*

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225 Here Kepler finally switches from the cumbersome "[defined] by a smaller/larger circle" way of describing the curvature of the surfaces to "larger/smaller semi-diameter".

This important proposition was very intricate, for the reason that if the proportion of AC *to* CG *had been the same as that of* DF *to* FI*, then* AG *being longer than* DI *would have made no difference for the magnification of visible objects. For everything would have been equal in both cases, as per LXXXIII.*

CXVI. PROBLEM

To show visible objects in whatever enlargement is desired.

63 *By CXIII and CXV, it is evident that if the proportion of the circles of concavity and convexity is increased, visible objects will be enlarged.*

CXVII. PROBLEM

To show visible objects with the same magnification for different separations of the lenses, that is, for tubes of different lengths.

By CXIII and CXV, make it so that there is the same proportion, both of the concavities and convexities, and of the distances between the lenses, but with the convex lenses being dissimilar to each other.

CXIIX. PROBLEM

To show [visible objects] as larger with shorter tubes.

If the convex lens has a smaller semidiameter, but the proportion between the convexity and concavity is greater than in a longer instrument, objects will be shown as larger by the shorter instrument, as per CXIII and CXV.

CXIX. PROPOSITION

For a given concave lens, visible objects are shown as brighter or stronger with a larger or wider convex lens than with a smaller one.

For more of the light from a point is sprinkled (see the adjacent diagram) through the larger width AB *than through the smaller one* DE*. However,* *all of it is gathered into one point,* C *or* F. *Thus, the picture is stronger at* C *than at* F, *and the eye receives more densely crowded rays in* GH *than in* IK.

Here, as per XXX, the size of the convex lens is understood to refer to the body of the lens, not the shape.

CXX. PROPOSITION

For a given convex lens, visible objects are shown as brighter or stronger through a concave lens of a larger circle than through one of a smaller circle.

For a small lens placed near the eye causes the correct divergence of rays **64**
with only a small part of itself. Therefore, although many rays from one point radiate onto the concave lens, coming from a large part of the convex lens, many of these rays stray around the eye due to excessive refraction caused by the sides or the edge of the concave lens (as in the diagram of prop. C, where F and G *stray around* HI, *the width of the pupil). Indeed, only a few rays, very close to perpendicular and therefore approaching from a small part of the convex lens, will enter the eye. Thus, by CXIX, vision will be weak through a concave lens defined by a small circle. The same also occurs for a portion of a concavity, defined by a large circle, that is narrow and smaller than the pupil.*

CXXI. PROPOSITION

Of the part of the hemisphere seen through the lenses, the central part close to the perpendicular [ray] is seen as brighter and stronger than the edge around it.[226]

The reason for this can be seen in the diagram of prop. LXX, where QG *may be taken as the width of the pupil. The eye placed at* QG, *whether*

226 In modern terms, this is a consequence of the location of the exit pupil inside the tube in telescopes of the Galilean design, which severely limits the field of view. The Keplerian design, in which the exit pupil can be made to coincide with the pupil of the eye, does not suffer from this issue, and can thus provide a much larger field of view, free of vignetting. This is indeed one of the primary reasons it became preferred over the Galilean variety. See e.g. H. A. Hughes & P. F. Everitt (1920).

naked or with a concave lens in front, intercepts all the rays from the middle point E *between* EAQ *and* EBG. *However, from point* C, *it intercepts not all rays but only a part of the pencil* CAFBC, *that is, that part which is between* CAI *and* CHG. *What instead is between* CHG *and* CBF *will stray outside the pupil* QG. *Therefore, since* E *is seen through* AB *and* C *through* AH, E *is seen as stronger and brighter than* C, *as per CXIX.*

CXXII.

With a narrow part of a convex lens, other things being equal, visible objects are shown as more distinct, and with a wide part, they are shown as more confused.

For objects that radiate into the eye through a wide part of the convexity radiate more strongly, as per CXIX, and by their power, they first excite the colours of the rainbow and then haze. For the hollow and retiform tunic of the eye is full of spirit, and although it may be touched by only
65 *one single point, the spirits in some part of the retiform around this point nevertheless become imbued with contagion of the penetrating effect if that point is excessively bright owing to the concurrence of many rays; see LXI. Therefore, according to what is suitable for the eye, the instrument, and the light of the day or the night,*[227] *the convex lens of the instrument is either widened and uncovered or narrowed and covered. This can be done directly at the convex lens or at an intermediate location between the lenses, using a diaphragm with a hole pierced in it, or by bending and narrowing the neck of the instrument inwards, or by extension of the tube beyond the convex lens, so that the opening of its cylinder is farther away, and therefore, as per LXVII, is seen at a smaller angle, thus appearing somewhat narrower. Nature has provided for the widening of the uveal aperture for night-time illumination and its narrowing for daylight.*

A diaphragm also serves to create darkness inside. The same purpose is also served by applying black colour inside and by the shape of a trumpet[228] *with flexible sides bent outwards towards the ends and inwards in the middle so that rays entering near the convex lens are not scattered backwards and forwards and cause glare.*

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227 The order should probably be "night or day", but I have kept it as in the original text.

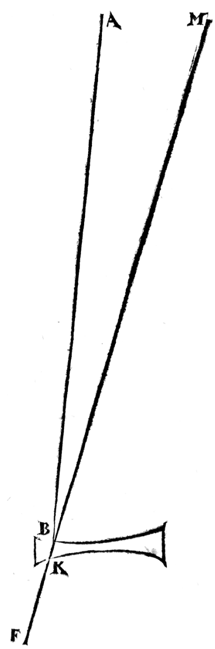
228 The Latin word *lituus* used here can refer to various wind instruments.

Extension of the tube well beyond the convex lens serves the same purpose, avoiding the convex lens being irradiated by the lateral parts of the hemisphere.

CXXIII. PROBLEM.

To see a visible object raised, lowered, to the right, or to the left, or wherever you wish.

This will be made possible if the diameter of the concave lens is wider than the pupil of the eye and large enough so that the eye can move outwards by an appropriate distance from the centre towards the sides. For all of the pencils are then also refracted **66** *obliquely in the sides of the concave lens: those on the left, towards the left, and those on the right, towards the right. For in the diagram of prop. C, let* ABKF *be the middle line of one pencil entering the centre of the pupil. It will be refracted outwards towards the left through points* B *and* K *because* BK *is also the left part of the lens. Thus, if the eye is shifted from the centre of the concave lens to the left side* K, *then the point* A *seen via the straight line* FKM *will be perceived to be at* M, *further to the right, as per XIX.*



CXXIV. PROBLEM.

To skilfully determine the magnitude of the enlarged *species*.

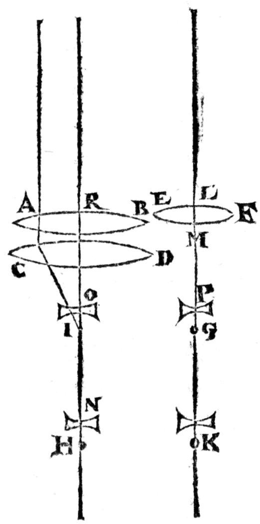
Let the left eye be aimed at a visible object freely without the instrument, while the right eye views the same object through the lenses. With the left eye aimed at the visible object, the right eye will always voluntarily remain parallel to the left eye if it is covered, as it is covered here by the instrument, for a parallel direction of the eyes is, in fact, natural, as per LVII. Thus, it is as if the right eye were aimed at the visible object itself, whether the species *appears lower or higher to it through the instrument than the object itself as seen through the left eye. For by LXII, the right eye will see* *an amplified* species *of the object towards which the eye is directed through its association with the left eye, but it will not always see it with the same part of itself as the part by which the left eye sees it.*

If the two species *are seen to differ in their locations, it is easily accomplished, by moving the convex lens of the instrument here and there and sometimes applying the concave lens slightly differently to the eye, that both* species *of the same visible object coincide with each other in the same location. Thus, the excess of one above the other will be evident when the two* species *are placed near each other.*

So much about the simple instrument, now about concealment.[229]

CXXV. PROPOSITION

67 For a given concave lens, two similar convex lenses placed close to each other, instead of one, will nearly halve the length of an instrument which has only one of those convex lenses and will, at the same time, reduce the size of the *species*.



Let AB *and* CD *be the two similar convex lenses, and let* H *be the centre of the circle* ARB. *Let the semidiameter* HR *be bisected at point* I. *Then, for one convex lens* AB, *the point of concurrence will be near* H, *as per XXXIX. And the concave lens should, therefore, be placed not far within* H, *as per CIX.*

I say that if CD *is placed near* AB, *the concave lens must be placed within* I. *I first prove this in a simple manner.*

Because parallel rays refracted in AB *concur towards* H *due to refraction but are then intercepted by* CD *and are therefore refracted again in* CD, *they will concur closer to* AB. *For they undergo a greater refraction in* CD *than in* AB *since they strike it more obliquely; they are indeed parallel as*

229 The word is the Greek "κρύψις" (krypsis).

they fall upon AB, but they are already converging when falling upon CD. Hence, it is clear that the concurrence of the rays will occur much closer [to AB], and the concave lens must therefore approach the convex lenses AB and CD, as per CIX. It is then also evident that the concave lens must be brought within point I, which halves the semidiameter HR of the convexity AB. For let GL be equal to the half, HI [of HR], and by a circle of this semidiameter, let a lens with convexities ELF and EMF be constructed and let LG be equal to GK. Therefore, by LXXIX, if there were only one surface EMF, it would have the same effect as two surfaces AB, making parallel rays concur at K, which has the same distance from EF as the point of concurrence H has from AB. But the lens EF has two such surfaces. And just as the surface EMF is equivalent to both convexities of AB, so the other surface ELF is equivalent to both convexities of CD because AB and **68** CD are similar, just like ELF and EMF. But the lens EF, which is equally convex on both sides, makes parallel rays concur at the centre G, as per XXXIX. That is, at the distance LG, which is half of the semidiameter of AB. Therefore, the joined and adjacent lenses AB and CD will also gather parallel rays at a distance that is half of this semidiameter, that is, near point I. The concave lens must, therefore, be placed within the point of concurrence, that is, within I, as per CIX. I also say that a smaller species is produced by the two convex lenses AB and CD joined together than by the single lens AB.

For since the concave lens is the same in both cases, it will cause the same divergence of the rays. Thus, it will be at the same distance from the concurrence at H caused by one lens AB as from the concurrence at I caused by both lenses AB and CD together, as per CXI. Let this distance be HN, IO, and GP. But the same part is in greater proportion to IR, the half, than to HR, the double. Therefore, AB and CD joined together are closer to O (or EF, which is equivalent to them, to P in proportion to its semidiameter LG) than AB alone is to N in proportion to its semidiameter RH. Therefore, EF shows visible objects as smaller through the concave lens P than AB alone does through the same concave lens N, as per XXCIII. Therefore, the two lenses AB and CD joined together also show them as smaller than AB alone.

CXXVI. PROPOSITION

For dispersing or dividing rays, a single concave surface defined by a small circle is nearly equivalent to two concave surfaces taken from a circle twice as large.

This is proven from LXXIX and III.

CXXVII. PROPOSITION

In order to achieve distinct vision, two concave lenses joined together must be situated at a slightly greater distance from the convex lens than a single one of them, but they greatly and nearly doubly enlarge the *species* of a visible object.

For the parallel rays, which were made to converge by the convex lens,
69 *thus fall converging upon the concave lens, and after passing through it and evading the concurrence, they diverge again towards the eye, as per CVII. For it is supposed that the instrument and the placement of the concave lens within it serve this purpose. However, if another concave lens is placed between the eye and the previous concave lens and intercepts the diverging rays, it will cause the rays, upon passing through it, to diverge further, as per XCIV. Therefore, they err due to the excess of divergence and cause confusion as per XCV and XCIX. Hence, it will be necessary to increase the contrary convergence from the convex lens so that the defects balance each other and cancel out mutually, as per CIV. However, the convergence and the confusion arising from it are increased if the convex lens is moved farther away from the eye placed within the point of concurrence, as per LXXI. Therefore, the two concave lenses with the eye adhering closely to them should be farther away from the convex lens than a single one of them. Or, by CXXVI, two concave lenses defined by a larger circle are equivalent to a single one defined by a smaller circle. And by CX, a concave lens defined by a smaller circle is farther from the [convex] lens than a single concave lens defined by a larger circle. Therefore, two concave lenses defined by a larger circle are also more distant than one of them alone.*

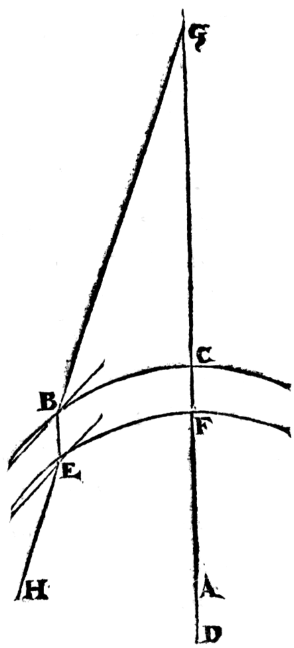
I also say that visible objects are shown as larger by two concave lenses than by one of them placed near the eye. This is demonstrated (as previously) from CXIII and CXXVI.

Moreover, a very small increase in distance causes a great increase in the size of the species, *as per CXIV.*

CXXIIX. PROPOSITION

For a lens that has a convex surface on one side and a concave surface on the other side, defined by equal circles, all rays parallel to the perpendicular which advance within the body are refracted by equal angles at both surfaces and the refracted rays retain the same divergence or parallelism.

Let the convex surface of the lens be defined by the circle BC *with centre* A, *and the concave surface by the circle* EF *with centre* D. *Let a straight line* DA *pass through the centres, intersecting the surfaces perpendicularly at* F *and* C. *Let some line be drawn parallel to it, and let* BE *be such a line. It is then demonstrated geometrically, especially by Ptolemy*[230] *and by Astronomers, that just as* CF *and* BE *are equal,* CB *and* FE *are also equal. Hence, the inclination of* BE *above both surfaces is the same, that is, with respect to the tangents of the surfaces at the points of incidence* B *and* E. *For these tangents are parallel. Therefore, the refraction is also the same, and the rays refracted from the dense body into the space on both sides will be parallel, like* BG *and* EH. *Thus, the divergences or convergences of the emerging rays* EH *are the same as those of the incoming rays* GB, *as long as* BE *and* CF *are parallel within the body.* *70*



230 E.g. the *Almagest*, Book 3, sect. 3, H226 (fig. 3.5, p. 149 in Toomer's translation).

CXXIX. PROPOSITION

If rays from one point are incident upon a lens with convex and concave surfaces defined by the same circle, they will converge after passing through the lens if the point is distant. If the point is closer to the lens than the diameter of the circle, the rays will diverge more than at the origin.

For the rays from a distant point are parallel, as per XXIII. And parallel rays incident upon the convex surface of a denser medium will converge within the body, as per XXXIV.

Let G *be a distant point,*[231] GB *and* GC *parallel rays, and* BE *and* CF *the converging rays. Therefore,* EF *will be shorter than* BC. *Therefore, the incidence of* BE *upon* EF *is less inclined than the incidence upon* BC. *Therefore, the refraction at point* E *is less than at* B. *Hence, the angle* GBE *is smaller than* BEH. *Thus,* GB *and* EH *are not parallel. But* GB *and* GC *are assumed to be parallel. Accordingly, the refracted rays* EH *and* FA *converge and will finally concur.*

Let now, instead, G *be a radiant point closer [to the lens] than the diameter of the circle. The rays* GB *and* GC *will, therefore, be diverging. Thus, as they travel through the denser convex body, they diverge less but nevertheless do diverge, as per XXXVII.*

71 *Since* BE *and* CF, *therefore, diverge towards* EF, *the concave boundary of the dense body,* EF *will be greater than* BC. *Therefore, the incidence of BE at point E is more oblique than at B, and therefore, the former refraction is greater than the latter. Therefore, the angle* GBE *is greater, and the angle* BEH *smaller, and, therefore,* GB *and* EH *are not parallel but would concur mutually if extended towards* H. *Therefore, the refracted rays* EH *and* FA *diverge more from each other than the original rays* GB *and* GC.

CXXX. PROPOSITION

If the concave surface is defined by a greater circle than the convex surface, rays from a distant point will converge after traversing the lens. They will, moreover, converge more (that is, after a shorter distance

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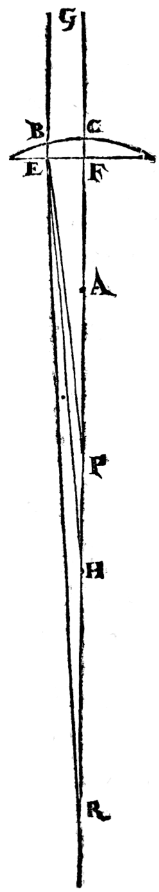
231 Referring still to the previous figure (prop. CXXIIX).

than for a single convex surface) if the circle of the concavity is more than three times as large as the circle of the convexity, but they will converge less (and after a greater distance) if it is less than three times.



Or

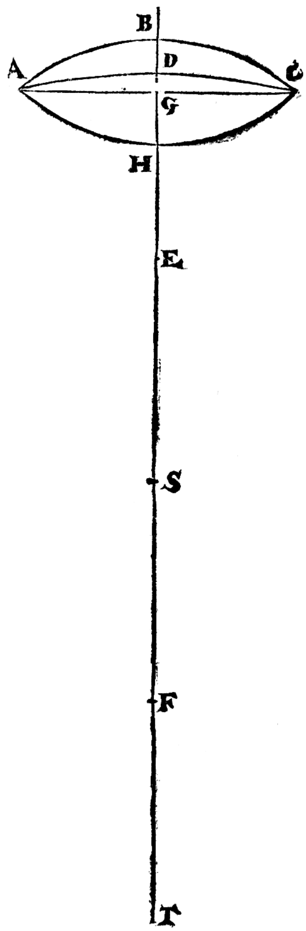
When the concavity of a larger circle diminishes the convexity of a smaller one, this produces the effect of the convexity of a very large circle. Let this be called a Meniscus.[232] It is equivalent to a purely convex lens.

Let CF *and* BE *be refracted rays within the body, coming from the distant point* G. *They will therefore converge towards* EF, *as per XXXIV, and* EF *will therefore be shorter than* BC. *But at the same time, its circle is also larger. Therefore* BE *will fall less obliquely upon* E *than upon* B. *Therefore, the refraction at* E *will be less than at* B. *Therefore, the angle* BEH *will be greater than* EBG. *Therefore,* HE *and* BG *are not mutually parallel but will concur if extended, and thus,* EH *and* FH *will converge mutually towards* H.

Let now A *be the centre of the circle* BC *and let* CH *be three times* CA. *72*
And let the point R *lie beyond* H. *But if* BC were alone, *then* BE *and* CF *would converge towards* H, *as per XXXIV. Let now* R *be the centre of the circle* EF. *And with the normal* ER *drawn,* BE *will be refracted away from* ER, *as per II, and will concur with* CH *before* H, *as in the point* P. *Thus,* EP *and* FP *converge more than* BE *and* CF, *and the distance* CP *of the point of concurrence* P *is less than* CH.

Let now, instead, the centre of the circle EF *be above* H, *for example, at the point* P, *and let the normal* EP *be drawn. The ray* BE *will then be*

232 From Greek, μηνίσκος, diminutive of "moon", "crescent-shaped". Kepler appears to have been the first to use this term in the context of optics.



refracted away from the normal at E, *per II, and the refracted ray will concur with FH at a greater distance than* EH, *below* H. *Let the concurrence be at* R. *Therefore, the divergence*[233] *of* ER *and* FR *will be less than that of* BE *and* CF. *And the point of concurrence* R *will be shifted to the distance* CR, *which is greater than* CH. *But if the centre of* EF *is at* H, *one and a half times the diameter beyond* C, *then the point of concurrence will also be at* H, *and thus* EF *will neither help nor hinder* BC.

CXXXI. PROPOSITION. PROBLEM

To find the point of concurrence for the meniscus. Or, the concurrence is extended by as much as the lens is thinned.

Let ABCD *be the meniscus and* E *and* F *the centres. But if only the convex surface* ABC *were to cause refraction, the concurrence would be after three times the semidiameter* BE, *as per XXXIV. This will be the case if the circle of the concavity* ADC *is three times as large as the circle of convexity* ABC, *that is, if* BF *is three times* BE. *For since the concurrence*
73 *is after three semidiameters* BE, *the concurrence will be at the centre* F *of the circle* ADC *because the rays will fall perpendicularly upon ADC after passing through the body ABC so that they are not refracted. The point of concurrence of the lens* ABCD *is, therefore, after three semidiameters.*

On the other hand, if the lens is equally convex on both sides, like ABC *and* AHC, *the point of concurrence is at one semidiameter* BE *from* B, *at* E, *as per XXXIX.*

233 Divergence instead of convergence may be a typo here.

Third, if the lens is AGCH, *with the side* AGC *being plane, parallel rays are not refracted at* AGC *and concur after two semidiameters, per XXXV, as at point S.*

Fourth, by CXXV, if two lenses are joined, the point of concurrence is at half of the distance EB.

From these steps, it appears that the distance of the point of concurrence from the lens increases in nearly the same proportion by which the thickness BD *of the lens is diminished. For when the thickness was twice* BH, *the distance was half of* BE. *When the former was equal to* BH, *the latter was equal to* BE, *and when the former was half, that is,* GH, *the latter was twice* BE, *that is* BS. *Now, if* GH *or* BG *is reduced by slightly less than a third part, a third semidiameter SF is added to the other two,* BE *and* ES.

That DG *is less than a third part of* GB, *or* GH, *is proven as follows:*

Let AB *be either* 30° *or* 0°30′, *from an abundance of possible cases, as per VII. Then*

AG *will be either*	*5000000*	*or*	*87265*[234]
and GB *will be either*	*1339746*	*or*	*381*[235]
of which BE *is*	*10000000,*	*while*	
DF *is approximately*	*30000000*		

But as DF *is to* BE, *that is, as 3 to 1, so is* AG *to the sine of the arc* AD. *Therefore,*

The sine is either	*1666667*	*or*	*29088*[236]
of which the arc is either	9°36′	*or*	0°10′[237]
and the complement either	80°24′	*or*	89°50′
The versed sine is either	*140039*	*or*	*41*[238]

But as the total sine[239] *is to the versed sines, so is DF 30000000 to DG.*

234 In modern notation: sin(AB) × BE (for Kepler the multiplication by BE=10000000 is implicit).

235 [1-cos(AB)]×BE, where 1-cos(AB) is the versed sine of AB.

236 That is, AG/3 = sin(AD)×BE.

237 $\sin^{-1}$(1666667/BE) etc.

238 [1-cos($\sin^{-1}$(1666667/BE))]×BE. More exact values are 139867 and 42.3.

239 That is, 10000000.

DG is thus either	*420117*	*or*	*123*[240]
But BG was either	*1339746*	*or*	*381*

Therefore, you see that DG *is less than a third part of* BG.

74 *Fifth, it is reasonable that if a fourth part of* DB *is removed, a fourth part of* BE *is added, so that again, with* BG *already reduced by slightly less than a third part and then by a quarter of what remains, that is by half of the total, the distance of the point of concurrence will be four of the semidiameters* BE *instead of two, like* BT.

For if you remove	*420117*	*or*	*123*
from	*1339746*	*or*	*381*
what remains is	*919629*	*or*	*258*
of which a quarter is	*229907*	*or*	*64*
and when removed, what remains is	*689722*	*or*	*194*

which is approximately half of BG.

Therefore, the distance of the point of concurrence is increased by approximately as much as the lens is thinned.

CXXXII. PROPOSITION

If the concavity is defined by a smaller circle than the convexity, then the rays from a point located at a distance of one diameter behind the convex surface will diverge more after passing through the lens. Or, when the convexity of a larger circle diminishes the concavity of a smaller one, this will produce the effect of a concavity of a very large circle.

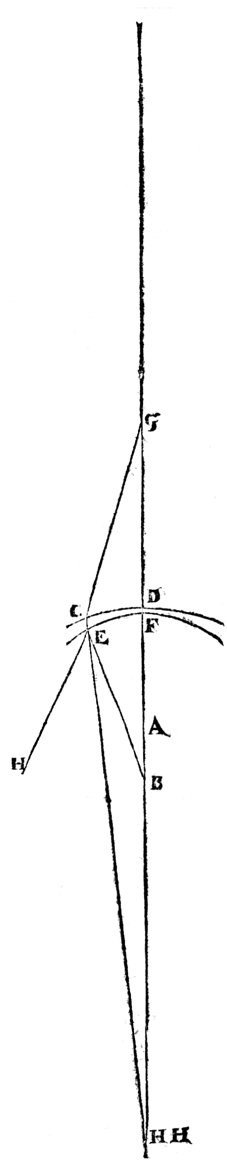
For within the body, the rays CE *and* DF, *coming from point* G, *will be parallel if the distance of the point from the convex lens is equal to the diameter of its surface, as per XXXV. Therefore, they intersect the concave surface* EF *more obliquely than the convex surface* CD. *The rest is as in CXXIX. But if* G *is instead closer, then* CE *and* DF *will diverge inside the body towards* EF, *as per XXXVII, while the refracted rays* EH *and* FB *will diverge more in the air, as per XCII.*

240 30000000×[1-cos(9°36′)] etc. More exact values are 419601 and 126.9.

CXXXIII. PROPOSITION

75

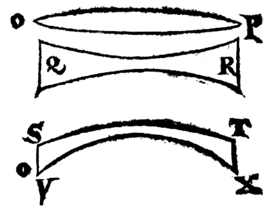
If the concave surface of a lens with one convex surface has its centre closer to the lens than the centre of the convex surface, then the lens will also cause divergence of rays from a distant point. Such a lens is equivalent to a purely concave lens of a very large circle.



For let G *be a distant point; its rays* GC *and* GD *are then parallel, as per XXIII. Therefore,* CE *and* DF *converge within the body, as per XXXIV, as if they were going to concur at a distance of one and a half times the diameter of the convexity at* HH. *But if a smaller circle were drawn through E around the centre* B *[of the convex surface], then* EB *and* FB *would cut off a larger part of it than* CD *with respect to its circle. This is evident, for since* CE *aims towards* HH, *point* E *lies below the line* CB. *In fact, only* CB *(and not* CHH*) would then cut off similar parts. Therefore,* EF *will, even more so, be a greater portion of its circle when its centre is above* B, *such as at* A. *Because* EF *is thus a greater portion than* CD, *the inclination of* CE *with respect to* EF *is greater than with respect to* CD. *The outwards refraction at* E *is, therefore, greater than the inwards refraction at* C *towards* BDG, *as per II. Therefore,* GC *and* EH *are not parallel. And since* GC *and* GD *are assumed to be parallel, the rays* DB *and* EH *belonging to them will diverge after refraction at the points* E *and* F *at the concave boundary of the dense body.*

CXXXIV. PROPOSITION

Pure lenses of different types, joined together contiguously with each other, are equivalent to a lens of mixed type and, ultimately, to a pure lens.



This is demonstrated nearly as in CXXV. For let OP *be a convex lens and* QR *a concave lens, and let the two convex surfaces of* OP *be converted to one convex surface* ST, *as per LXXIX.*

By CXXVI, however, the two concavities of QR *may also be converted to one,* VX, *and a lens* STXV *of mixed type may be formed. If the concave part* VX *dominates, that is, if its circle is smaller, then the mixed lens is equivalent to a pure concave lens, as per CXXXIII. Hence, the joined lenses* OP *and* QR *of*
76 *different types are equivalent to a purely concave lens of a very large circle. But if instead, the convex surface* ST *dominates because of a smaller circle, as in the meniscus in the diagram of prop. CXXXI, where the convexity* ABC *is greater and the concavity* ADC *smaller, then the mixed lens* SX, *and therefore also the two lenses* OP *and* QR *joined together, are equivalent to one pure convex lens, as per CXXX.*

CXXXV. PROBLEM

To furnish an instrument with a convex lens defined by a large circle, which is shorter than those who construct common ones would expect.

Let the single convex lens be duplicated, and let one of the two be hidden inside, unknown to the observer. See CXXV.

CXXXVI. PROBLEM

To furnish an instrument with a concave lens defined by a large circle (which may even exceed the circle of the convex lens), which shows visible objects as larger than those who construct common instruments would expect.

Instead of one, let there be a second concave lens, unknown to the observer. See CXXVII.

CXXXVII. PROBLEM

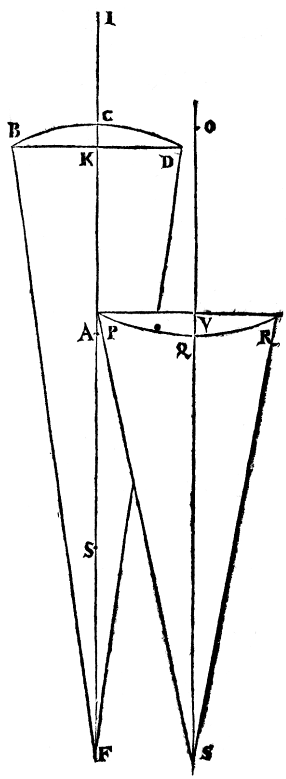
From a convex lens defined by a small circle, and smaller even than the circle of the concave lens near the eye (which seems absurd as per CVII), make a very long instrument, and show visible objects as huge.

Either combine, with a certain adjustment, the convex lens defined by a small circle with a concave lens defined by a larger circle, hidden inside and unseen, and the result will follow, as per CXXXIV. Or use a mixed lens with an outer convex surface defined by a smaller circle and an inner concave surface defined by a larger circle, as per CXXX. And find the place of the other concave lens, to be placed near the eye, as per CXXXI. Also try something from CXXIIX.

CXXXIIX. PROPOSITION 77

With the same distance of the lens from the eye maintained and a line passing from the eye through the middle of the lens and through the centres of convexity or concavity, nearly the same refractions will occur no matter which of the two dissimilar surfaces of the lens you turn towards the eye.

This seems absurd and contrary to prop. XXXIV and prop. XXXV. For in the diagram of prop. XXXIV, the convex surface BCD *of the dense body, turned towards the parallel rays, collects them at* F *at a distance of one and a half diameters. But in XXXV, the surface* PQR *of the dense body turned away from the parallel rays collects them at* S *at a distance of one diameter. But you must remember that the discussion in those propositions is about a single surface, while any lens necessarily has two. Besides, in prop. XXXIV, the parallel rays are considered in air, while in prop. XXXV they are considered within the dense body, and thus [the two cases] cannot be treated equally. But if both lenses are also bounded by a second surface in such a way that the points of concurrence* F *and* S *remain, the difference will become apparent. For let the other surface*[241] *be defined by the portion* BKD *of a circle with centre* F *and radius* FB, *intersecting the line* IAF *at* K *and* BCD *at the points* B *and* D *so that*



241 Referring first to the case considered in XXXIV.

78 *the rays concurring at* F *are all perpendicular to* BKD *and hence are not refracted in* BKD. *In the second diagram, from prop. XXXV, let the arc* PQR *be equal and similar to the arc* BCD, *with* Q *as its middle point, and let the end-points* P *and* R *be connected by a straight line intersecting the perpendicular* OQS *at the point* V, *which defines the second, plane surface of the lens, to which all rays parallel to* OQ *are perpendicular. Therefore, no rays are refracted in* PVR, *and the point of concurrence remains at* S. *Hence, it is already clear that the lenses, of which one causes parallel rays to concur at one and a half diameters* CF, *and the other at one diameter* QS, *are of different thickness, although their convexities are similar and equal. The former has a smaller thickness* CK, *and the latter has a greater [thickness]* QV. *The difference between them is the versed sine of the curve* BK. *Therefore, it is no wonder that in the former case, the distance of concurrence contains three semidiameters, and in the latter, only two, as per CXXXI. However, the truth of the proposition also becomes clear as follows. In the diagram of prop. XXXIV, let the circles* BCD *and* BKD *be turned away from the parallel rays while points* B *and* D *remain in the same place, such that the parallel rays will then first fall upon the concave surface* BKD *of the dense body. They will then diverge as they pass through the body towards the convex surface* BCD, *as per XC. If they had remained parallel within the body, as in the diagram of prop. XXXV, they would have concurred at two semidiameters after the convexity, as per XXXV. But because they diverge towards* BCD *(as if, in the other diagram, they converged towards* PQR*), it is reasonable that they concur at a longer distance after* S, *namely at* F, *as per XI. The same is also easy to demonstrate from the diagram of prop. XXXV. For if* PQR *is turned towards the parallel rays, they will converge within the body, as if they wished to concur after three semidiameters, as when refracted in* BCD *towards* F. *But since they converge as they propagate through the body and are inclined with respect to its plane boundary as they fall upon it, they are also refracted at the plane surface, each of them away from the normal at its own point. And since, looking back at the whole lens, the rays repel each other mutually and their normals within the body, the refracted rays, therefore, unite all the more outside in the air as they retreat individually from their normal.*[242] *And it*
79 *is thus no wonder that they unite sooner than after three semidiameters,*

242 This whole passage is somewhat oddly phrased. Kepler seems to have deliberately avoided his regular optical terminology here.

that is to say, at S. And this demonstration affirms the proposition in a general sense. Nevertheless, there is a small difference, and for this reason, an accurate demonstration is not given. However, anyone who wishes can explore the subtlety by means of numbers, as I myself have also done above in p. XXXIV.

CXXXIX. PROBLEM

That the effect may nevertheless follow when each glass is concave, both that facing the eye and that facing the visible object.

Instead of a single convex lens, either place a concave lens facing outwards towards the visible object with an adjacent convex lens defined by a correspondingly smaller circle hidden inside, as in prop. CXXXVII. Or use a mixed lens there, as in prop. CXXXVII, with the concave surface facing outwards. For by CXXXIIX, it works in the same manner whichever way you turn it.

CXL.

Prepare a tube with two convex glasses, both that facing the eye and that facing the visible object, such that the effect will follow nonetheless.[243]

Instead of one concave glass near the eye, combine a convex glass with a concave defined by a smaller circle, and let the convex glass defined by a larger circle face outwards towards the eye, with the concave hidden inside, as per CXXXIV. Or use a mixed lens near the eye, with the convexity defined by a large circle facing outwards, and the concavity defined by a correspondingly smaller circle inwards, as per CXXXIII.

CXLI. PROBLEM **80**

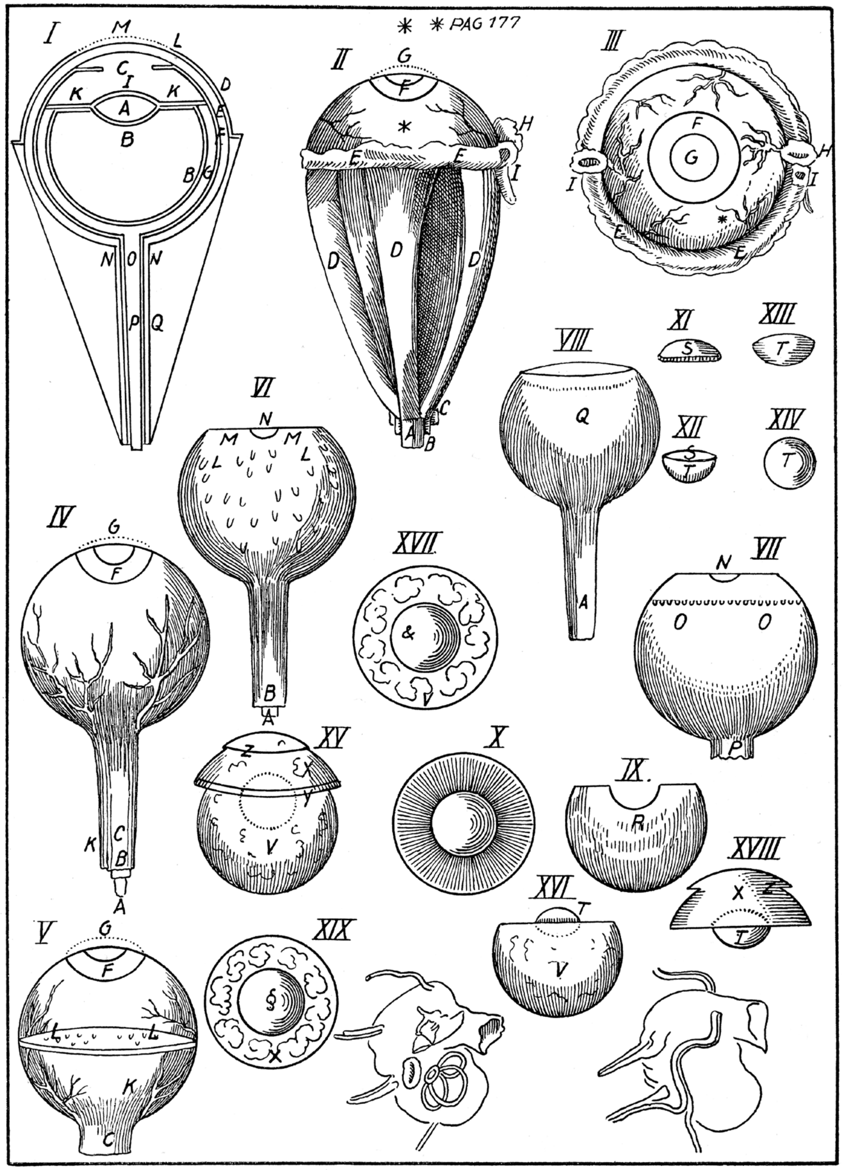
Prepare a tube with a convex glass facing the eye and a concave glass facing the visible object.

243 It is interesting to note that Kepler does not here refer back to Problems XXCVI-XXCIX.

This is arranged according to CXXXIX and CXL. For what came about there for each glass separately, must here come about for both together.

THE END

Felix Platter's illustrations of the anatomy of the eye



The figure is discussed in proposition 64 and included in Kepler's *Optics*. It is not included in the *Dioptrice* itself but has been added here for convenience of reference. Reproduced from KGW vol. II, p. 159.

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Kepler's *Dioptrice* is a seminal work in the fields of optics and astronomy. Written in response to the groundbreaking telescopic discoveries announced by Galileo Galilei in March 1610, the *Dioptrice* contained the first theory of the telescope–a theory that Kepler was uniquely qualified to develop–and provided an essential foundation for modern geometrical optics.

In the preface, Kepler highlights several examples of the close links which, then as now, exist between optics and astronomy. His excitement about Galileo's new observations is palpable, especially the discoveries of the four moons orbiting Jupiter and the phases of Venus. Both discoveries lent crucial support to the Copernican system, of which Kepler had been an ardent supporter since his days as a student in Tübingen.

The *Dioptrice* is available here in a complete English translation for the first time, along with an introduction and numerous notes to help guide the reader through Kepler's text.

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